Solutions

Chapter 1

Concept Application Exercises

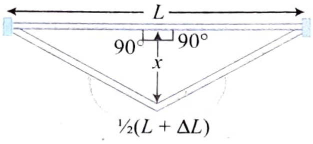
ercise 1.1

1. As the coefficient of thermal expansion of the brass is greater than that of steel, the length of brass strip will be more than that of steel strip. Therefore, brass strip will be on convex side.

2. Consider the diagram.

Applying Pythagoras theorem in right angled triangle in figure,

$$\left(\frac{L+\Delta L}{2}\right)^2=\left(\frac{L}{2}\right)^2+x^2$$



$$\Rightarrow \quad x=\sqrt{\left(\frac{L+\Delta L}{2}\right)^2-\left(\frac{L}{2}\right)^2}$$

$$x=\frac{1}{2}\sqrt{(L+\Delta L)^2-L^2}$$

$$\Rightarrow \quad x=\frac{1}{2}\sqrt{(L^2+\Delta L^2+2L\Delta L)-L^2}=\frac{1}{2}\sqrt{(\Delta L^2+2L\Delta L)}$$

As increase in length L is very small, therefore, neglecting $(L)^2$, we get

$$x=\frac{1}{2}\times\sqrt{2L\,\Delta L} \qquad \text{...(i)}$$

But $\Delta L = L\alpha\,\Delta t$ ...(ii)

Substituting value of ΔL in Eq. (i) from Eq. (ii), we get

$$x=\frac{1}{2}\sqrt{2L\times L\,\alpha\,\Delta t}$$

$$=\frac{1}{2}L\sqrt{2\alpha\,\Delta t}$$

$$=\frac{10}{2}\times\sqrt{2\times1.2\times10^{-5}\times20}$$

$$=5\times\sqrt{4\times1.2\times10^{-4}}=20\sqrt{30}\times10^{-3}\text{ cm}$$

Given, decrease in temperature, $\Delta t = 57 - 37 = 20°\text{C}$

Coefficient of cubical expansion of copper, $\gamma_{\text{Cu}} = 3\alpha_{\text{Cu}} = 5.1 \times 10^{-5}/°\text{C}$

Let initial volume of the cavity be V. Also, let its volume increase by ΔV due to increase in temperature.

$$\therefore \quad \Delta V = \gamma V \Delta t$$

$$\Rightarrow \quad \frac{\Delta V}{V}=\gamma\,\Delta t$$

Thermal stress produced = B × Volumetric strain

$$=B\times\frac{\Delta V}{V}=B\times\gamma\,\Delta t$$

$$=140\times10^9\times(1.5\times10^{-5}\times20)=42\times10^6\text{ N/m}^2$$

This is about 10^3 times of atmospheric pressure.

Since copper expands more than iron, the length of the iron rod must be greater than that of the copper rod. Let their lengths be l_1 nd l_2. Then $l_1 - l_2 = 10$ cm.

ince the difference is always same at all temperatures, their crease in length must be the same whatever the rise in temperature

$$\Delta l = l_1\,\alpha_{\text{Fe}}\,\Delta T = l_2\alpha_{\text{Cu}}\,\Delta T$$

or $\quad \dfrac{l_1}{l_2}=\dfrac{\alpha_{\text{Cu}}}{\alpha_{\text{Fe}}}=\dfrac{17}{11}$

Also $l_1 - l_2 = 10$ cm

Solving for l_1 and l_2

$l_1 = 28.3$ cm $\quad l_2 = 18.3$ cm

5. $\Delta l = l\,\alpha\,\Delta T$ or $0.075 = 30 \times \alpha_A \times 100$

or $\quad \alpha_A = 25 \times 10^{-6}\,°\text{C}^{-1}$

and $\quad 0.045 = 45 \times \alpha_A \times 100$

or $\quad \alpha_B = 10 \times 10^{-6}\,°\text{C}^{-1}$

Let l_1 and l_2 be the lengths of the two sections of C.

Then $l_1 + l_2 = 45$ and $0.04 = l_1 \times 25 \times 10^{-6} \times 50 + l_2 \times 10 \times 10^{-6} \times 50$

or $\quad 160 = 5l_1 + 2l_2$

solving $l_1 = 23.3$ cm

$l_2 = 21.7$ cm

6. $T = 2\pi\sqrt{\dfrac{l}{g}}$

$$2\times\frac{86400}{86400+5}=2\pi\sqrt{\frac{l_{15}}{g}}\ \text{ and }\ 2\times\frac{86400}{86400-10}=2\sqrt{\frac{l_{30}}{g}}$$

Only dividing, we get $\sqrt{\dfrac{l_{30}}{l_{15}}}=\dfrac{86405}{86390}$

$$\text{or} \quad \sqrt{\frac{l_{15}(1+\alpha\times15)}{l_{15}}}=1+\frac{15}{86390}$$

$$\text{or} \quad (1+15\alpha)^{\frac{1}{2}}=1+\frac{15}{86390}$$

$$\text{or} \quad 1+\frac{1}{2}\times15\alpha=1+\frac{15}{86390} \quad (\alpha\text{ is small})$$

or $\quad \alpha = 23 \times 10^{-6}\text{ K}^{-1}$

7. $\Delta V = V_{Hg} - V_V$

$1.6 = 10^3\,(\gamma_L \times 10 - 10^3 \times 3 \times 6.5 \times 10^{-6} \times 10)$

$\gamma_L = (1.6 + 0.195) \times 10^{-4} = 1.795 \times 10^{-4}/°\text{C}$

8. $\gamma_M < \gamma_l$

$$\text{So,}\ \frac{F_B'}{F_B}=\frac{[1+\gamma_S\Delta\theta]}{[1+\gamma_l\Delta\theta]}\Rightarrow F_B'<F_B$$

So apparent weight increased and $W_2 > W_1$.

9. Since the scale is graduated at 10°C, 1 cm of the scale at 10°C = exactly 1 cm

$\therefore$ 1 cm of the scale at 30°C = exactly $(1 + 18 \times 10^{-6} \times 20)$ cm

$\therefore$ 60 cm of the scale at 30°C = exactly $60.00\,(1 + 36 \times 10^{-5})$

$= 60.02$ cm

10. Apparent expansion of mercury $= \dfrac{\pi}{4}\,(6 \times 10^{-4})^2 \times 10 \times 10^{-2}$

$= 9\pi \times 10^{-9}\text{ m}^3$

Volume of mercury $= \dfrac{43\times10^{-3}}{13.6\times1000} = 3.16 \times 10^{-6}\text{ m}^3$

$\therefore \quad 9\pi \times 10^{-9} = 3.16 \times 10^{-6} \times \gamma_a \times 50 \qquad (\Delta V = V\gamma \times \Delta T)$

or $\quad \gamma_a = 18 \times 10^{-5}$

$$\gamma_r = \gamma_a + \gamma_g = \gamma_a + 3\alpha_g \ (\gamma_g = 3\alpha_g)$$

$$\therefore \quad \gamma_r = 18 \times 10^{-5} + 3 \times 9 \times 10^{-6} = 20.7 \times 10^{-5}\ \mathrm{K}^{-1}$$

11. By the law of flotation $266.5 = \dfrac{4\pi}{3}\left(\dfrac{7}{2}\right)^3 \times \rho_{35}$

where ρ_{35} = density of the liquid at 35°C

$$\rho_{35} = 1.4839\ \mathrm{g/cm^3}$$

But $\quad \rho_0 = \rho_{35}(1 + \gamma \times 35)$

$$\therefore \quad 1.527 = 1.4839(1 + 35\gamma) = 8.3 \times 10^{-4}\ {}^\circ\mathrm{C}^{-1}$$

12. Apparent increase in volume $= V \times (18 \times 10^{-5} - 27 \times 10^{-6}) \times 100$

Also apparent increase $= 15 \times 10^{-4} \times 30$

$$\therefore \quad 15 \times 10^{-4} \times 30 = V \times (18 \times 10^{-5} - 27 \times 10^{-6}) \times 100$$

or $\quad V = 2.94$ cc

13. Since the scale is correct at 0°C, 1 cm at 0°C exactly 1 cm.

$\therefore \quad$ 1 cm at t°C = exactly $(1 + \alpha t)$ cm

$\therefore \quad H_1$ cm at t°C $= H_1(1 + \alpha t)$ cm

Since pressure is the same, namely, the atmospheric pressure,

$$\rho_0 g H_0 = \rho g H_1 (1 + \alpha t)$$

or $\quad \rho_1 (1 + \gamma t) H_0 = H_1 \rho_1 (1 + \alpha)$

or $\quad H_0 \approx H_1 (1 + \alpha)(1 - \gamma t)$

14. Volume of the space above the mercury will remain constant if increase in volume of the vessel is equal to the increase in the volume of the mercury for any increase in temperature. Let V' be the volume of mercury and V the volume of the vessel.

Then $\quad V \times 27 \times 10^{-6} \times \Delta T = V' \times 1.8 \times 10^{-4}\,\Delta T$

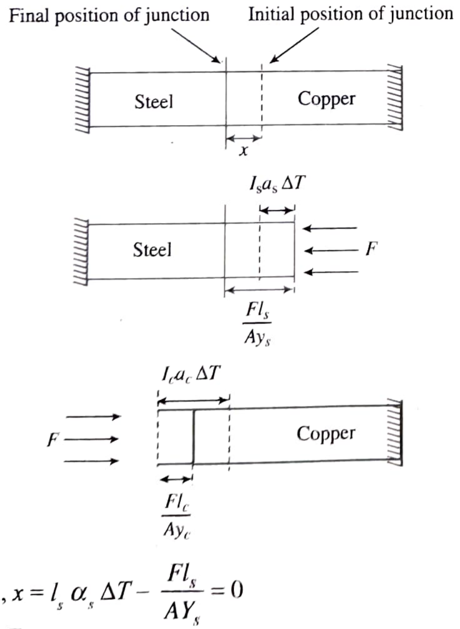
or $$\frac{V'}{V} = \frac{27 \times 10^{-6}}{1.8 \times 10^{-4}} = \frac{3}{20}$$

15. $\Delta l = l_s \alpha_s \Delta T + l_c \alpha_c \Delta T$

$$0.002 = [1.5\,\alpha_s + 0.5 \times 1.6 \times 10^{-5}] \times 100$$

$$\alpha_s = \frac{1.2 \times 10^{-5}}{1.5} = 8 \times 10^{-6}\ {}^\circ\mathrm{C}$$

there is no change in component length



For steel, $x = l_s \alpha_s \Delta T - \dfrac{Fl_s}{AY_s} = 0$

$$\frac{F}{AY_s} = \alpha_s \Delta T \qquad \text{...(i)}$$

For copper, $x = \dfrac{Fl_c}{Ay_c} - l_c \alpha_c \Delta T = 0$

$$\frac{F}{Ay_c} = \alpha\,\Delta T \qquad \text{...(ii)}$$

On dividing Eq. (i) by Eq. (ii), we get $\dfrac{Y_s}{Y_c} = \dfrac{\alpha_c}{\alpha_s}$

$$Y_s = Y_c \times \frac{\alpha_c}{\alpha_s} = \frac{1.5 \times 10^{13} \times 1.6 \times 10^{-5}}{8 \times 10^{-6}}$$

$$Y_s = 3 \times 10^{13}\ \mathrm{N/m^2}$$

Exercise 1.2

1. Mass of the child, $M = 30$ kg $= 30 \times 10^3$ g

It is given that specific heat of human body is equal to specific heat of water, hence specific heat of human body, $c = 1$ cal/g-°C.

Fall in temperature of the body, $\Delta T = 101 - 98 = 3^\circ\mathrm{F} = 3 \times \dfrac{5}{9} = \dfrac{5}{3}{}^\circ\mathrm{C}$

Time taken for lowering temperature of the child's, $t = 20$ min

Latent heat of evaporation of water, $L = 580$ cal/g

Let m be the mass of the sweat that evaporates from the child's body in 20 min.

Heat gained by the sweat $= mL$ ...(i)

Heat lost by the child $= Mc\,\Delta T$ ...(ii)

From equations (i) and (ii), using principle of calorimetry,

$$mL = Mc\Delta T$$

or $$m = \frac{Mc\Delta T}{L} = \frac{(30 \times 10^3) \times 1 \times (5/3)}{580} = 86.2\ \mathrm{g}$$

Rate of evaporation $= \dfrac{m}{\Delta t} = \dfrac{86.2}{20} = 4.3$ g/min

2. For changing ice at –20°C to steam at 100°C the complete process is achieved in following four different stages:

(i) The rise in temperature of ice from –20°C to 0°C (i.e., melting point of ice).

Heat required for this process,

$$\Delta Q_1 = (50\ \mathrm{g})(0.5\ \mathrm{cal/g\text{-}^\circ C})(0 + 20)^\circ\mathrm{C} = 500\ \mathrm{cal}$$

(ii) The melting of ice at 0°C.

Heat required for this process,

$$\Delta Q_2 = (50\ \mathrm{g})(80\ \mathrm{cal/g}) = 4000\ \mathrm{cal}$$

(iii) The rise in temperature of melted ice (i.e., water) from 0°C to 1000°C (i.e., B.P. of water).

Heat required for this process,

$$\Delta Q_3 = (50\ \mathrm{g})(1\ \mathrm{cal/g\text{-}^\circ C})(100\ {}^\circ\mathrm{C}) = 5000\ \mathrm{cal}$$

(iv) The vaporisation of water at 100°C.

Heat required for this process,

$$\Delta Q_4 = (50\ \mathrm{g})(540\ \mathrm{cal/g}) = 27000\ \mathrm{cal}$$

$\therefore$ Total heat required, $\Delta Q = \Delta Q_1 + \Delta Q_2 + \Delta Q_3 + \Delta Q_4$

$$= 36500\ \mathrm{cal}$$

3. The final temperature is obviously 0°C.

$$600 \times 10^{-3} \times 0.5 \times 10 + (600 - 550) \times 10^{-3} \times 80\ \mathrm{kcal}$$

$$= m \times 0.1 \times 350\ \mathrm{kcal}$$

or $\quad m = 200 \times 10^{-3}$ kg or 200 g

4. The small ice crystal acts as the centre of condensation and so water begins to freeze instantaneously.

$$mL = Mc\{0 - (-t)\} = Mct$$

or $$m = \frac{Mct}{L} = \frac{1 \times 4200 \times 8}{336 \times 10^3}\ \mathrm{kg} = 100\ \mathrm{g}$$

Now $\quad ML = Mct \Rightarrow t = \dfrac{L}{c} = \dfrac{336 \times 10^3}{4200} = 80\ {}^\circ\mathrm{C}$

5. Energy supplied by heater $= Pt = 54 \times 3 \times 60 = 9720$ J

Energy absorbed by water $= (650 \times 10^{-3}) \times 4200 \times 3.4 = 9282$ J

Therefore, energy that passes out in the form of radiant heat

$$= 9720 - 9282 = 438 \text{ J}$$

∴ Percentage loss = 438/9720 × 100 = 4.5%

Let m kg of water freeze on it.

Then heat lost = $m \times 80 \times 1000$ cal

Heat gained by ice = $50 \times 10^{-3} \times 500 \times \{0 - (-10)\}$ cal

Now, Heat loss = Heat gain,

$$m = 3.125 \times 10^{-3} \text{ kg} = 3.125 \text{ g}$$

Let t be the temperature of the mixture.

Net loss of heat by all

$$= (V\rho_1)\, c_1\, (t_1 - t) + (V\rho_2) c_2\, (t_2 - t) + (V\rho_3) c_3\, (t_3 - t)$$

But net loss = 0

$$V\rho_1 c_1\, (t_1 - t) + V\rho_2 c_2\, (t_2 - t) + V\rho_3 c_3\, (t_3 - t) = 0$$

or $$t = \frac{\rho_1 c_1 t_1 + \rho_2 c_2 t_2 + \rho_3 c_3 t_3}{\rho_1 c_1 + \rho_2 c_2 + \rho_3 c_3}$$

Consider mass m of water falling.

$$mgy = mc\Delta t$$

We express both sides in joules by noting $c = 1$ kcal/kg,

$$K = 4184\, \Delta t$$

Then $9.8(122) = 4184\, \Delta t$

and $\Delta t = 0.29$ K

Let the specific heat capacities of A, B, C be s_1, s_2, s_3.

$$\theta_{A+B} = 19 = \frac{10s_1 + 25s_2}{s_1 + s_2} \quad \text{or} \quad s_1 = \frac{2}{3}s_2$$

$$\theta_{B+C} = 35 = \frac{25s_2 + 40s_3}{s_2 + s_3} \quad \text{or} \quad s_3 = 2s_2$$

$$\theta_{A+B+C} = \frac{10s_1 + 25s_2 + 40s_3}{s_1 + s_2 + s_3}$$

$$= \frac{10 \times \frac{2}{3}s_2 + 25s_2 + 40 \times 2s_2}{\frac{2}{3}s_2 + s_2 + 2s_2} = 30.5°\text{C}$$

Here water at the surface is evaporated at the cost of the water in the vessel losing heat.

Heat lost by the water in the vessel

$$= (9.5 + 0.5) \times 1000 \times (30 - 20) = 10^5 \text{ cal}$$

Let t be the required time in seconds.

Heat gained by the water at the surface = $(t \times 10^{-3}) \times 540 \times 10^3$

$$(L = 540 \text{ cal/g} = 540 \times 10^3 \text{ cal/kg})$$

∴ $10^5 = 540t$

or $t = 185$ s = 3 min 5 s

In the first interval of 1 min ice is heated to its melting point from its initial temperature which is below the melting point. In the next 4 min there is phase change. In the next 2 minutes there is heating of water

Let Q be the rate of supply of heat in joule per minute

∴ $4Q = mL = m \times 336 \times 10^3$

and $2Q = m \times 4200 \times \theta$

On dividing

$$\frac{4Q}{2Q} = \frac{m \times 336 \times 10^3}{m \times 4200 \times \theta}$$

$$\Rightarrow \quad \theta = \frac{336 \times 10^3}{2 \times 4200} = 40°\text{C}$$

12. Heat available on steam (changes into steam to water)

$$= mL = 1 \times 540 = 540 \text{ cal}$$

Heat gained by ice to change into water and then rise its temperature to 100°C

$$= m_{\text{ice}} L + m_{\text{wat}}\, c\Delta T = 1 \times 80 + 1 \times 1 \times (100 - 0) = 180 \text{ cal}$$

The above calculations show that some part of steam will condense to change the ice into water at 100°C. Let m is the mass of steam condensed, then

$$m \times 540 = 180$$

or $$m = \frac{80}{540} = \frac{1}{3} \text{ g}$$

Final contents: ice = 0 g

Water = 1 + 1/3 = 4/3 g

Steam = 1 – 1/3 = 2/3 g

13. $\dfrac{\rho_1}{\rho_2} = \dfrac{3}{4}, \ \dfrac{s_1}{s_2} = \dfrac{4}{3}$

$$\theta = \frac{m \times s}{m/\rho} \quad \Rightarrow \quad \frac{\theta_1}{\theta_2} = \frac{s_1}{s_2} \times \frac{\rho_1}{\rho_2} = 1:1$$

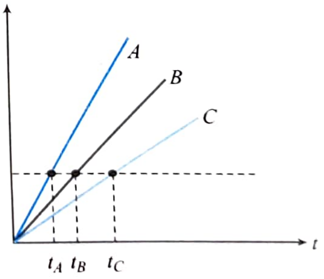
Exercises

Single Correct Answer Type

1. (3) On heating the system, x, r and d all increase, since the expansion of isotropic solids is similar to true photographic enlargement.

2. (2) If the sheet is heated then both d_1 and d_2 will increase since the thermal expansion of isotropic solid is similar to true photographic enlargement.

3. (3) Substances having more specific heat take longer time to get heated to a higher temperature and longer time to get cooled.

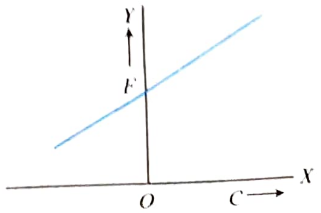


If we draw a line parallel to the time axis then it cuts the given graphs at three different points. Corresponding points on the times axis shows that

$$t_C > t_B > t_A \Rightarrow C_C > C_B > C_A$$

4. (1) $\dfrac{C}{5} = \dfrac{F - 32}{9} \Rightarrow C = \left(\dfrac{5}{9}\right)F - \dfrac{20}{3}$. Hence graph between °C and °F will be a straight line with positive slope and negative intercept.

5. (1) We know that, $\dfrac{C}{100} = \dfrac{F - 32}{180}$ or $F = \dfrac{9}{5}C + 32$



Equation of straight line is, $y = mx + c$

Hence, $m = 9/5$ (positive) and $c = 32$ (positive). The graph is shown in figure.

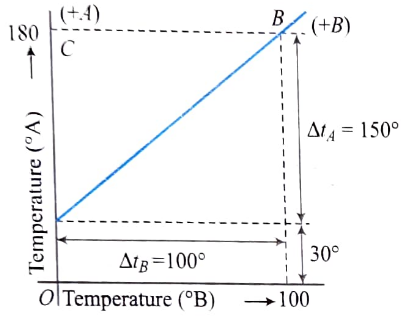
6. (2) Relation between Celsius and Fahrenheit scales of temperature is:

$$\frac{C}{5} = \frac{F-32}{9} \Rightarrow C = \frac{5}{9}F - \frac{160}{9}$$

Equating above equation with standard equation of line $y = mx + c$, we get slope of the line AB as $m = \frac{5}{9}$.

7. (2) It is clear from the graph that lowest point for scale A is 30° and lowest point for scale B is 0°. Highest point for the scale A is 180° and for scale B is 100°. Hence, correct relation is

$$\frac{t_A - (\text{LFP})}{(\text{UFP})_A - (\text{LFP})_A} = \frac{t_B - (\text{LFP})_B}{(\text{UFP})_B - (\text{LFP})_B}$$



$$\Rightarrow \quad \frac{t_A - 30}{180 - 30} = \frac{t_B - 0}{100 - 0}$$

$$\Rightarrow \quad \frac{t_A - 30}{150} = \frac{t_B}{100}$$

where LFP → Lower fixed point

and UFP → Upper fixed point

8. (1) Temperature on any scale can be converted into other scale by

$$\frac{x - \text{LFP}}{\text{UFP} - \text{LFP}} = \text{Constant for all scales.}$$

So, $\frac{x - 20}{150 - 20} = \frac{60}{100}$

$\Rightarrow \quad x = 98°\text{C}$

9. (2) $\gamma_r = \gamma_a + \gamma_v$; where γ_r = coefficient of real expansion, γ_a = coefficient of apparent expansion and γ_v = coefficient of expansion of vessel.

For copper: $\gamma_r = C + 3\alpha_{Cu} = C + 3A$

For silver: $\gamma_r = S + 3\alpha_{Ag}$

$\Rightarrow \quad C + 3A = S + 3\alpha_{Ag}$

$\Rightarrow \quad \alpha_{Ag} = \frac{C - S + 3A}{3}$

10. (2) When length of the liquid column remains constant, then the level of liquid moves down with respect to the container, thus γ must be less than 3α.

Now we can write $V = V_0(1 + \gamma\Delta T)$

Since $V = Al_0 = [A_0(1 + 2\alpha\Delta T)]l_0 = V_0(1 + 2\alpha\Delta T)$

Hence $V_0(1 + \gamma\Delta T) = V_0(1 + 2\alpha\Delta T) \Rightarrow \gamma = 2\alpha$.

11. (3) Since in the region AB, temperature is constant, at this temperature, phase of the material changes from solid to liquid and $(H_2 - H_1)$ heat will be absorbed by the material. This heat is known as the heat of melting of the solid.

Similarly in the region CD, temperature is constant. Therefore at this temperature, phase of the material changes from liquid to gas and $(H_4 - H_3)$ heat will be absorbed by the material. Th heat is known as the heat of vaporisation of the liquid.

12. (3) From given curve, Melting point for A = 60°C and melti point for B = 20°C

Time taken by A for fusion = (6 – 2) = 4 minutes

Time taken by B for fusion = (6.5 – 4) = 2.5 minutes

$$\therefore \quad \frac{H_A}{H_B} = \frac{6 \times 4 \times 60}{6 \times 2.5 \times 60} = \frac{8}{5}$$

13. (3) Since specific heat = 0.6 kcal/g × °C = 0.6 cal/g × °C

From graph, it is clear that in a minute, the temperature is rais from 0°C to 50°C.

$\Rightarrow$ Heat required for a minute = 50 × 0.6 × 50 = 1500 cal

Also from graph, Boiling point of wax is 200°C.

14. (4) Let the quantity of heat supplied per minute be Q. Then

Quantity of heat supplied in 2 min = $mC(90 - 80)$

Quantity of heat supplied in 4 min = $2mC(90 - 80)$

$$\therefore \quad 2mC(90 - 80) = mL \Rightarrow \frac{L}{C} = 20$$

15. (3) Energy $= \frac{1}{2}mv^2 = mc\,\Delta\theta$

$\Rightarrow \quad \Delta\theta \propto v^2$

Temperature does not depend upon the mass of the balls.

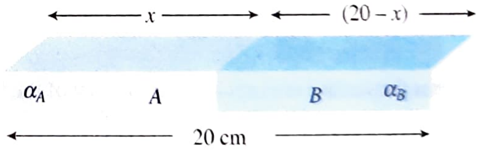
$$\therefore \quad \frac{mL}{t} = P$$

or $\quad L = \frac{Pt}{m}$

16. (2) $\Delta L = L_0\alpha\Delta\theta$

Rod A: $0.075 = 20 \times \alpha_A \times 100 \Rightarrow \alpha_A = \frac{75}{2} \times 10^{-6}/°\text{C}$

rod B: $0.045 = 20 \times \alpha_B \times 100 \Rightarrow \alpha_B = \frac{45}{2} \times 10^{-6}/°\text{C}$



For composite rod: Taking x cm of A and $(20 - x)$ cm of B. have

$0.060 = x\,\alpha_A \times 100 + (20 - x)\,\alpha_B \times 100$

$$= x\left[\frac{75}{2} \times 10^{-6} \times 100 + (20 - x) \times \frac{45}{2} \times 10^{-6} \times 100\right]$$

On solving, we get x = 10 cm.

17. (4) Coefficient of volume expansion

$$\gamma = \frac{\Delta\rho}{\rho\,\Delta T} = \frac{(\rho_1 - \rho_2)}{\rho\,(\Delta\theta)} = \frac{(10 - 9.7)}{10 \times (100 - 0)} = 3 \times 10^{-4}$$

Hence, coefficient of linear expansion

$$\alpha = \frac{\gamma}{3} = 10^{-4}/°\text{C}$$

18. (1) $\frac{h_1}{h_2} = \frac{\rho_1}{\rho_2} = \frac{(1 + \gamma\theta_1)}{(1 + \gamma\theta_2)}$ $\quad \left[\because \rho = \frac{\rho_0}{(1 + \gamma t}\right.$

$$\Rightarrow \quad \frac{50}{60} = \frac{1 + \gamma \times 50}{1 + \gamma \times 100} \Rightarrow \gamma = 0.005/°\text{C}$$

19. (3) It is given that the volume of air in the flask remains the sa This means that the expansion in volume of the vessel is exa equal to the volume expansion of mercury.

i.e., $\Delta V_g = \Delta V_L$

or $V_g \gamma_g \Delta\theta = V_L \gamma_L \Delta\theta$

$$\therefore \quad V_L = \frac{V_g \gamma_g}{\gamma_L} = \frac{1000 \times (3 \times 9 \times 10^{-6})}{1.8 \times 10^{-4}} = 150\,\text{cc}$$

(3) Due to volume expansion of both mercury and flask, the change in volume of mercury relative to flask is given by

$$\Delta V = V_0[\gamma_L - \gamma_g]\Delta\theta = V[\gamma_m - 3\alpha_g]\Delta\theta$$

$$= 50[180 \times 10^{-6} - 3 \times 9 \times 10^{-6}](38 - 18) = 0.153\,\text{cc}$$

21. (4) $V = V_0(1 + \gamma\Delta\theta)$

$\Rightarrow$ Change in volume, $V - V_0 = \Delta V = A.\Delta l = V_0 \gamma\, \Delta\theta$

$$\Rightarrow \quad \Delta l = \frac{V_0 \gamma \Delta\theta}{A} = \frac{10^{-6} \times 18 \times 10^{-5} \times (100 - 0)}{0.004 \times 10^{-4}}$$

$$= 45 \times 10^{-3}\, m = 4.5\,\text{cm}$$

22. (1) $\gamma_{\text{real}} = \gamma_{\text{app.}} + \gamma_{\text{vessel}}$

So $(\gamma_{\text{app.}} + \gamma_{\text{vessel}})_{\text{glass}} = (\gamma_{\text{app.}} + \gamma_{\text{vessel}})_{\text{steel}}$

$\Rightarrow$ $153 \times 10^{-6} + (\gamma_{\text{vessel}})_{\text{glass}} = 144 \times 10^{-6} + (\gamma_{\text{vessel}})_{\text{steel}}$

Further, $(\gamma_{\text{vessel}})_{\text{steel}} = 3\alpha = 3 \times (12 \times 10^{-6}) = 36 \times 10^{-6}/^\circ\text{C}$

$\Rightarrow$ $153 \times 10^{-6} + (\gamma_{\text{vessel}})_{\text{glass}} = 144 \times 10^{-6} + 36 \times 10^{-6}$

$\Rightarrow$ $(\gamma_{\text{vessel}})_{\text{glass}} = 3\alpha = 27 \times 10^{-6}/^\circ\text{C}$

$\Rightarrow$ $\alpha = 9 \times 10^{-6}/^\circ\text{C}$

23. (2) Due to volume expansion of both liquid and vessel, the change in volume of liquid relative to container is given by

$$\Delta V = V_0[\gamma_L - \gamma_g]\Delta\theta$$

Given, $V_0 = 1000$ cc, $\alpha_g = 0.1 \times 10^{-4}/^\circ\text{C}$

$\therefore$ $\gamma_g = 3\alpha_g = 3 \times 0.1 \times 10^{-4}/^\circ\text{C} = 0.3 \times 10^{-4}/^\circ\text{C}$

$\therefore$ $\Delta V = 1000\,[1.82 \times 10^{-4} - 0.3 \times 10^{-4}] \times 100 = 15.2$ cc

24. (3) As the coefficient of cubical expansion of metal is less as compared to the coefficient of cubical expansion of liquid, we may neglect the expansion of metal ball. So when the ball is immersed in alcohol at 0°C, it displaces some volume V of alcohol at 0°C and has weight W_1.

$W_1 = W_0 - V\rho_0 g$ where W_0 = weight of ball in air

Similarly, $W_2 = W_0 - V\rho_{50} g$ where ρ_0 = density of alcohol at 0°C and ρ_{50} = density of alcohol at 50°C

As $\rho_{50} < \rho_0$, therefore, $W_2 > W_1$ or $W_1 < W_2$

25. (3) Initial diameter of tyre = (1000 – 6) mm = 994 mm

So, initial radius of tyre, $R = \dfrac{994}{2} = 497$ mm

and change in diameter, $\Delta D = 6$ mm. So $\Delta R = \dfrac{6}{2} = 3$ mm

After increasing temperature by $\Delta\theta$, tyre will fit onto wheel.

Increment in the length (circumference) of the iron tyre,

$$\Delta L = L \times \alpha \times \Delta\theta = L \times \frac{\gamma}{3} \times \Delta\theta \qquad \left[\text{As } \alpha = \frac{\gamma}{3}\right]$$

$$\Rightarrow \quad 2\pi\,\Delta R = 2\pi\, R\left(\frac{\gamma}{3}\right)\Delta\theta$$

$$\Rightarrow \quad \Delta\theta = \frac{3}{\gamma}\frac{\Delta R}{R} = \frac{3 \times 3}{3.6 \times 10^{-5} \times 497}$$

$\Rightarrow$ $\Delta\theta = 500^\circ\text{C}$

26. (1) With rise in temperature (same 20°C for both), steel scale and copper wire both expand.

Hence, length of copper wire w.r.t. steel scale or apparent length of copper wire after rise in temperature,

$$L_{\text{app}} = L'_{\text{cu}} - L'_{\text{steel}} = [L_0(1 + \alpha_{Cu}\Delta\theta) - L_0(1 + \alpha_s\Delta\theta)$$

$$\Rightarrow \quad L_{\text{app}} = L_0(\alpha_{\text{Cu}} - \alpha_s)\Delta\theta$$

$$= 80(17 \times 10^{-6} - 11 \times 10^{-6}) \times 20 = 80.0096\text{ cm}$$

27. (2) Loss of weight at 27°C = 46 – 30 = 16

$\Rightarrow$ $V_1 \times 1.24\,\rho_l \times g = 16$ ...(i)

Loss of weight at 42°C = 46 – 30.5 = 15.5

$\Rightarrow$ $V_2 \times 1.2\,\rho_l \times g = 15.5$ ...(ii)

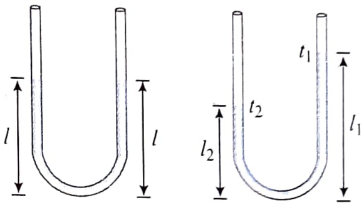
Now, dividing (i) by (ii), we get $\dfrac{16}{15.5} = \dfrac{V_1}{V_2} \times \dfrac{1.24}{1.2}$

$$\therefore \quad \frac{V_2}{V_1} = 1 + 3\alpha(t_2 - t_1) = \frac{15.5 \times 1.24}{16 \times 1.2} = 1.001042$$

$\Rightarrow$ $3\alpha(42^\circ - 27^\circ) = 0.001042$

$\Rightarrow$ $\alpha = 2.316 \times 10^{-5}/^\circ\text{C}$

28. (1) Suppose, height of liquid in each arm before rising the temperature is l.

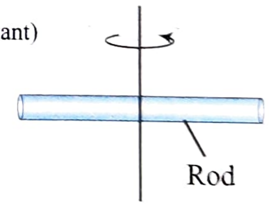


With temperature rises, height of liquid in each arm increases i.e., $l_1 > l$ and $l_2 > l$.

Also, $l = \dfrac{l_1}{1 + \gamma t_1} = \dfrac{l_2}{1 + \gamma t_2}$

$$\Rightarrow \quad l_1 + \gamma\, l_1 t_2 = l_2 + \gamma\, l_2 t_1 \Rightarrow \gamma = \frac{l_1 - l_2}{l_2 t_1 - l_1 t_2}$$

29. (2) As the rod is heated, it expands. No external torque is acting on the system, so angular momentum should be conserved.



L (Angular momentum) = $I\omega$ (constant)

$\Rightarrow$ $I_1\omega_1 = I_2\omega_2$

Due to expansion of the rod $I_2 > I_1$

$\therefore$ $\dfrac{\omega_2}{\omega_1} = \dfrac{I_1}{I_2} < 1 \Rightarrow \omega_2 > \omega_1$

So, angular velocity (speed of rotation) decreases.

30. (1) Density of water is maximum at 4°C, this is because of anomalous expansion of water.

Let volume of the sphere be V and ρ be its density, then buoyant force,

$F = V\rho G$ (g = acceleration due to gravity)

$\Rightarrow$ $F \propto \rho$ ($\because$ V and g are almost constant)

$\Rightarrow$ $\dfrac{F_{4^\circ\text{C}}}{F_{0^\circ\text{C}}} = \dfrac{\rho_{4^\circ\text{C}}}{\rho_{0^\circ\text{C}}} > 1$ ($\because \rho_{4^\circ\text{C}} > \rho_{0^\circ\text{C}}$)

$\Rightarrow$ $F_{4^\circ\text{C}} > F_{0^\circ\text{C}}$

Hence, buoyancy will be less in water at 0°C than that in water at 4°C.

31. (3) Since specific heat of lead is given in Joules, hence use $W = Q$ instead of $W = JQ$.

$$\text{So, } \frac{1}{2} \times \left(\frac{1}{2}mv^2\right) = m.c.\Delta\theta \quad \Rightarrow \Delta\theta = \frac{v^2}{4c} = \frac{(300)^2}{4 \times 150} = 150^\circ\text{C}$$

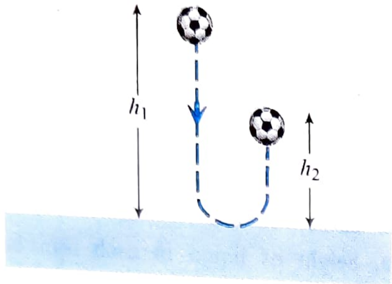
32. (1) $W = JQ$

$$\frac{1}{2}(mgh) = J \times mc\Delta\theta$$

$$\Rightarrow \quad \Delta\theta = \frac{gh}{2JS} \quad \Rightarrow \quad \Delta\theta = \frac{9.8 \times 84}{2 \times 4.2 \times 1000} = 0.098°\text{C}$$

$$\left(\because S_{\text{water}} = 1000 \frac{\text{cal}}{\text{kg} \times °\text{C}}\right)$$

33. (2) According to energy conservation, change in potential energy of the ball appears in the form of heat which raises the temperature of the ball.



i.e., $mg(h_1 - h_2) = mc\Delta\theta$

$$\Rightarrow \quad \Delta\theta = \frac{g(h_1 - h_2)}{c} = \frac{10(10 - 5.4)}{460} = 0.1°\text{C}$$

34. (1) Heat taken by ice to melt at 0°C,

$$Q_1 = mL = 540 \times 80 = 43200 \text{ cal}$$

Heat given by water to cool upto 0°C,

$$Q_2 = ms\Delta\theta = 540 \times 1 \times (80 - 0) = 43200 \text{ cal}$$

Hence heat given by water is just sufficient to melt the whole ice and final temperature of mixture is 0°C.

35. (2) Heat lost by hot water = Heat gained by cold water in beaker + Heat absorbed by beaker

$$\Rightarrow \quad 440(92 - \theta) = 200 \times (\theta - 20) + 20 \times (\theta - 20)$$

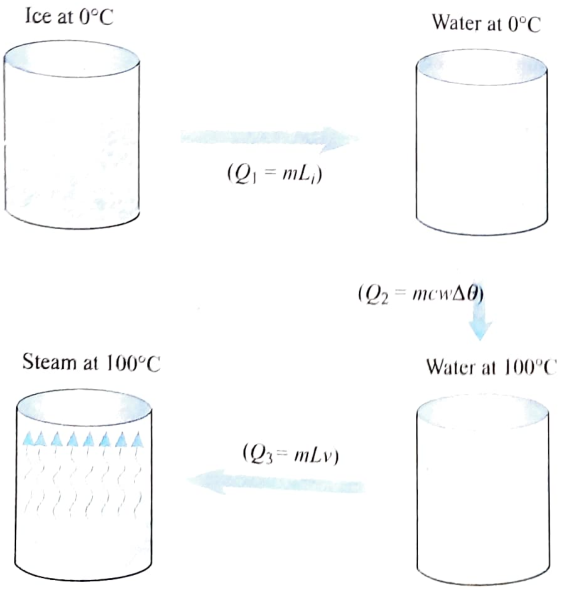
$$\Rightarrow \quad \theta = 68°\text{C}$$

36. (4) Heat released to convert x gram of steam at 100°C to water at 100°C be $x \times 540$ cal.

If y g of ice is to be converted from 0°C to water at 100°C, then heat required is $y \times 80 + y \times 1 \times 100 = 180\, y$ gram.

$$\therefore \quad x \times 540 = 180\, y \quad \text{or} \quad \frac{y}{x} = \frac{540}{180} = \frac{3}{1}$$

37. (3) Ice (0°C) converts into steam (100°C) in following three steps.

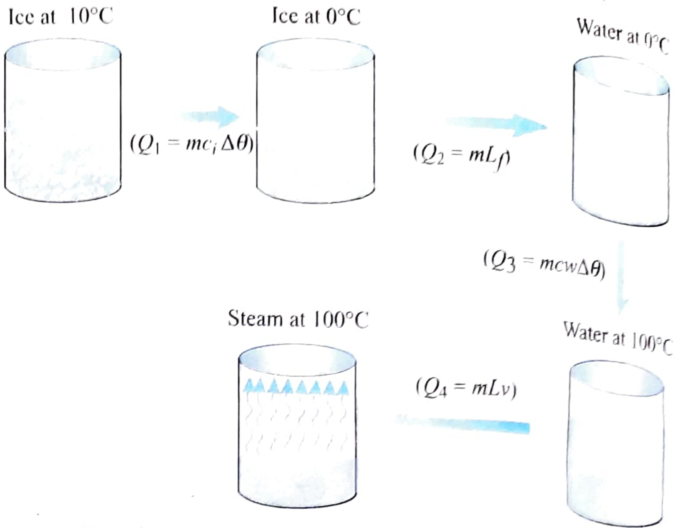


Total heat required $Q = Q_1 + Q_2 + Q_3$

$$= 5 \times 80 + 5 \times 1 \times (100 - 0) + 5 \times 540 = 3600 \text{ cal}$$

38. (1) Ice (−10°C) converts into steam as follows:

(c_i = Specific heat of ice, c_W = Specific heat of water)

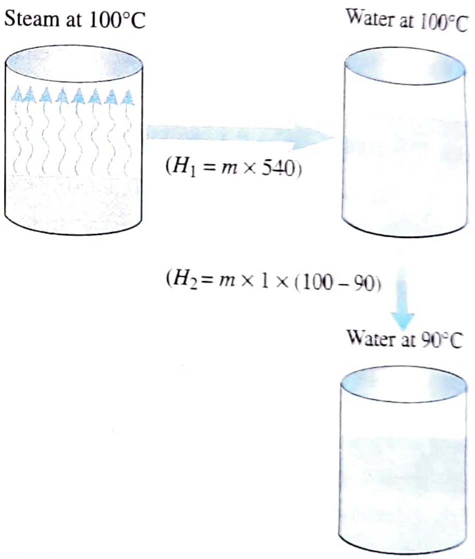


Total heat required, $Q = Q_1 + Q_2 + Q_3 + Q_4$

$$\Rightarrow Q = 1 \times 0.5(10) + 1 \times 80 + 1 \times 1 \times (100 - 0) + 1 \times 540 = 725 \text{ cal}$$

Hence work done, $W = JQ = 4.2 \times 725 = 3045$ J

39. (1) Let m gram of steam get condensed into water (by heat loss). This happens in following two steps:



Heat gained by water (20°C) to raise its temperature upto 90° = $22 \times 1 \times (90 - 20)$

Hence, in equilibrium, Heat lost = Heat gain

$$\Rightarrow \quad m \times 540 + m \times 1 \times (100 - 90) = 22 \times 1 \times (90 - 20)$$

$$\Rightarrow \quad m = 2.8 \text{ g}$$

Net mass of the water present in the mixture = 22 + 2.8 = 24.8 g

40. (3) Heat given by water, $Q_1 = 10 \times 10 = 100$ cal

Heat taken by ice to melt,

$$Q_2 = 10 \times 0.5 \times [0 - (-20)] + 10 \times 80 = 900 \text{ cal}$$

As $Q_1 < Q_2$, so ice will not completely melt and final temperature = 0°C.

As heat given by water in cooling up to 0°C is only just sufficient to increase the temperature of ice from −20°C to 0°C, hence mixture in equilibrium will consist of 10 g ice and 10 g water at 0°C.

41. (1) If mass of the bullet is m gram, then total heat required for bullet to just melt down,

$$Q_1 = mc\,\Delta\theta + mL = m \times 0.03\,(327 - 27) + m \times 6$$

$$= 15m \text{ cal} = (15m \times 4.2) \text{ J}$$

Now when bullet is stopped by the obstacle, the loss in its mechanical energy $= \frac{1}{2}(m \times 10^{-3})v^2$ J (As m gram $= m \times 10^{-3}$ kg)

As 25% of this energy is absorbed by the obstacle, the energy absorbed by the bullet,

$$Q_2 = \frac{75}{100} \times \frac{1}{2} mv^2 \times 10^{-3} = \frac{3}{8} mv^2 \times 10^{-3} \text{ J}$$

Now the bullet will melt if $Q_2 \geq Q_1$.

i.e., $\frac{3}{8} mv^2 \times 10^{-3} \geq 15m \times 4.2 \Rightarrow v_{\text{min}} = 410$ m/s

(3) Heat gain = Heat lost

$$\Rightarrow \quad C_A(16 - 12) = C_B(19 - 16) \Rightarrow \frac{C_A}{C_B} = \frac{3}{4}$$

$$\text{and} \quad C_B(23 - 19) = C_C(28 - 23) \Rightarrow \frac{C_B}{C_C} = \frac{5}{4}$$

$$\Rightarrow \quad \frac{C_A}{C_C} = \frac{15}{16} \qquad \text{...(i)}$$

If θ is the temperature when A and C are mixed, then

$$C_A(\theta - 12) = C_C(28 - \theta) \Rightarrow \frac{C_A}{C_C} = \frac{28 - \theta}{\theta - 12} \qquad \text{...(ii)}$$

On solving equation (i) and (ii), $\theta = 20.2$°C.

(2) Suppose m kg of steam is required per hour.

Heat is released by steam in following three steps

(i) When 150°C steam $\xrightarrow[Q_1]{}$ 100°C steam

Then $Q_1 = mc_{\text{Steam}}\,\Delta\theta = m \times 1\,(150 - 100) = 50\,m$ cal

(ii) When 150°C steam $\xrightarrow[Q_2]{}$ 100°C water

Then $Q_2 = mL_V = m \times 540 = 540\,m$ cal

(iii) When 100°C water $\xrightarrow[Q_3]{}$ 90°C water

Then $Q_3 = mc_W\,\Delta\theta = m \times 1 \times (100 - 90) = 10\,m$ cal

Hence, total heat given by the steam,

$$Q = Q_1 + Q_2 + Q_3 = 600 \text{ mcal} \qquad \text{...(i)}$$

Heat taken by 10 kg water,

$$Q' = mc_W\Delta\theta = 10 \times 10^3 \times 1 \times (80 - 20) = 600 \times 10^3 \text{ cal}$$

Hence $Q = Q' \Rightarrow 600$ m $= 600 \times 10^3$

$\Rightarrow m = 10^3$ g $= 1$ kg

44. (1) $W = JQ \Rightarrow \frac{1}{2} I\omega^2 = J(MS\Delta\theta)$

$$\Rightarrow \quad \frac{1}{2}\left(\frac{2}{5}MR^2\right)\omega^2 = J(MS\Delta\theta) \Rightarrow \Delta\theta = \frac{R^2\omega^2}{5Js}$$

45. (4) Since tension in the two rods will be same,

$$A_1Y_1\alpha_1\Delta\theta = A_2Y_2\alpha_2\Delta\theta$$

$$A_1Y_1\alpha_1 = A_2Y_2\alpha_2$$

46. (3) The change in length of wire $= l_1\,\alpha_{\text{Al}}\,\Delta\theta + l_2\,\alpha_{\text{st}}\,\Delta\theta$ associated with temperature change of $\Delta\theta$, where α_{Al} and α_{St} are the coefficient of linear expansion of aluminium and steel, respectively

$$\alpha_{\text{Al}} = 23 \times 10^{-6}/°\text{C}$$

$$\alpha_{\text{st}} = 12 \times 10^{-6}/°\text{C}$$

The effective coefficient of linear expansion of the two segments of wire $= 19 \times 10^{-6}$/°C

$$l_1\alpha_{\text{Al}}\Delta\theta + l_2\alpha_{\text{st}}\Delta\theta = (l_1 + l_2)\alpha\Delta\theta$$

$$\frac{l_1}{(l_1 + l_2)} = \frac{\alpha - \frac{l_2}{(l_1 + l_2)}\alpha_{\text{st}}}{\alpha_{\text{Al}}} = \text{Fraction length of aluminium}$$

$$\left[\frac{l_1}{l_1 + l_2} = x \text{ and } \left(\frac{l_2}{l_1 + l_2}\right) = 1 - x\right]$$

$$\text{Hence, } x = \frac{\alpha - (1 - x)\alpha_{\text{st}}}{\alpha_{\text{Al}}}$$

$$x = \frac{\alpha - \alpha_{\text{st}}}{\alpha_{\text{Al}} - \alpha_{\text{st}}} = \frac{19 \times 10^{-6} - 12 \times 10^{-6}}{23 \times 10^{-6} - 12 \times 10^{-6}} = \frac{7}{11}$$

47. (1) $Q_4 = mL_w = 540$ cal

$Q_3 = ms_w\,(100 - 0) = 100$ cal

$Q_2 = mL_{\text{ice}} = 80$ cal

$Q_1 = ms_{\text{ice}}\,(20 - 0) = 20$ cal

$Q_4 > Q_3 > Q_2 > Q_1$

48. (3) Fraction of wooden block immersed at 0°C,

$$\frac{V_1}{V_0} = \frac{(\rho_{\text{wood}})_{0°\text{C}}}{(\rho_{H_2O})_{0°\text{C}}}$$

$$f_1 = \frac{V_0 - V_1}{V_0} = \frac{(\rho_{H_2O})_{0°\text{C}} - (\rho_{\text{wood}})_{0°\text{C}}}{(\rho_{H_2O})_{0°\text{C}}}$$

V_1 = Volume of wood immersed in water at 0°C

V_0 = Volume of wood

$(\rho_{\text{wood}})_{0°\text{C}}$ = Density of wood at 0°C

When the temperature is raised to 10°C, the volume of wood immersed in water changes to V_2.

$$\frac{V_2}{V_0} = \frac{(\rho_{\text{wood}})_{10°\text{C}}}{(\rho_{H_2O})_{10°\text{C}}}$$

$$f_2 = \frac{V_0 - V_2}{V_0} = \frac{(\rho_{H_2O})_{10°\text{C}} - (\rho_{\text{wood}})_{10°\text{C}}}{(\rho_{H_2O})_{10°\text{C}}}$$

From 0°C to 4°C, the density of water increases, and from 4°C to 10°C the density of water decreases.

But for wood density decreases as temperature increases. The volume of block above water level will first increase and then decrease.

49. (2) Power sent to heat the water in the calorimeter $P' = \frac{ms\Delta\theta}{t}$

$$= \frac{V\rho s\Delta\theta}{t} = \frac{10^3 \times 10^{-6} \times 10^3 \times 4200 \times 4}{420} = 40 \text{ W}$$

Required ratio $= \frac{P - P'}{P} = \frac{54 - 40}{54} = \frac{14}{54} = 26\%$

50. (3) Thermal stress that is produced in an elastic wire is $Y\,\alpha\,\theta$ per unit area, where Y is Young's modulus, α coefficient of linear expansion and θ change of temperature.

Thus, tension developed in the wire is

T = Thermal stress × area of cross section

$= Y\alpha \times (\Delta T)[\pi r^2]$ N

$= 0.91 \times 10^{11} \times 2 \times 10^{-5} \times [27 - (-33)]\,[\pi \times 1 \times 10^{-6}]$ N

$= 0.91 \times 2 \times 6 \times 3.14 \times 10$ N

$= 34.4 \times 10$ N $= 0.34$ kN

51. (4) Specific heat of lead = 0.03 kcal/kg°C. Gravitational potential energy is converted into thermal energy which is absorbed by the lead shot. If T_0 is the rise in temperature of the shot, then

$$(100)\ [mgh] = (m \times s \times T)\ \text{J}$$

$$T = \frac{100 \times g \times h}{Js} = \frac{100 \times 9.8 \times 1.5}{0.03 \times 4.18 \times 10^3}\,°\text{C} = 11.3°\text{C}$$

52. (2) The entire kinetic energy of the fragment is changed to heat. Expressing mass m in kilograms everywhere, we have

$$\frac{1}{2}mv^2 = [m(30) + m(0.11)(1535° + 100°)]4184$$

$$v^2 = (8368)\ [30 + 180] \simeq 1.76 \times 10^6$$

Hence, $v = (\sqrt{1.76}) \times 10^3 = 1.32$ km/s

53. (1) This is a problem on 'flow calorimeter' used to measure specific heat of a liquid.

Amount of heat supplied to the water per second by the heating coil, $Q_s = 250\ \text{J} = \frac{250}{4186}$ kcal

The volume of liquid flowing out per second = 8.0 cm³ = 8×10^{-6} m³

Mass of this liquid = $(0.85) \times 1000 \times 8 \times 10^{-6}$ kg

Temperature rise of this mass of liquid = 15°C

Hence, $\frac{250}{4186} = mst = 0.85 \times 8 \times 10^{-3} \times s \times 15$

Hence, $s = \frac{250 \times 10^3}{4186 \times 0.85 \times 8 \times 15} = 0.6$ kcal/kg K

54. (3) We have $\frac{\Delta V}{V} = \gamma\Delta T = (1.82 \times 10^{-4})(100 - 20) = 1.46 \times 10^{-2}$

Now the bulk modulus, $P = B\left(\frac{\Delta V}{V}\right)$, where P is the pressure.

$P = (2500) \times 1.46 \times 10^{-2}$ MPa = 36 MPa

55. (4) It is clear that at desired temperature, T°C, the densities of the wood and benzene must be equal for the wood to just sink.

i.e., $\rho_w(T) = \rho_B(T)$

If m is the mass of wood (which is assumed to be constant) then, if $(V_0)_w$ and $(V_0)_B$ are the respective volumes at 0°C of mass m of wood and benzene,

$$(\rho_0)_w(V_0)_w = (\rho_0)_B(V_0)_B = m$$

$$(\rho_0)_w = 880\ \text{kg/m}^3 \text{ and } (\rho_0)_B = 900\,\text{kg/m}^3$$

Hence, $(V_0)_w = \frac{m}{880}(m^3)$

End, $(V_0)_B = \frac{m}{900}(m^3)$

We then have, $(V_T)_W = (V_0)_W(1 + 1.2 \times 10^{-3}T)$

$$(V_T)_B = (V_0)_B(1 + 1.5 \times 10^{-3}T)$$

Thus $\frac{(V_T)_W}{(V_T)_B} = \frac{(\rho_B)_T}{(\rho_W)_T} = 1 = \frac{(V_0)_W 1 + 1.2 \times 10^{-3}T}{(V_0)_B 1 + 1.5 \times 10^{-3}T}$

Thus $\frac{(V_0)_W}{(V_0)_B} = \frac{900}{880} = \frac{90}{88} = \frac{1 + 1.5 \times 10^{-3}T}{1 + 1.2 \times 10^{-3}T}$

Solving for T, we have $T = 83.2$°C.

56. (1) Here we assume that all the work done on the bullet inside the wood is converted into thermal energy.

Thermal energy produced in the first case is

$$H_1 = \frac{1}{2}(5 \times 10^{-3})(200)^2\ \text{J}$$

Since the resistance in the second plank is the same as the first and also the thickness is equal, the same amount of work will be done on the bullet as in the first case and hence the thermal energy $H_2 = H_1$. The ratio is 1:1.

57. (2) Let m be the mass of ice.

Rate of heat given by the burner is constant.

In the first 50 min

$$\frac{dQ}{dt} = \frac{mL}{t_1} = \frac{m\ \text{kg} \times (80 \times 4.2 \times 10^3)\,\text{J/kg}}{(50\ \text{min})} \quad \text{...(i)}$$

From 50 min to 60 min

$$\frac{dQ}{dt} = \frac{(m+5)S_{H_2O}\Delta\theta}{t_2}$$

$$= \frac{(m+5)\ \text{kg}\ (4.2 \times 10^3)\ \text{J/kg} \times 2°\text{C}}{10\ \text{min}} \quad \text{...(ii)}$$

From Eqs. (i) and (ii)

$$\frac{80m}{50} = \frac{2(m+5)}{10}$$

$$7m = 5 \Rightarrow m = \frac{5}{7}\text{kg} \simeq 0.7\ \text{kg}$$

58. (3) Let l_s and l_b be the initial lengths of the steel and brass rod, respectively, and l'_s and l'_b be the corresponding lengths at any other temperature, then,

$$l_s - l_b = l'_s - l'_b = 5\ \text{cm} \quad \text{...(i)}$$

or, $l_s - l_b = l_s(1 + \alpha_s\Delta T) - l_b(1 + \alpha_b\Delta T)$

$\Rightarrow$ $l_s\alpha_s = l_b\alpha_b$

or, $\frac{l_s}{l_b} = \frac{\alpha_b}{\alpha_s} = \frac{18 \times 10^{-6}/°\text{C}}{12 \times 10^{-6}/°\text{C}} = \frac{3}{2}$ ...(ii)

Solving Eqs. (i) and (ii), we get, $l_b = 10$ cm.

59. (1) The increase in volumes of the two liquids due to an increase in temperature ΔT will be

$$\Delta V_1 = V_1\gamma_1\Delta T$$

and $\Delta V_2 = V_2\gamma_2\Delta T$

$\therefore$ Total volume expansion $\Delta V_1 + \Delta V_2 = (V_1\gamma_1 + V_2\gamma_2)\Delta T$

The average increase in volume per unit volume per degree rise in temperature will be

$$\frac{(\Delta V_1 + \Delta V_2)}{(V_1 + V_2)\Delta T}$$

$$\gamma = \frac{V_1\gamma_1 + V_2\gamma_2}{V_1 + V_2} \quad \text{...(i)}$$

But, since the cubical expansion of the two liquids compensates that of the container, γ for the container will be given by Eq. (i).

$\therefore$ $\gamma = \frac{200 \times 1.4 \times 10^{-5} + 500 \times 2.1 \times 10^{-5}}{200 + 50} = 1.9 \times 10^{-5}/°\text{C}$

60. (4) Let V be the volume of the liquid and ΔT the rise in temperature. Since apparent expansion = true expansion – expansion of vessel

$\therefore$ $V\gamma_1\Delta T = V\gamma\Delta T - V(3\alpha_1)\Delta T$ ...(i)

or, $\gamma_1 = \gamma - 3\alpha_1$

(for vessel A)

and $\gamma_2 = \gamma - 3\alpha_2$

...(ii)
(for vessel B)

where γ is the coefficient of real expansion of the liquid.

Subtracting Eq. (ii) from Eq. (i),

$$\gamma_1 - \gamma_2 = 3(\alpha_2 - \alpha_1)$$

or, $(\alpha_2 - \alpha_1) = (\gamma_1 - \gamma_2)/3$

or, $\alpha_2 = [(\gamma_1 - \gamma_2)/3] + \alpha_1$

Hence, correct option is (4).

(4) Let V be the volume of the glass vessel. Then volume of mercury will be $(3/5)V$.

Expansion of mercury = $(3/5)\ V \times \gamma_m \Delta T$

and that of glass = $V\gamma_g \Delta T$

Net (effective or apparent) expansion of mercury

$$\Delta V = V\left(\frac{3\gamma_m}{5} - \gamma_g\right)\Delta T$$

∴ Coefficient of apparent expansion will be

$$\frac{\Delta V}{(3/5)V \times \Delta T} = \left(\frac{3}{5}\gamma_m - \gamma_g\right)\frac{5}{3}$$

(4) Given $\frac{dR}{R} \times 100 = \frac{(507.5 - 500.0)}{500.0} \times 100 = 1.5\%$

Now, $K = \frac{1}{2}I\omega^2 = \frac{1}{2I}(I\omega)^2 = \frac{L^2}{2I}$

or, $K = \frac{L^2}{2(MR^2/2)} = \frac{L^2}{M}(R^{-2})$

Taking log and differentiating, $\frac{dK}{K} = -\frac{2dR}{R}$

or, % change in $K = \frac{dK \times 100}{K} = -\frac{2dR}{R} \times 100 = -3\%$

(1) $\frac{V'}{V} = \frac{\rho}{\sigma}$ = fraction of volume of sphere submerged = η (say)

To find % change in η, i.e., $\frac{\eta' - \eta}{\eta} \times 100$

or, $\left(\frac{\eta'}{\eta} - 1\right) \times 100 = \left[\frac{(\rho/\sigma)'}{(\rho/\sigma)} - 1\right] \times 100$

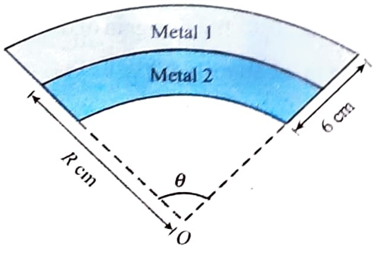
$$= \left[\frac{(1 + \gamma_m \Delta T)}{1 + \gamma_P \Delta T} - 1\right] \times 100 \simeq (\gamma_m - \gamma_p)\Delta T \times 100$$

$$= (182 - 27) \times 10^{-6} \times 100 = 1.24\%$$

(4) The slope of lines are

$$\left(\frac{\Delta L}{\Delta T}\right)_1 = L\alpha_1 = \frac{4 - 0}{4 - 0} = 1 \qquad \text{(For the first metal)}$$

and $\left(\frac{\Delta L}{\Delta T}\right)_2 = L\alpha_2 = \frac{2 - 0}{4 - 0} = \frac{1}{2}$ (For the second metal)



$$L_0(1 + \alpha_1 T) = (R + 1.5)\ \theta$$

and $L_0(1 + \alpha_2 T) = (R - 1.5)\ \theta$

On dividing, we get $\frac{1 + \alpha_1 T}{1 + \alpha_2 T} = \frac{R + 1.5}{R - 1.5}$

or, $(\alpha_1 - \alpha_2)T \simeq \frac{3}{R}$ and $\alpha_1 - \alpha_2 = \frac{3}{RT}$

From the graph [shown in the given Fig. (b) in the question]

$$\alpha_1 - \alpha_2 = \frac{3}{7500 \times 50} = 8 \times 10^{-6}/°\text{C}$$

But $\alpha_1/\alpha_2 = 2$

∴ $\alpha_1 = 16 \times 10^{-6}/°\text{C}$ and $\alpha_2 = 8 \times 10^{-6}/°\text{C}$

65. (2) Given $\frac{\text{Latent heat of fusion}}{\text{Latent heat of vapourization}} = \frac{\lambda}{1} = \frac{L_f}{L_v}$ (say)

i.e., $L_f = \lambda L_v$

Now, if k is required fraction.

Then, $(1 - k)\ L_f = kL_v$

or, $\frac{(1-k)}{k} = \frac{1}{\lambda}$ or, $\frac{1}{k} = \frac{1}{\lambda} + 1$

or, $k = \frac{\lambda}{\lambda + 1}$

66. (3) Efficiency $= \frac{0.54 \times 746}{500} = 0.80$ or 80%

(0.5 kW = 500 W and 0.54 hp = 0.54 × 746 W)

∴ 80% of the electrical energy is converted to mechanical energy and the rest 20% is converted to heat energy.

∴ $\frac{20}{100} \times 500 = 100$ W of power is converted to heat

∴ Heat produced in 1 h (or 3600 s) = 100 × 3600 = 36×10^4 J

$$= \frac{36 \times 10^4}{4.18}\ \text{cal} = 8.6 \times 10^4\ \text{cal}$$

67. (2) As, $\frac{dL}{L_0} = \alpha(T)\,dT$

$$\int_{L_0}^{L} dL = L_0 \int_{T_0}^{T} \alpha(T)\,dT$$

$$L - L_0 = L_0 \int_{T_0}^{T} \alpha(T)\,dT$$

$$L = L_0\left[1 + \int_{T_0}^{T} \alpha(T)\,dT\right]$$

68. (1) As, $\gamma_1 = \frac{\Delta V_1}{V_1 \Delta T}$ and $\gamma_2 = \frac{\Delta V_2}{V_2 \Delta T}$

⇒ $\Delta V_1 = \gamma_1 V_1 \Delta T$

and $\Delta V_2 = \gamma_2 V_2 \Delta T$

Fraction of volume submerged before temperature is raised is given by $f = \rho_1/\rho_2$.

Fraction of volume submerged after the temperature is raised is given by $f' = \rho_1'/\rho_2'$.

$$\alpha' = \left(\frac{\rho_1}{1 + \gamma_1 \Delta T}\right)\left(\frac{1 + \gamma_2 \Delta T}{\rho_2}\right) = \left(\frac{\rho_1}{\rho_2}\right)\left(\frac{1 + \gamma_2 \Delta T}{1 + \gamma_1 \Delta T}\right)$$

$$\Rightarrow \quad \alpha' = f\left(\frac{1+\gamma_2\Delta T}{1+\gamma_1\Delta T}\right)$$

$$\frac{\alpha'}{f} = \frac{1+\gamma_2\Delta T}{1+\gamma_1\Delta T}$$

69. (3) From the conservation principle of angular momentum

$$L_i = L_f$$

$$I_0\omega_0 = I\omega$$

Here, $I = I_0\,[1 + 2\alpha\,(\theta_2 - \theta_1)]$

$$\therefore \quad \omega = \frac{I_0\omega_0}{I_0[1+2\alpha(\theta_2-\theta_1)]} = \frac{\omega_0}{[1+2\alpha(\theta_2-\theta_1)]}$$

70. (2) The change in natural length $= \Delta l_t = l\alpha t$

The natural length of rod at temperature t°C $= l + l\alpha t$

The decrease in natural length due to developed stress $= \Delta l$

But the length of rod remains constant.

$$\therefore \quad \Delta l_t - \Delta l = 0$$

$$\therefore \quad \Delta l = \Delta l_t = l\,\alpha\, t$$

$$\therefore \quad E = \frac{\text{stress}}{\text{strain}} = \frac{F/A}{\dfrac{-\Delta l}{l+\Delta l_t}}$$

$$\therefore \quad F = -\frac{EA\Delta l}{l+\Delta l_t} = -\frac{EA\Delta l_t}{l+\Delta l_t} = -\frac{EAl\alpha t}{l+l\alpha t} = -\frac{EA\alpha t}{(1+\alpha t)}$$

71. (2) Expansion in mercury $= V_0\,\gamma_m T$

Expansion in glass bulb $= V_0\,(3\alpha)T$.

Apparent expansion in mercury $= V_0\,\gamma_m T - V_0\,(3\alpha)T$

$$\therefore \quad \Delta l = \frac{V_0\gamma_m T - V_0 3\alpha T}{A_0(1+2\alpha_g T)}$$

Length of mercury column in capillary is

$$h = \Delta l = \frac{V_0 T}{A_0}\frac{(\gamma_m - 3\alpha_g)}{(1+2\alpha_g T)}$$

72. (1) $P = W/t$

Total work done by drill machine in 2.5 × 60 s

$= (10 \times 10^3)\,(2.5 \times 60) = 15 \times 10^5$ J

$\therefore$ Energy lost = 50% of 15×10^5 J $= 7.5 \times 10^5$ J

Energy taken by its surroundings, i.e., aluminium block

$\Delta Q = mc\Delta t = 8 \times 10^3 \times 0.91 \times \Delta T$ J

Energy given = Energy taken

$$\Rightarrow \quad 7.5 \times 10^5 = 8 \times 10^3 \times 0.91 \times \Delta T$$

$$\Rightarrow \quad \Delta T = 103°\text{C}$$

73. (1) Thermal capacity of ball $= mc = 10$ cal/°C

Let T be the furnace temperature.

Water eq. of vessel and contents $= mc = 200$ g.

Resultant temperature = 40°C.

According to principle of calorimetry.

Heat lost by hot body = heat gained by cold body.

$$m_1c_1(T - 40) = m_2c_2(\,40 - 10)$$

$$10\,(T - 40) = 200 \times 30$$

$$T = 640°\text{C}$$

74. (1) $m_1 \times 1 \times (50-30) = m_2 \times 1 \times (80-50)$

$m_1 \times 20 = m_2 \times 30$ or $\dfrac{m_1}{m_2} = \dfrac{3}{2}$

Mass of water from tank $A = \dfrac{3}{5} \times 40 = 24$ kg

Mass of water from tank $B = \dfrac{2}{5} \times 40 = 16$ kg

Multiple Correct Answers Type

1. (1), (3), (4)

True value = scale reading $[1 + \alpha(\theta' - \theta)]$

(1) If $\theta' > \theta$, then

Scale reading < true value

(2) $\Delta t = \dfrac{1}{2}\alpha\,\Delta\theta t$

If $\theta' > \theta$ or $\Delta\theta$ is positive, i.e., clock will lose time, i.e., will become slow. Hence, not correct.

(3) With rise in temperature, upthrust decreases. As weight remains same, hence, a floating body sinks a little more.

(4) As upthrust on the body decreases with rise in temperature, hence, weight of a body in a liquid ($= W_0 - Th$) increases.

2. (3), (4)

This situation arises during melting (alternative (3)) or freezing (alternative (4)) of the mass.

3. (2), (3)

Melting involves an increase in potential energy and hence an increase in internal energy.

4. (3), (4)

The rate of flow of heat is given by $\dfrac{Q}{t} = \dfrac{KA(\theta_2 - \theta_1)}{l}$

where K is the thermal conductivity. A is the cross-sectional area and l the length of the rod. Hence, the correct choices are (3) and (4).

5. (3), (4)

Thermal expansion is like photographic enlargement.

6. (1), (3), (4)

$$\frac{\Delta A}{A} \times 100 = 2\left(\frac{\Delta l}{A}\right) \times 100$$

% increase in area $= 2 \times 0.2 = 0.4$

$$\frac{\Delta V}{V} \times 100 = 3 \times 0.2 = 0.6\%$$

Since $\Delta l = l\,\alpha\,\Delta T$

$$\frac{\Delta l}{l} \times 100 = \alpha\ \Delta T \times 100 = 0.2$$

$$\alpha = 0.25 \times 10^{-4}/\ \text{C}$$

7. (1), (4)

$$\Delta V_L = \Delta V_V$$

$$\gamma_L V_L = \gamma_V V_V \quad \text{or} \quad \frac{\gamma_L}{\gamma_V} = \frac{V_V}{V_L}$$

$$V_V > V_L \quad \Rightarrow \gamma_L > \gamma_V$$

8. (2), (3)

$$T = 2\pi\sqrt{\frac{l}{g}} = 2\pi\sqrt{\frac{l_0 + \alpha l_0\Delta\theta_0}{g}} = T_0\left(1+\frac{1}{2}\alpha\Delta\theta\right)$$

At 30°C, fraction loss of time $= \dfrac{T_{30°} - T_{20°}}{T_{20°}} = 5\alpha = 5 \times 19 \times 10^{-6}$

Time lost in 24 hours $= 86400 \times 95 \times 10^{-6} = 8.2$ s

On a cold day at 10°C, fraction gain of time $= \dfrac{T_{10°} - T_{20°}}{T_{20°}} = -5\alpha$

Time gained in 24 hours = 8.2 s

9. (1), (2)

Let M and m be masses of water and ice, initially at temperatures of 40°C and −40°C, respectively.

To attain a temperature of 0°C, the heat lost by the water would be 4 units.

$$(Mg)(1\text{ cal/g-°C})(40-0°\text{C}) = 4\text{ units}$$

or $\quad M\text{ cal} = \dfrac{1}{10}\text{unit} \qquad \ldots\text{(i)}$

Similarly, to attain a temperature of 0°C, the heat gained by ice would be 1 unit.

$$(mg)\left(\frac{1}{2}\text{ cal/g°C}\right)(0+40°\text{C}) = 1\text{ unit}$$

or $\quad m\text{ cal} = \dfrac{1}{20}\text{unit} \qquad \ldots\text{(ii)}$

From (i) and (ii), $\dfrac{M}{m} = 2$

Heat required for the complete ice to melt at 0°C will be (mg) (80 cal/g) = 80 m cal = 4 unit.

By the time the temperature of the entire water had dropped to 0°C, the amount of heat ejected would be 4 unit, out of which 1 unit would be consumed by the ice to get heated up from –40°C to 0°C and the remaining heat of 3 units, would be consumed to melt.

Since the total mass of ice require a heat of 4 unit to melt completely, a heat of 3 units will be able to melt only (3/4)th of the ice.

10. (2), (3)

The horizontal parts of the curve, where the system absorbs heat at constant temperature must depict changes of state. Here the latent heats are proportional to lengths of the horizontal parts. In the sloping parts, specific heat capacity is inversely proportional to the slopes.

11. (1), (4)

During phase change process, temperature of the system remains constant. During the process AB temperature of the system is 0°C. Hence, it represents phase change that is transformation of ice into water while temperature remains 0°C.

BC represents rise in temperature of water from 0°C to 100°C (at C). Now, water starts converting into steam which is represented by CD.

12. (2), (4)

According to question

$T_x = T_y \qquad$ ($\because$ x and y are in thermal equilibrium)

$T_x \neq T_z \qquad$ ($\because$ x is not in thermal equilibrium with z)

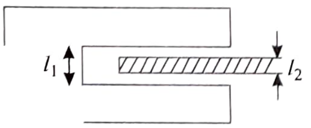
Clearly, $T_y \neq T_z$

Hence, y and z are not in thermal equilibrium, (4) Given, $T_x \neq T_y$ and $T_x \neq T_z$

We cannot say about equilibrium of Y and Z, they may or may not be in equilibrium.

13. (1), (2)

Change in length l_1 due to increase in temperature, $\Delta l_1 = l_1\alpha\,\Delta\theta$



Change in length l_2 is $\Delta l_2 = l_2\alpha\,\Delta\theta$

Since $l_1 > l_2 \quad \therefore \quad \Delta l_1 = \Delta l_2$

Gap indicated by x will increase.

It is trivial to understand that y will decrease.

14. (1), (3)

The COM will not move if the height of liquid column in the container does not change.

Let original volume of liquid, area of cross section of the container and height of liquid be V_0, A_0 and H_0 respectively.

$$H_0 = \frac{V_0}{A_0}$$

If temperature increases by $\Delta\theta$

$V = V_0[1+\gamma_0\Delta\theta]$ and $A = A_0[1+2\alpha_g\Delta\theta]$

$\therefore$ Height of liquid $H = \dfrac{V_0[1+\gamma_0\Delta\theta]}{A_0[1+2\alpha_g\Delta\theta]} = \dfrac{H_0[1+\gamma_0\Delta\theta]}{[1+2\alpha_g\Delta\theta]}$

If $\quad H = H_0$ then $1+\gamma_0\Delta\theta = 1+2\alpha_g\Delta\theta$

$\Rightarrow \quad \gamma_0 = 2\alpha_g$

Let original value of the liquid and the container be V_{l0} and V_{C0}.

At increased temperature

$$V_l = V_{l0}[1+\gamma_0\Delta\theta] \text{ and } V_C = V_{C0}[1+3\alpha_g\Delta\theta]$$

$$\frac{V_l}{V_C} = \frac{V_{l0}[1+\gamma_0\Delta\theta]}{V_{C0}[1+3\alpha_g\Delta\theta]}$$

For $\dfrac{V_l}{V_C} = \dfrac{V_{l0}}{V_{C0}}$ we must have

$\Rightarrow \quad 1+\gamma_0\Delta\theta = 1+3\alpha_g\Delta\theta$

$\gamma_0 = 3\alpha_g$

15. (2), (3)

As it is given that $L_1\alpha_1 = L_2\alpha_2$ are the expansion in both the rods at same difference of temperature will be equal hence their difference in lengths will also remain equal.

16. (1), (4)

If $\alpha_b < \alpha_p$ then pipe will expand more than that of bolt and a tensile stress will be developed in the bolt and if $\alpha_b > \alpha_p$ then bolt will expand more than that of pipe and will remain loose so no stress will be developed in it.

17. (1), (3), (4)

Length $l = l_0(1+\alpha\Delta T) = l_0(1+20\alpha)$

Area $A = A_0(1+\beta\Delta T) = 6l_0^2(1+40\alpha)$

Volume $V = V_0(1+\gamma\Delta T) = l_0^3(1+3\alpha\Delta T) = l_0^3(1+60\alpha)$

Density $\rho = \dfrac{\rho_0}{1+\gamma\Delta T} = \dfrac{\rho_0}{1+60\alpha}$

Linked Comprehension Type

For Problems 1–3

1. (2) In 7 min, temperature of 100 g of water is raised by (100°C – 16°C) = 84°C. The amount of the heat provided by heater

$$Q_W = C_W m_W \Delta T = (1\text{ cal/g°C})(100\text{ g})(84°\text{C})$$
$$= 8.4\times10^3\text{cal} = (8.4\times10^3\times4.186)\text{J}$$
$$\simeq 3.5\times10^4\text{ J}$$

$$\text{Power of heater } = \frac{Q_W}{t_1} = \frac{8.4\times10^3\text{cal}}{(7\times60)\text{s}} = 20\text{cal/s}$$
$$= (20\times4.18)\text{ J/s} = 83.6\text{ W} \approx 84\text{ W}$$

2. (2) With 200 g of alcohol in the vessel, the temperature rises from 16°C to 78°C in 6 min 12 s.

Quantity of heat absorbed by alcohol

$$Q_a = C_a m_a(\theta_f - \theta_i) = C_a\times(200\text{g})(78-16)\text{ C}$$

Heat given by heater $Q_a = Pt_2 = (20\text{ cal/s})\times372\text{ s}$

Specific heat of alcohol $C_a = \dfrac{20\times372}{200\times62} = 0.6\text{ cal/g°C}$

$$= \frac{0.6\times4.186\text{J}}{10^{-3}\text{kg C}} = 2.5\times10^3\text{ J/kg C} = 2.5\text{ K/g°C}$$

3. (2) Heat required to vapourize it at 78°C

$$Q_v = Pt_3 = (20\,\text{cal})(306\,\text{s})$$

If L is the heat of vapourization of alcohol,

$$Q_v = m_3 L$$

$$L = \frac{(20 \times 306)\,\text{cal}}{30\,\text{g}} = 204\,\text{cal/g} = 854\ \text{J/g}$$

$$= 854 \times 10^3\ \text{J/kg}$$

For Problems 4–6

4. (3) $\Delta V = \gamma V \Delta T = 3\alpha V \Delta T = 3 \times 1 \times 10^{-3} \times \left(\frac{5\ \text{kg}}{50\ \text{kg/m}^3}\right) \times 50$
$= 15 \times 10^{-3}\ \text{m}^3$

$$W = P\Delta V = \left(1 \times 10^5\ \frac{\text{N}}{\text{m}^2}\right) \times (15 \times 10^{-3}\ \text{m}^3) = 15 \times 10^2 = 1500\ \text{J}$$

5. (2) $Q = mC\Delta T = (5\ \text{kg}) \times (200\ \text{J/kg°C}) \times (50°\text{C}) = 50000\ \text{J}$

6. (2) $\Delta U = Q - W = 50000 - 1500 = 48500\ \text{J}$

For Problems 7–9

7. (3) $l_1 = l_0(1 + \alpha t)$

$$\frac{2\pi \times 6}{2} = l_0(1 + \alpha t)$$

$$\frac{2\pi \times 5.98}{2} = l_0(1 + 20\alpha)$$

$$\frac{6}{5.98} = \frac{1 + \alpha t}{1 + 20\alpha} = \frac{(1 + 17 \times 10^{-6} t)}{1 + 20 \times 17 \times 10^{-6}}$$

$$\therefore \quad t = 216.8°\text{C} \approx 217°\text{C}$$

8. (3) Tensile stress $= Y\alpha\Delta\theta$

$$= 11 \times 10^{10} \times 17 \times 10^{-6} \times (217 - 20) = 3.68 \times 10^{+8}\ \text{N/m}^2$$

9. (3) $\alpha L \Delta T + L\frac{F/A}{Y} = 0$

$$\Delta T = -\frac{(F/A)}{\alpha Y} = -\frac{230 \times 10^6\ \text{N/m}^2}{(17 \times 10^{-6}\ \text{K}^{-1})(110 \times 10^9\ \text{N/m}^2)}$$

$$= -123\ \text{K} = -123\ \text{C}$$

Final temperature at which the copper collar breaks

$$T_f = T_i + \Delta T = 217°\text{C} - 123°\text{C} = 94°\text{C}$$

For Problems 10–12

10. (1) In 40 min temperature of water has come down by 40°C.

Therefore rate $P = \frac{mS\Delta T}{t} = \frac{0.60 \times 4200 \times 40}{40 \times 60} = 42\ \text{W}$

11. (3) Sample of ice has been receiving heat at constant rate P from water. Its temperature has increased by 30°C in time 60 min.

Therefore $\frac{m_i s_i \Delta T_i}{P} = 60$ min.

$$\Rightarrow \quad m = \frac{(60 \times 60s) \times (42\,W)}{(2100\ \text{J/kg}) \cdot (30°\text{C})} = 2.4\ \text{kg}$$

12. (2) Thermal equilibrium reaches after 60 min. Ice conversion takes place for 20 min. During this time water at 0°C continues to give heat at rate P.

$$m \times L_f = P \times (20 \times 60\text{s})$$

$$\Rightarrow \quad m = \frac{42 \times 20 \times 60}{3.3 \times 10^5} = 0.15\ \text{kg}$$

For Problems 13–15

13. (2) $V = V_0(1 + \gamma_L \Delta T) = (10)(100)\left[1 + 5 \times 10^{-5} \times 20\right]$

$$= 1000(1 + 0.001) = 1001\ \text{cm}^3 = 1001\ \text{cc}$$

14. (2) Cross sectional area of vessel at 40°C

$$A = A_0(1 + 2\alpha_g \Delta T) = 100(1 + 2 \times 10^{-5} \times 20) = 100.04\ \text{cm}^2$$

Actual height of liquid

$$= \frac{\text{Actual volume of liquid}}{\text{cross} - \text{sectional area of vessel}}$$

$$= \frac{1001}{100.04} = (1001)(100 + 0.04)^{-1}$$

$$= \left(\frac{1001}{100}\right)\left(1 + \frac{0.04}{100}\right)^{-1} = \frac{(1001)}{100}\left(1 - \frac{0.04}{100}\right) = \frac{1}{100}$$

$$(1001 - 0.4) = 10.006\ \text{cm}$$

15. (2) $\because\ TV = SR(1 + \alpha_g \Delta T) = (TV)(1 - \alpha_g \Delta T)$

$$\therefore\ SR = (TV)(1 + \alpha g \Delta T)^{-1} = (10.006)(1 - 10^{-5} \times 20)$$

$$= 10.006 - 0.002 = 10.004\ \text{cm}$$

Matrix Match Type

1. i. → c.; ii. → a.; iii. → d.; iv. → b.

Let Q be the heat required to convert 100 g of water at 20°C to 100°C

Then $\quad mc\Delta\theta = (100)\,(1)\,(100 - 20)$

$$Q = 8000\ \text{cal}$$

Now suppose m_0 mass of steam converts into water to liberate this much amount of heat. Then

$$m_0 = \frac{Q}{L} = \frac{8000\ \text{cal}}{540\ \text{cal/g}} = 14.8\ \text{g}$$

Since it is less than m = 20 g, the temperature of the mixture is 100°C.

Mass of steam in the mixture = (20 – 14.8) = 5.2 g

Mass of water in the mixture = (100 + 14.8) = 114.8 g

If m = 10 g, the amount of heat liberated by steam

$= mL = 10 \times 540 = 5400$

Let θ be the final temperature of the mixture.

$$m_{H_2O} s_{H_2O}(\theta - 20) = m_{\text{steam}} L + m s_{H_2O}(100 - \theta)$$

$$100 \times 1(\theta - 20) = 10 \times 540 + 10 \times 1(100 - \theta)$$

$$110\theta = 5400 + 1000 + 2000$$

$$\theta = 76.4°\text{C}$$

2. i. → b.; ii. → a.; iii. → d.

Fraction of volume submerged $f = \frac{V_i}{V} = \frac{\rho_1}{\rho_2}$

After increasing the temperature

$$f' = \frac{V_i'}{V'} = \frac{\rho_1(1 - \gamma_1 \Delta T)}{\rho_2(1 - \gamma_2 \Delta T)} > f \text{ (because } \gamma_2 > \gamma_1)$$

If $\gamma_2 = \gamma_1$, then $f' = f$

If $\gamma_2 < \gamma_1$, then $f' < f$; it means fraction of volume submerged decreases and solid lifts up.

3. i. → a.; ii. → c.; iii. → d.; iv. → b.

(i) Buoyant force = Mg = constant = $V_{\text{sub}} = \frac{Mg}{\rho_l g}$.
volume of displace fluid = constant
∴ density of fluid must be constant.

(ii) $(V_{\text{solid}} - V_{\text{sub}}) = \text{constant}$

$$\Rightarrow \quad \left(V_{\text{solid}} - \frac{M_{\text{solid}}}{\rho_{\text{liquid}}}\right) = \text{constant}$$

$$V \times 3\alpha \times \Delta T = \frac{M\gamma\Delta T}{d} \Rightarrow \gamma = 3\alpha\frac{d}{\rho}$$

(iii) $Ah_{\text{in}}\, d_{\text{liquid}} = A(h_{\text{in}} + h_{\text{out}})\, \rho_{\text{solid}} = M$ (mass of solid)

$$h_{\text{out}} = -\frac{M}{A\rho_{\text{solid}}}$$

$$h_{\text{in}} = \frac{M}{A\rho_{\text{solid}}} - \frac{M}{Ad_{\text{liquid}}} = \text{constant}$$

$$\frac{M(1+3\alpha\Delta T)}{A(1+2\alpha\Delta T)\rho} - \frac{M(1+\gamma\Delta T)}{A(1+2\alpha\Delta T)d} = \frac{M}{A\rho} - \frac{M}{Ad}$$

$$\gamma = 2\alpha + \alpha\frac{d}{\rho}$$

(iv) $Ah_{\text{in}}\, d_{\text{liquid}}\, g$ = Buoyant force = constant = Mg

$$A_0(1+2\alpha\,\Delta T)h_{\text{in}}\frac{d}{1+\gamma\Delta T} = \text{constant}$$

$$h_{in} = \frac{M}{A_0 d}(1+(\gamma-2\alpha)\Delta T)\ \gamma = 2\alpha$$

4. **i. → e.; ii. → c.; iii. → b.; iv. → e.**

When A and B are mixed

$$ms(T-20) = (2m)s(40-T) \Rightarrow T = \frac{100}{3} = 33.3°\text{C}$$

When A and C are mixed $ms(T-20) = (3m)s(60-T) \Rightarrow T = 50°\text{C}$

When B and C are mixed $(2m)s(T-40) = (3m)s(60-T) \Rightarrow T = 52°\text{C}$

When A, B and C are mixed

$$ms(T-20) + (2m)s(T-40) = (3m)s(60-T)$$

$$\Rightarrow \quad T = 46.67°\text{C}$$

Numerical Value Type

1. **(8)** Suppose m gram ice melts, then heat required for its melting $= mL = m \times 80$ cal

Heat available with steam for being condensed and then brought to 0°C $= 1 \times 540 + 1 \times 1 \times (100-0) = 640$ cal

Now, Heat lost = Heat taken

$$\Rightarrow \quad 604 = m \times 80 \quad \Rightarrow m = 8 \text{ g}$$

2. **(6)** Energy with 5 kg of H_2O at 20°C to become water at 0°C,

$$E_1 = 5000 \times 1 \times 20 = 100000 \text{ cal}$$

Energy to raise the temperature of 2 kg ice from −20°C to 0°C,

$$E_2 = 2000 \times 0.5 \times 20 = 20000 \text{ cal}$$

$(E_1 - E_2) = 80000$ cal is available to melt ice at 0°C.

So only 1000 g or 1 kg of ice would have melt.

So, the amount of water available 1 + 5 = 6 kg

3. **(3)** Energy released by water from 25°C to 0°C

$$= 2500 \times 1 \times 25 = 62500 \text{ cal}$$

Energy to bring ice to 0°C $= 2000 \times \frac{1}{2} \times 15 = 15000$ cal

Energy used to melt ice of m gram $= m80$ cal

$$\therefore \quad \text{Ice melt } m = \left(\frac{62500 - 15000}{80}\right) = 593.75 \text{ g}$$

So, mass of water = (2500 + 593.75) g = 3093.75 g ≃ 3 kg

4. **(8)** Let power lost to surrounding be Q.

$$16 - Q = \left(\frac{dm}{dt}\right)S(10)$$

and $\quad 32 - Q = 3\left[\left(\frac{dm}{dt}\right)S(10)\right]$

$$\Rightarrow \quad \frac{32-Q}{16-Q} = 3 \quad \Rightarrow Q = 8 \text{ W}$$

5. **(2)** Loss or gain per day $= dT = \frac{1}{2}\alpha dt \times 86400$
Since $T = 86400$ s for each day

At 15°C, $5 = \frac{1}{2}\alpha(t-15) \times 86400$

At 30°C, $10 = \frac{1}{2}\alpha(30-t) \times 86400$

$$\therefore \quad \frac{30-t}{t-15} = 2 \Rightarrow 3t = 60°\text{C} \Rightarrow t = 20°\text{C}$$

$$\therefore \quad \alpha = \frac{10}{(t-15)\times 86400} = \frac{10}{5\times 86400} = 0.000023$$

$$= 2.3 \times 10^{-5}/°\text{C} = 2$$

6. **(1)** Because the sliding plug stays in the connecting pipe, the pressure in both the vessels at the level of the pipe must be the same.

$$h_{20}d_{20} = h_{80}d_{80} \Rightarrow \frac{h_{20}}{h_{80}} = \frac{d_{80}}{d_{20}}$$

$$\Rightarrow \quad \frac{d_0(1-80\gamma)}{d_0(1-20\gamma)} = 0.94$$

7. **(4)** $(OR)^2 = (PR)^2 - (PO)^2 = l^2 - \left(\frac{l}{2}\right)^2$

$$\Rightarrow \quad (\text{OR})^2 = [l(1+\alpha_2 t)]^2 - \left[\frac{l}{2}(1+\alpha_1 t)\right]^2$$

$$\Rightarrow \quad l^2 - \frac{l^2}{4} = l^2(1+\alpha_2^2 t^2 + 2\alpha_2 t) - \frac{l^2}{4}(1+\alpha_1^2 t^2 + 2\alpha_1 t)$$

Neglecting $\alpha_2^2 t^2$ and $\alpha_1^2 t^2$, we get

$$0 = l^2(2\alpha_2 t) - \frac{l^2}{4}(2\alpha_1 t) \Rightarrow 2\alpha_2 = \frac{2\alpha_1}{4} \Rightarrow \alpha_1 = 4\alpha_2$$

8. **(0.076)**

It is given that at 10°C, volume of beer is 500 cm^3 and the area of cross section of can is 125 cm^2. Thus height of beer level is

$$h = \frac{500}{125} = 4 \text{ cm}$$

Now at 80°, volume of beer becomes

$$V_{80°\text{C}} = 500\,(1 + 3.2 \times 10^{-4} \times 70) = 511.2 \text{ cm}^3$$

At 80°, area of cross section of can becomes

At $\quad 80°\text{C} = 125[1 + 2\alpha_{\text{Al}} \times 70] = 125[1 + 2 \times 2.3 \times 10^{-5} \times 70]$

$$= 125.402 \text{ cm}^2$$

Thus, new height of beer level at 80°C is

$$h' = \frac{V_{80°\text{C}}}{A_{80°\text{C}}} = \frac{511.2}{125.402} = 4.076 \text{ cm}$$

Thus, rise in level of beer is $\Delta h = h' - h = 4.076 - 4.0 = 0.076$ cm

9. **(500)** If temperature is increased by $\Delta\theta$ then $\Delta l = l\alpha\Delta\theta$

$$\Rightarrow \quad \Delta\theta = \frac{\Delta l}{l\alpha}$$

$$E_1 = (\rho Al)S\Delta\theta = \rho AlS\frac{\Delta l}{l\alpha}$$

When stretched, stress $= Y\frac{\Delta l}{l}$

$$E_2 = \frac{1}{2}\left(Y\frac{\Delta l}{l}\right)\left(\frac{\Delta l}{l}\right) \times Al = \frac{Y(\Delta l)^2 A}{2l}$$

So, $$\frac{E_1}{E_2}=\frac{2\rho Sl}{\alpha(\Delta l)Y}=500$$

10. (1) Increase in length of $dx=\alpha l_0\Delta T=\left(\frac{x}{5000}\right)(dx)(100)=\frac{x}{50}dx$

$\Rightarrow$ Total thermal expansion $=\int_0^1\frac{x}{50}dx=\left(\frac{x^2}{100}\right)_0^1=\frac{1}{100}m=1\text{ cm}$

11. (6) $T=2\pi\sqrt{\frac{l}{g}}\Rightarrow\frac{\Delta T}{T}=\frac{1}{2}\frac{\Delta l}{l}=\frac{1}{2}\alpha\Delta\theta$

$$\Rightarrow\Delta T=\frac{T}{2}(\alpha\Delta\theta)=\left(\frac{10\times8.64\times10^4}{2}\right)$$
$(7\times10^{-7})(20)=6\text{ s}$

12. (50) Supplied heat $=(22)(0.5)(8)+(22)(80)+(22)(1)(16)$
$=88+1760+352=2200\,\text{cal}$

Heat capacity of the body $=\frac{2200\text{ cal}}{44°\text{C}}=50\text{ cal/C}$

Water equivalent of the body

$$=\frac{\text{Heat capacity of the body}}{\text{spcific heat capacity of water}}=\frac{50\text{ cal/°C}}{1\text{ cal/g°C}}=50\text{ g}$$

13. (120)

Stress $=Y(\text{strain})=Y_S(\alpha_b-\alpha_s)\Delta T$
$=(2\times10^{11})(0.6\times10^{-5})(100)=120\times10^6\text{ Nm}^{-2}$

14. (160)

If there is no air resistance, velocity of ball on reaching ground is
$V=\sqrt{2gh}=\sqrt{2\times10\times2000}=200\text{ m/s}$

Kinetic energy of ball is $K=\frac{1}{2}mv^2$

$=\frac{1}{2}\times m\times(200)^2=2\times10^4\times m$ Joule [if mass m is in kg]

If all the kinetic energy is absorbed by the ball as heated due to this temperature of ball is increased by ΔT, we have

$K=ms\Delta T$

or $2\times10^4\times m=m\times125\times\Delta T$

or $\Delta T=\frac{2\times10^4}{125}=160°\text{C}$

15. (60)

$Q=\frac{3}{4}mL$

$2Q=mL+ms(50-20)$

$\Rightarrow\quad\frac{3}{2}mL=mL+30\text{ ms}\quad\Rightarrow\frac{L}{2}=30\text{ s}\quad\Rightarrow\frac{L}{s}=60$

16. (2) Work done by friction, $W=\frac{1}{2}Mv^2=\frac{1}{2}\times13.44\times(10)^2=672\text{ J}$

If m mass of ice will melt, we have $\frac{672}{4.2}=m(80)$

The mass of ice that has melted $m=\frac{672}{80\times4.2}=2.0\text{g}$

17. (12) Heat gained by ice $=mS_{ice}\times10+mS_m\times20+mL_f=105$ m cal.
Heat lost by calorimeter $=10\times1\times20=200$ cal.

Heat lost by liquid $=-50\int_{40}^{20}\left(1+\frac{\theta}{500}\right)d\theta=50\left[\theta+\frac{\theta^2}{1000}\right]_{20}^{40}$

$=50\left[\left(40+\frac{1600}{1000}\right)-\left(20+\frac{400}{1000}\right)\right]$

$=50\times21.2=1060$ cal.

$\therefore\quad 105\,m=1060+200\quad\Rightarrow\quad m=12\text{ g}$

Archives

JEE Main

Single Correct Answer Type

1. (4) From statement I: Given $R=R_0(1+\alpha\Delta t)$
Resistance at 27°C, $100=R_0(1+\alpha\times27)$ ...(i)
Resistance at 227°C, $150=R_0(1+\alpha\times227)$ ...(ii)

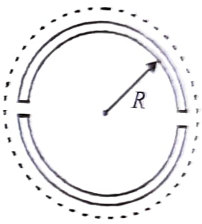
From (i) and (ii), $\frac{100}{150}=\frac{R_0(1+27\alpha)}{R_0(1+227a)}$

Which gives $\alpha=268\times10^{-3}/°\text{C}$

Hence, statement II is correct as the value of α is constant if the change in temperature Δt is small.

2. (2) $\Delta Q=MS\Delta T=100\times10^{-3}\times4184\times20=8.4\times10^3$
$\Delta Q=8.4$ kJ and $\Delta W=0$
$\Delta Q=\Delta U+\Delta W\Rightarrow\Delta U=8.4$ kJ

3. (4) $L\Rightarrow S$



$\Delta L=L\,\alpha\,\Delta T$

$\frac{F}{A}=\frac{\Delta L}{L}Y$

$F=\alpha\Delta T\,YS$

So $T=2F$

$T=2\alpha\Delta T\,YS$

4. (3) Thermal strain in the wire, $\frac{\Delta l}{l}=\alpha\,\Delta T$

Thermal stress in rod is the pressure due to the thermal strain

Thermal stress $=Y\times$ strain $=Y\left(\frac{\Delta l}{l}\right)$

Thermal stress $=\frac{F}{A}=$ pressure $=Y\alpha\,\Delta T$

Given, $\Delta T=100°\,C$, $Y=2\times10^{11}\text{ N m}^{-2}$
$\alpha=1.1\times10^{-5}\text{ K}^{-1}$
Pressure $=2\times10^{11}\times1.1\times10^{-5}\times100=2.2\times10^8$ Pa

5. (1) Let at temperature θ, clock gives correct time

$\Delta T=\left(\frac{1}{2}\alpha\Delta\theta\right)T,\ T=1\text{ day}=86400\text{s}$

$12=\frac{1}{2}\alpha(40-\theta)T$ ...(i)

$4=\frac{1}{2}\alpha(\theta-20)T$ ...(ii)

$\frac{12}{4}=\frac{40-\theta}{\theta-20}\Rightarrow3\theta-60=40-\theta$

Hence, $\theta=25°\text{C}$

From (ii) $4=\frac{1}{2}\alpha(25-20)\times86400$

$\alpha=\frac{8}{5\times86400}=1.85\times10^{-5}\,°\text{C}$

(3) Due to thermal expansion $\frac{\Delta V}{V} = 3\alpha\Delta T$ ...(i)

Volumetric strain due to external pressure $\frac{\Delta V}{V} = \frac{P}{K}$ ...(ii)

Equating both equations we get $3\alpha\Delta T = \frac{P}{K} \Rightarrow \Delta T = \frac{P}{3\alpha K}$

(4) Heat given = Heat taken

$(100)(0.1)(T-75) = (100)(0.1)(45) + (170)(1)(45)$

$10(T-75) = 450 + 7650 = 8100$

$T - 75 = 810$

$T = 885°\text{C}$

E Advanced

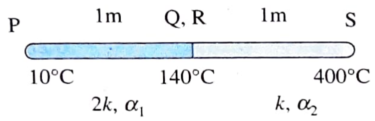
ingle Correct Answer Type

1. (2) $3000 - P = (120 \times 1)(4.2 \times 10^3)\frac{dT}{dt}$

$$\frac{dT}{dt} = \frac{20}{60 \times 60 \times 3}$$

$P = 2067$ W

2. (1)



$dl = dx\alpha_1(\theta - 10)$

$$\Delta\ell = \int d\ell \quad \because \left(\frac{\theta - 10}{x}\right) = \frac{130}{1} \Rightarrow \theta = 10 + 130x$$

$$\Delta\ell = \int_0^1 (130x)\alpha_1 dx \quad \Delta\ell = 130\alpha_1\frac{x^2}{2}$$

$$\Delta\ell = 130 \times 1.2 \times 10^{-5} \times \frac{1}{2} = 78 \times 10^{-5} = 0.78 \text{ mm}$$

Multiple Correct Answers Type

1. **(1), (2), (3), (4)**

Option (1) is correct because the graph between 0–100 K appears to be a straight line upto a reasonable approximation.

Option (2) is correct because area under the curve in the temperature range 0–100 K is less than in range 400–500 K.

Option (3) is correct because the graph of C versus T is constant in the temperature range 400–500 K.

Option (4) is correct because in the temperature range 200–300 K specific heat capacity increases with temperature.

Numerical Value Type

1. **(3)** $\frac{F}{A} = Y\frac{\Delta L}{L} \Rightarrow \frac{mg}{A} = Y(\alpha\Delta\theta)$

$$m = \frac{Ay\alpha(\Delta\theta)}{g} = \frac{\pi r^2 y\alpha(\Delta\theta)}{g}$$

$$= \frac{\pi(10^{-3})^2 \times 10^{11} \times 10^{-5} \times 10}{10} = \pi \approx 3 \text{ kg}$$

Chapter 2

Concept Application Exercises

Exercise 2.1

1. $mL = \dfrac{KA\Delta\theta\, t}{\Delta x}$

$$\Rightarrow \quad 500 \times 80 = \frac{0.0075 \times 75 \times (40-0)t}{5}$$

$$\Rightarrow \quad t = 8.9 \times 10^3 \text{ sec} = 2.47 \text{ hr}$$

2. $\dfrac{dQ}{dt} = \dfrac{KA\Delta\theta}{l}$, For both rods K, A and $\Delta\theta$ are same.

$$\Rightarrow \quad \frac{dQ}{dt} \propto \frac{1}{l}$$

So $\dfrac{(dQ/dt)_{\text{semi circular}}}{(dQ/dt)_{\text{straight}}} = \dfrac{l_{\text{straight}}}{l_{\text{semicircular}}} = \dfrac{2r}{\pi r} = \dfrac{2}{\pi}$

3. Let each side of the cube measure x cm.

As area (A) of each face is 4 cm², we have

$$A = x^2 = 4\text{cm}^2 \text{ or } x = 2 \text{ cm}$$

Further, with usual notation, $(T_1 - T_2) = (100 - 0) = 100°\text{C}$

(T_1= Temperature of steam, T_2 = temperature of melting ice)

$k = 0.2$ cgs unit, $t = 10$ min $= 10 \times 60$ s $= 600$ s

Now, $Q = \dfrac{kA(T_1 - T_2)t}{x}$

$$\therefore \quad Q = \frac{0.2 \times 4 \times 100 \times 600}{2} = 24,000 \text{ cal}$$

If m gram of ice is melted with this heat and L is the latent heat of fusion of ice, then $Q = mL$

or $m = \dfrac{Q}{L} = \dfrac{24,000}{80}$ (as $L = 80$ cal/g)

or $m = 300$ g

4. Area of six faces of the box $= 6l^2 = 6 \times (0.30)^2 = 0.54$ m²

and $L = 5.0$ cm $= 0.05$ m

time, $t = 6$ hr $= 6 \times 3600$ s

$T_1 - T_2 = 45 - 0 = 45°\text{C}$

Total heat entering into the box in 6 hr

$$Q = \frac{KA(T_1 - T_2)}{L} = \frac{0.01 \times 0.54 \times 45 \times 6 \times 3600}{0.05} = 104976 \text{ J}$$

If m be the amount of ice melted, then $Q = mL$

or $m = \dfrac{Q}{L} = \dfrac{104976}{335 \times 10^3} = 0.313$ kg

Mass of the ice left after 6 hr = (4 – 0.313) kg = 3.687 kg

5. Let T_1 be the temperature of the part of the flame in contact with boiler. The amount of heat flows into water in 1 min

$$Q = KA\frac{T_1 - T_2}{L}t = \frac{109 \times 0.15 \times (T_1 - 100) \times 60}{0.01} \text{ J}$$

Mass of the water boiled in 1 min = 6 kg = 6000 g

Heat required to boil the water $Q = mL = 6000 \times 2256$ J

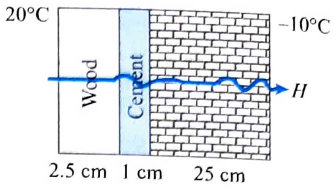
$$\therefore \quad \frac{109 \times 0.15 \times (T_1 - 100) \times 60}{0.01} = 6000 \times 2256$$

or $T_1 - 100 = 138$

or $T_1 = 238°\text{C}$

6. Equivalent thermal conductivity of the wall

$$K = \frac{l_1 + l_2 + l_3}{\dfrac{l_1}{K_1} + \dfrac{l_2}{K_2} + \dfrac{l_3}{K_3}}$$



$$= \frac{0.025 + 0.01 + 0.25}{\left(\dfrac{0.025}{0.125} + \dfrac{0.01}{1.5} + \dfrac{0.25}{1.0}\right)} = \frac{0.285}{0.457} = 0.624 \text{ W/m°C}$$

The rate of flow of heat is given by $H = KA\dfrac{T_1 - T_2}{L}$

$$= 0.624 \times 137 \times \frac{[20-(-10)]}{0.285} = \frac{0.624 \times 137 \times 30}{0.285} = 9000 \text{ W}$$

7. Let θ_1 and θ_2 be the temperatures at the ends of the copper bar. Heat transfer per second through the system is

$$\frac{dH}{dt} = \frac{A(100-0)}{\left(\dfrac{0.001}{K_w} + \dfrac{100}{K_{Cu}} + \dfrac{0.01}{K_w}\right)}$$

Heat transfer per second through copper bar $= \dfrac{K_{Cu}A(\theta_1 - \theta_2)}{100}$

As the rods are in series, heat transfer per second must be same through each part.

$$\frac{A(100-0)}{\left(\dfrac{0.01}{K_W} + \dfrac{100}{K_{Cu}} + \dfrac{0.01}{K_W}\right)} = \frac{K_{Cu}A(\theta_1 - \theta_2)}{100}$$

Putting $K_{Cu} = 1.04$ and $K_w = 0.0014$ we get

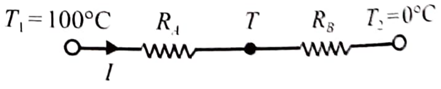
Temperature gradient $= \dfrac{(\theta_1 - \theta_2)}{100} = 0.87°\text{C cm}^{-1}$

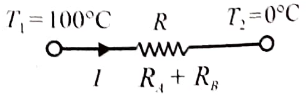
8. (i) Thermal resistance is defined as $R = l/kA$

For rod A $R_A = \dfrac{l}{k_A A} = \dfrac{l}{3kA}$

For rod B $R_B = \dfrac{l}{k_B A} = \dfrac{l}{kA}$

(ii) Using electrical analogy, the given problem may be modified as





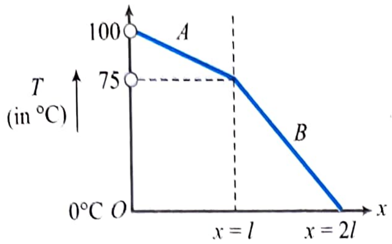
The equivalent resistance is $R = R_A + R_B = \dfrac{4}{3}\dfrac{l}{kA}$

The heat current is given by $I = \dfrac{T_1 - T_2}{R} = \dfrac{100-0}{\dfrac{4}{3}\dfrac{l}{kA}} = 73\left(\dfrac{kA}{l}\right)$

(iii) The temperature of the junction is given by

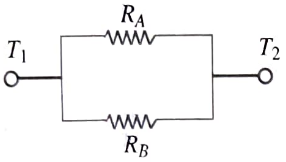
$$T = T_1 - IR_A = 100 - \frac{75kA}{l}\left(\frac{l}{3kA}\right) = 75°\text{C}$$

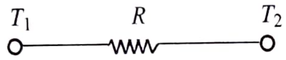
(iv) The variation of temperature along the length of the conductor is shown in the following figure.



9. (i) The thermal resistance of the two rods are

$$R_A = \frac{l}{3kA}; \quad R_B = \frac{l}{kA}$$





The equivalent resistance is $\frac{1}{R} = \frac{1}{R_A} + \frac{1}{R_B} = \frac{3kA}{l} + \frac{kA}{l} = \frac{4kA}{l}$

or $\quad R = \frac{l}{4kA}$

(ii) The heat current through each rod is

$$I_A = \frac{T_1 - T_2}{R_A} = \frac{100-0}{l/3kA} = 300\left(\frac{kA}{l}\right)$$

$$I_B = \frac{T_1 - T_2}{R_B} = \frac{100-0}{l/kA} = 100\left(\frac{kA}{l}\right)$$

Total heat current is $I = I_A + I_B = 400\frac{kA}{l}$

10. Let us find the amount of heat absorbed by ice per minute,

$$A = 1 \times 10^{-4}\ \text{m}^2; \quad \Delta t = 60\ \text{s}; \quad K = 110\ \text{W/m K};$$

$$L = 2\ \text{m}; \quad \Delta T = 100 - 0 = 100°\text{C}$$

Using $\Delta Q = KA\Delta T\Delta t/L$ we have

$$\Delta Q = (110)(1 \times 10^{-4})\frac{(100)(60)}{2} = 33\ \text{J}$$

Now, if m be the mass of ice melting per minute, then, from $\Delta L = mL$,

$$m = \frac{\Delta Q}{L} = \frac{33\ \text{J}}{80 \times 4.2\ \text{J/g}} = 0.098\ \text{g}$$

11. Let the thermal conductivities of the rod AB, BC and BD be K, $2K$, and $3K$ respectively. Also, let the length be $2L$, L and L.

If T be the required temperature of the junction B and assuming $T_1 > T > T_2$ and $T > T_3$, we have

$$\left.\frac{\Delta Q}{\Delta t}\right]_{AB} = \left.\frac{\Delta Q}{\Delta t}\right]_{BC} + \left.\frac{\Delta Q}{\Delta t}\right]_{BD} \quad \text{(Point rule)}$$

$$\frac{KA(T_1 - T)}{2L} = \frac{2KA(T - T_2)}{L} + \frac{3KA(T - T_3)}{L}$$

$$\frac{T_1 - T}{2} = 2(T - T_2) + 3(T - T_3)$$

$$T = \frac{1}{11}(T_1 + 4T_2 + 6T_3)$$

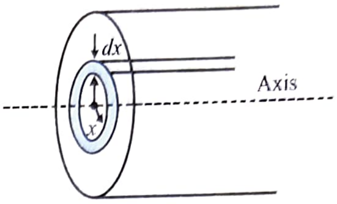
12. Here $A_1 = \pi R^2$; $K_1 = K$

And $A_2 = 4\pi R^2 - \pi R^2 = 3\pi R^2$; $K_2 = 2K$

Using, the expression for the effective thermal conductivity for slabs in parallel,

$$K_{\text{eff}} = \frac{A_1K_1 + A_2K_2}{A_1 + A_2} = \frac{\pi R^2(K) + 3\pi R^2(2K)}{\pi R^2 + 3\pi R^2} = \frac{7K}{4}$$

13. Let us subdivide the entire cylinder into a number of coaxial cylindrical shells of infinitesimally small thickness. Consider one such shell of radius x and thickness dx. Cross-sectional area of the shell $2\pi x dx$.



Using, the expression for the effective thermal conductivity

$$K_{\text{eff}} = \frac{\sum A_iK_i}{\sum A_i}$$

We have $K_{\text{eff}} = \frac{1}{\sum A_i}\int_0^R K(2\pi x dx)$

$$= \frac{1}{\pi R^2}\int_0^R (K_1x + K_2)2\pi x dx = \frac{2}{R^2}\int_0^R (K_1x^2 + K_2x)dx$$

$$= \frac{2}{R^2}\left[K_1\frac{x^3}{3} + K_2\frac{x^2}{2}\right]_0^R = \frac{1}{3}(2K_1R + 3K_2)$$

Exercise 2.2

1. Black bulb absorbs more heat in comparison with painted bulb. So air in black bulb expands more. Hence, the level of alcohol in limb X falls while that in limb Y rises.
2. Rate of cooling (here it is rate of loss of heat).

$$\frac{dQ}{dt} = (mc + W)\frac{d\theta}{dt} = (m_lc_l + m_cc_c)\frac{d\theta}{dt}$$

$$\Rightarrow \quad \frac{dQ}{dt} = (0.5 \times 2400 + 0.2 \times 900)\left(\frac{60-55}{60}\right) = 115\frac{\text{J}}{\text{sec}}.$$

3. According to Newton's law $\frac{\theta_1 - \theta_2}{t} = k\left[\frac{\theta_1 + \theta_2}{2} - \theta_0\right]$

Initially, $\frac{(80-64)}{5} = K\left(\frac{80+64}{2} - \theta_0\right)$

$$\Rightarrow \quad 3.2 = K[72 - \theta_0] \qquad \text{...(i)}$$

Finally, $\frac{(64-52)}{10} = K\left[\frac{64+52}{2} - \theta_0\right]$

$$\Rightarrow \quad 1.2 = K[58 - \theta_0] \qquad \text{...(ii)}$$

On solving equation (i) and (ii), we get $\theta_0 = 94°$C

4. Area under given curve represents emissive power and emissive power $\propto T^4 \Rightarrow A \propto T^4$

$$\Rightarrow \quad \frac{A_2}{A_1} = \frac{T_2^4}{T_1^4} = \frac{(273+327)^4}{(273+27)^4} = \left(\frac{600}{300}\right)^4 = \frac{16}{1}$$

5. The constant k depends on materials, shape, nature of the surface, and air currents. Its dimensions are T^{-1}.
6. If T is the surface temperature of the filament, then $\lambda_m T = b$

or $$T = \frac{b}{\lambda_m} = \frac{0.288}{2.16 \times 10^{-5}} = 13333.3 \text{ K}$$

The temperature of surrounding air, $T_0 = 13 + 273 = 286$ K

The net amount of energy radiated per unit area per second

$$E = \sigma(T^4 - T_0^4)$$

$$= 5.77 \times 10^{-5}\left[(13333.3)^4 - (286)^4\right] = 1.824 \times 10^{12} \text{erg s}^{-1}\text{cm}^{-2}$$

7. According to Stefan's law, the rate of cooling of a body is given by

$$\frac{-dT}{dt} = \frac{e\sigma A}{mc}(T^4 - T_0^4)$$

Here e, σ, m, c, T and T_0 are constants for all the three objects, therefore $\frac{-dT}{dt} \propto A$

Since the surface area of the plate is maximum and that of the sphere is minimum for the given mass, therefore, the circular plate cools fastest, and the sphere cools slowest.

8. From equation, the rate of cooling of a body is given by

$$\frac{-dT}{dt} = \frac{Ae\sigma}{\rho Vs}(T^4 - T_0^4)$$

Since, substance is same for both bodies, so $e/\rho s$ = constant.

Finally, they are allowed to cool under identical conditions, so $(T^4 - T_0^4)$ = constant.

$$\frac{-dT}{dt} \propto \frac{A}{V}$$

Let the edge of the cube or radius of the sphere be a, then, for cube; $A = 6a^2$ and $V = a^3$ so $A/V = 6a$

For the sphere; $A = 4\pi a^2$ and $V = (4/3)\pi a^3$; so $A/V = 3/a$

Evidently, the ratio A/V is more for cube, so, the cube cools at a faster rate.

9. It is given that the temperature of ball is constant. This means that the rate at which it loosing heat by radiation must be equal to the rate at which heat is supplied to this ball externally to keep its temperature constant.

The rate of heat loss by the ball is given as $\frac{dQ}{dt} = eA\sigma(T^4 - T_S^4)$

$$= 0.3 \times 4\pi(0.01)^2 \times 5.67 \times 10^{-8} \times [(1000)^4 - (300)^4] = 21.1 \text{ W}$$

Thus electrical energy must be supplied to the ball at a rate of 21.1 W.

10. If E_1 and E_2 are the maximum energy radiated by the body in a short wavelength interval at temperature T_1 and T_2 then according to Wien's fifth power law, we have

$$\frac{E_1}{E_2} = \left(\frac{T_1}{T_2}\right)^5 \qquad \text{...(i)}$$

According to Wien's displacement law, we have $\lambda_{m_1} T_1 = \lambda_{m_2} T_2$

or $$\frac{T_1}{T_2} = \frac{\lambda_{m_2}}{\lambda_{m_1}} \qquad \text{...(ii)}$$

Here λ_{m_1} and λ_{m_2} are the wavelengths at temperatures T_1 and T_2 at which the energy radiated in a small interval of wavelength is maximum.

Now from Eqs. (i) and (ii) we have $\frac{E_1}{E_2} = \left(\frac{\lambda_{m_2}}{\lambda_{m_1}}\right)^5$

or $$\frac{E_1}{E_2} = \left(\frac{2342 \times 10^{-10}}{4253 \times 10^{-10}}\right)^5 = 0.0506$$

11. According to Wien's displacement law, we have $\lambda_m T$ = constant

or $\lambda_{m_1} T_1 = \lambda_{m_2} T_2$

Thus $20000 \times 10^{-10} \times 1500 = 5500 \times 10^{-10} \times T_2$

or $$T_2 = \frac{200}{55} \times 1500 = 5454.54 \text{ K}$$

Exercises

Single Correct Answer Type

1. (3) The area of cross section is different at different section in the rod, it implies that the temperature gradient cannot be constant in the rod in steady state. In steady state the rate of flow of the heat is given by

$$\left|\frac{dQ}{dt}\right| = kA\left(\frac{dT}{dx}\right) \Rightarrow dT = \frac{1}{KA}\left|\frac{dQ}{dt}\right|dx \text{ or } dT \propto \frac{1}{A}$$

the area of cross section AB is more than that at CD, hence the temperature difference across AB will be less.

2. (2) The wavelength corresponding to maximum emission of radiation from the sun is $\lambda_{\text{max}} = 4753$ Å (close to the wavelength of violet colour of visible region). Hence, if temperature is doubled, λ_m is decreased $\left(\lambda_m \propto \frac{1}{T}\right)$ i.e., mostly ultraviolet radiations are emitted.

3. (2) According to Prevost theory, every body radiates heat at all temperatures, (except 0 K) and also, absorbs heat from surroundings.

Since $T_A > T$, object A emits radiations more than the radiations it absorbs.

Since $T_B < T$, object B, absorbs more radiations than it emits.

After a certain time all bodies attain a common temperature.

4. (2) $\lambda_m T = b$ where $b = 2.89 \times 10^{-3}$ mK

$$\Rightarrow \quad T = \frac{b}{\lambda_m} = \frac{2.89 \times 10^{-3}}{1.5 \times 10^{-6}} \approx 2000 \text{ K}$$

5. (1) According to Wien's displacement law,

$$\lambda_m \propto \frac{1}{T} \Rightarrow \lambda_{m_2} < \lambda_{m_1} \qquad (\because \ T_1 < T_2)$$

Therefore, I–λ graph for T_2 shows lesser wavelength (λ_m) and so, curve for T_2 will shift towards left side.

6. (3) According to Wien's law, $\lambda_m \propto \frac{1}{T} \Rightarrow \nu_m \propto T$. As the temperature of body increases, frequency corresponding to maximum energy in radiation (λ_m) increases which is shown in graph (3).

7. (4) $\frac{A_T}{A_{2000}} = \frac{16}{1}$ (given)

Area under e_λ–λ curve represents the emissive power of body.

Emissive power $\propto T^4$

$\therefore$ Area under e_λ–λ curve $\propto T^4$

$$\Rightarrow \quad \frac{A_T}{A_{2000}} = \left(\frac{T}{2000}\right)^4 \Rightarrow \frac{16}{1} = \left(\frac{T}{2000}\right)^4 \Rightarrow T = 4000 \text{ K}$$

8. (4) $\frac{d\theta}{dt} = \frac{\sigma A}{mc}(T^4 - T_0^4)$. If the liquids are put in exactly similar calorimeters and identical surroundings then we can consider T_0 and A constant and thus, $\frac{d\theta}{dt} \propto \frac{(T^4 - T_0^4)}{mc}$ ...(i)

If we consider that equal masses of liquid (m) are taken at the same temperature then

$$\frac{d\theta}{dt} \propto \frac{1}{c}$$

So for the same rate of cooling, c should be equal, which is not possible because liquids are of different nature.

Again from equation (i), we get

$$\frac{dT}{dt} \propto \frac{(T^4 - T_0^4)}{mc} \Rightarrow \frac{d\theta}{dt} \propto \frac{(T^4 - T_0^4)}{V\rho c}$$

Now, if we consider that equal volumes of liquid (V) are taken at the same temperature then

$$\frac{dT}{dt} \propto \frac{1}{\rho c}$$

So for same rate of cooling, multiplication of $\rho \times c$ for two liquids of different nature can be possible. Therefore, option (4) may be correct.

9. (2) Since the curved surface of the conductor is thermally insulated, in steady state, the rate of flow of heat at every section will be the same. Hence, the curve between H and x will be a straight line parallel to x-axis.

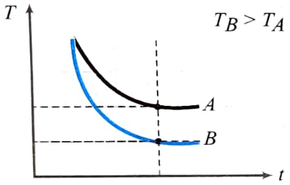
10. (4) Rate of cooling, $R_C = \dfrac{d\theta}{dt} = \dfrac{A\varepsilon\sigma(T^4 - T_0^4)}{mc}$

$$\Rightarrow \quad \frac{d\theta}{dt} \propto \frac{A}{V} \propto \frac{r^2}{r^3} \Rightarrow \frac{d\theta}{dt} \propto \frac{1}{r}$$

11. (2) As we know, Rate of cooling $\propto \dfrac{1}{\text{specific heat(c)}}$

$\therefore \quad c_{\text{oil}} < c_{\text{water}}$

$\Rightarrow \quad (\text{Rate of cooling})_{\text{oil}} > (\text{Rate of cooling})_{\text{water}}$



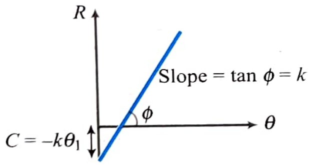
It is clear that at a particular time after cooling begins, temperature of oil will be less than that of water.

So, graph B represents the cooling curve of oil and A represents the cooling curve of water.

12. (3) Rate of loss of heat (R) $\propto$ temperature difference

$\Rightarrow \quad R \propto (\theta - \theta_0) \Rightarrow R = k(\theta - \theta_0) \Rightarrow k\theta - k\theta_0$ (k is constant)

On comparing it with $y = mx + c$, it is observed that the graph between R and θ will be a straight line with slope k and intercept $-k\theta_0$.



13. (3) $\dfrac{d\theta}{dt} = \dfrac{\varepsilon A\sigma}{mc} 4\theta_0^3\, \Delta\theta$

For given sphere and cube, $\dfrac{\varepsilon A\,\sigma}{mc} 4\theta_0^3\, \Delta\theta$ is constant. So, for both rates of fall of temperature, $\dfrac{d\theta}{dt}$ = constant.

14. (2) For θ-t plot, rate of cooling $= \dfrac{d\theta}{dt}$ = slope of the curve.

At P, $\dfrac{d\theta}{dt} = \tan\phi_2 = k(\theta_2 - \theta_0)$, where k is constant.

At Q, $\dfrac{d\theta}{dt} = \tan\phi_1 = k(\theta_1 - \theta_0)$

$$\therefore \quad \frac{\tan\phi_2}{\tan\phi_1} = \frac{\theta_2 - \theta_0}{\theta_1 - \theta_0}$$

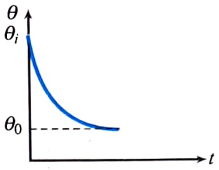
15. (1) Rate of cooling, $\dfrac{\Delta\theta}{t} = \dfrac{A\varepsilon\sigma(T^4 - T_0^4)}{mc}$

As surface area, material and temperature difference are same, so rate of loss of heat is same in both the spheres. Now in this case, rate of cooling depends on mass.

$\Rightarrow \quad$ Rate of cooling, $\dfrac{\Delta\theta}{t} \propto \dfrac{1}{m}$

Since $m_{\text{solid}} > m_{\text{hollow}}$, hollow sphere will cool at faster rate

16. (2) According to Newton's law of cooling,



Rate of cooling $\propto$ Temperature difference

$$\Rightarrow \quad -\frac{d\theta}{dt} \propto (\theta - \theta_0) \Rightarrow -\frac{d\theta}{dt} = \alpha(\theta - \theta_0) \qquad (\alpha = \text{constant})$$

$$\Rightarrow \quad \int_{\theta_i}^{\theta} \frac{d\theta}{(\theta - \theta_0)} = -\alpha \int_0^t dt \Rightarrow \theta = \theta_0 + (\theta_i - \theta_0)\, e^{-\alpha t}$$

This relation tells us that temperature of the body varies exponentially with time from θ_i to θ_0.

Hence graph (2) is correct.

17. (3) $\dfrac{dQ}{dt} = -KA\dfrac{d\theta}{dx}$

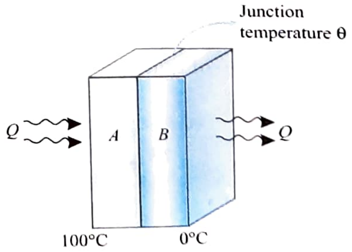
$\because \quad \dfrac{dQ}{dt}$, K and A are constants for all points

$\Rightarrow \quad d\theta \propto -dx$, i.e. temperature will decrease linearly with x.

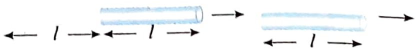
18. (1) It is given that $\dfrac{K_1}{K_2} = \dfrac{1}{3} \Rightarrow K_1 = K$ then $K_2 = 3K$.

The temperature of the junction in contact,

$$\theta = \frac{K_1\theta_1 + K_2\theta_2}{K_1 + K_2} = \frac{1 \times 100 + 3 \times 0}{1 + 3} = \frac{100}{4} = 25°\text{C}$$



19. (2) Let the heat transferred be Q.



When rods are joined end to end heat transferred by each rod,

$$Q = \frac{KA\Delta\theta}{2l} \times 12 \qquad \text{...(i)}$$

When rods are joined lengthwise, $Q = \frac{K2A\Delta\theta}{l}t$ ...(ii)

From Eqs. (i) and (ii) we get $t = 3$ s

20. (3) $\frac{Q}{At} = K\frac{\Delta\theta}{l} \Rightarrow K\frac{\Delta\theta}{l} = \text{constant} \Rightarrow \frac{\Delta\theta}{l} \propto \frac{1}{K}$

Hence, if $X_c > X_m > X_g$, then

$$\left(\frac{\Delta\theta}{l}\right)_c < \left(\frac{\Delta\theta}{l}\right)_m < \left(\frac{\Delta\theta}{l}\right)_g \Rightarrow X_c < X_m < X_g$$

This is because higher K implies lower value of the temperature gradient.

21. (1) $\frac{dQ}{dt} = \frac{K(\pi r^2)d\theta}{dl} \Rightarrow \frac{\left(\frac{dQ}{dt}\right)_s}{\left(\frac{dQ}{dt}\right)_l} = \frac{K_s \times r_s^2 \times l_l}{K_l \times r_l^2 \times l_s} = \frac{1}{2} \times \frac{1}{4} \times \frac{2}{1}$

$$\Rightarrow \left(\frac{dQ}{dt}\right)_s = \frac{\left(\frac{dQ}{dt}\right)_l}{4} = \frac{4}{4} = 1$$

22. (4) $Q = \frac{KA(\Delta\theta)t}{l}$

$\because$ Since Q and $\Delta\theta$ are same for both spheres,

$$K \propto \frac{l}{At} \propto \frac{l}{r^2 t} \Rightarrow \frac{K_{\text{larger}}}{K_{\text{smaller}}} = \frac{l_l}{l_s} \times \left(\frac{r_s}{r_l}\right)^2 \times \frac{t_s}{t_l}$$

It is given that $r_l = 2r_s, l_l = \frac{1}{4}l_s, t_1 = 25$ min and $t_s = 16$ min.

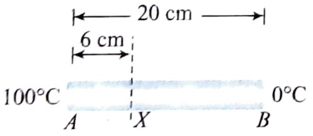
$$\Rightarrow \frac{k_{\text{larger}}}{k_{\text{smaller}}} = \left(\frac{1}{4}\right)\left(\frac{1}{2}\right)^2 \times \frac{16}{25} = \frac{1}{25}$$

23. (3) Let be the temperature at middle point C and in series, rate of heat flow be the same.

$\Rightarrow K(2A)(100 - \theta) = KA(\theta - 70)$

$\Rightarrow 200 - 2\theta = \theta - 70 \Rightarrow 3\theta = 270 \Rightarrow \theta = 90°\text{C}$

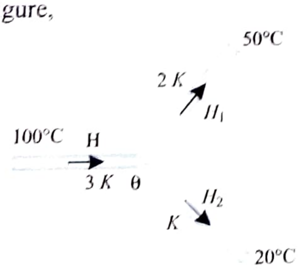
24. (2) In steady state, temperature gradient = constant



$$\Rightarrow \frac{(\theta_A - \theta_x)}{6} = \frac{(\theta_A - \theta_B)}{20} \Rightarrow (100 - \theta_x) = \frac{6}{20} \times (100 - 0)$$

$\Rightarrow \theta_x = 70°\text{C}$

25. (2) Let the temperature of junction be θ. Then according to following figure,

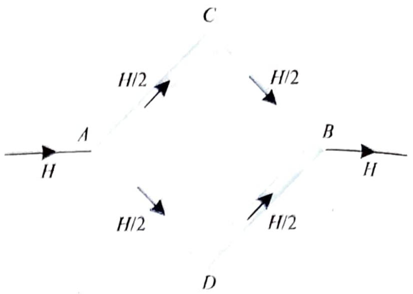


$H = H_1 + H_2$

$$\Rightarrow \frac{3K \times A \times (100 - \theta)}{l} = \frac{2KA(\theta - 50)}{l} + \frac{KA(\theta - 20)}{l}$$

$\Rightarrow 300 - 3\theta = 3\theta - 120 \Rightarrow \theta = 70°\text{C}$

26. (1) Suppose that temperature difference between A and B is 100°C and $\theta_A > \theta_B$.

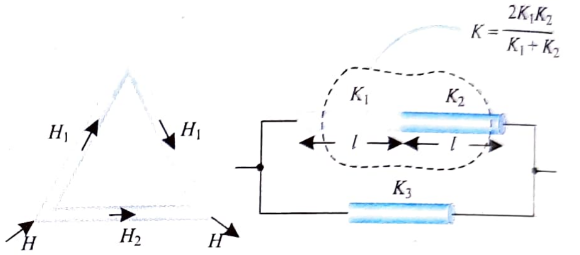


Heat current will flow from A to B via path ACB and ADB. Since all the rods are identical, $(\Delta\theta)_{AC} = (\Delta\theta)_{AD}$.

(Because heat current $H = \frac{\Delta\theta}{R}$; here R = same for all)

$\Rightarrow \theta_A - \theta_C = \theta_A - \theta_D \Rightarrow \theta_C = \theta_D$

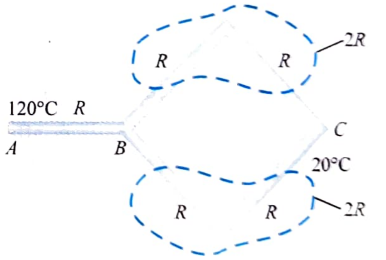
27. (3) The given arrangement of rods can be redrawn as follows:

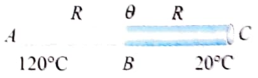


It is given that $H_1 = H_2$

$$\Rightarrow \frac{KA(\theta_1 - \theta_2)}{2l} = \frac{K_3A(\theta_1 - \theta_2)}{l} \Rightarrow K_3 = \frac{K}{2} = \frac{K_1K_2}{K_1 + K_2}$$

28. (3) If thermal resistance of each rod is considered R, then the given combination can be redrawn as follows:





(Heat current)$_{AC}$ = (Heat)

$$\frac{(120 - 20)}{R} = \frac{(120 - \theta)}{R} \Rightarrow \theta = 70°\text{C}$$

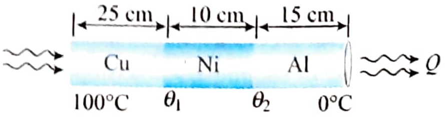
29. (2) Let us suppose that $K_{\text{Ni}} = K \Rightarrow K_{\text{Al}} = 3\,K$ and $K_{\text{Cu}} = 6\,K$

Since all metal bars are connected in series,

$$\left(\frac{Q}{t}\right)_{\text{Combination}} = \left(\frac{Q}{t}\right)_{\text{Cu}} = \left(\frac{Q}{t}\right)_{\text{Al}} = \left(\frac{Q}{t}\right)_{\text{Ni}}$$

and $\frac{3}{K_{\text{eq}}} = \frac{1}{K_{\text{Cu}}} + \frac{1}{K_{\text{Al}}} + \frac{1}{K_{\text{Ni}}} = \frac{1}{6K} + \frac{1}{3K} + \frac{1}{K} = \frac{9}{6K}$

$\Rightarrow K_{\text{eq}} = 2\,K$



Hence, if $\left(\frac{Q}{t}\right)_{\text{Combination}} = \left(\frac{Q}{t}\right)_{\text{Cu}}$

$$\Rightarrow \frac{K_{eq}A(100-0)}{l_{\text{Combination}}} = \frac{K_{Cu}A(100-\theta_1)}{l_{Cu}}$$

$$\Rightarrow \frac{2K\,A(100-0)}{(25+10+15)} = \frac{6K\,A(100-\theta_1)}{25} \Rightarrow \theta_1 = 83.33°\text{C}$$

Similarly, $\left(\frac{Q}{t}\right)_{\text{Combination}} = \left(\frac{Q}{t}\right)_{Al}$

$$\Rightarrow \frac{2K\,A(100-0)}{50} = \frac{3K\,A(\theta_2-0)}{15} \Rightarrow \theta_2 = 20°\text{C}$$

(1) Heat developed by the heater, $H = \frac{V^2}{R}\cdot\frac{t}{J} = \frac{(200)^2\times t}{20\times 4.2}$

Heat conducted by the glass, $H = \frac{0.2\times 1\times(20-\theta)t}{0.002}$

Hence, $\frac{(200)^2\times t}{20\times 4.2} = \frac{0.2\times(20-\theta)t}{0.002} \Rightarrow \theta = 15.24°\text{C}$

1. (2) Rate of cooling of a body, $R = \frac{\Delta\theta}{t} = \frac{A\varepsilon\sigma(T^4-T_0^4)}{mc}$

$\Rightarrow \quad R \propto \frac{A}{m} \propto \frac{\text{Area}}{\text{Volume}}$

$\Rightarrow$ For the same surface area, $R \propto \frac{1}{\text{Volume}}$

$\because$ Volume of cube < Volume of sphere

$\Rightarrow \quad R_{\text{Cube}} > R_{\text{Sphere}}$

i.e., cube cools down with faster rate

32. (2) Energy received per second i.e., power, $P \propto (T^4 - T_0^4)$

$\Rightarrow \quad P \propto T^4 \qquad (\because T_0 << T)$

Also energy received per sec, $P \propto \frac{1}{d^2}$ (inverse square law)

$$\Rightarrow \quad P \propto \frac{T^4}{d^2} \Rightarrow \frac{P_1}{P_2} = \left(\frac{T_1}{T_2}\right)^4 \times \left(\frac{d_2}{d_1}\right)^2$$

$$\Rightarrow \quad \frac{P}{P_2} = \left(\frac{T}{2T}\right)^2 \times \left(\frac{2d}{d}\right)^2 = \frac{1}{4} \Rightarrow P_2 = 4P$$

33. (1) Since $t = \frac{\rho L}{2k\theta}(x_2^2 - x_1^2)$

$$\therefore \quad t = \frac{\rho L}{2k\theta}(x^2-y^2) = \frac{\rho L(x+y)(x-y)}{2k\theta}$$

34. (4) The total energy radiated from a black body per minute, $Q \propto T^4$

$$\Rightarrow \quad \frac{Q_2}{Q_1} = \left(\frac{2T}{T}\right)^4 = 16 \Rightarrow Q_2 = 16Q_1$$

If m is the mass of water taken and S is its specific heat capacity, then $Q_1 = ms(20.5-20)$ and $Q_2 = ms(\theta - 20)$ where θ°C is final temperature of water.

$$\Rightarrow \quad \frac{Q_2}{Q_1} = \frac{\theta-20}{0.5} \Rightarrow \frac{16}{1} = \frac{\theta-20}{0.5} \Rightarrow \theta = 28°\text{C}$$

35. (3) Rate of cooling, $\frac{\Delta\theta}{t} = \frac{A\varepsilon\sigma(T^4-T_0^4)}{mc}$

$\Rightarrow \quad t \propto \frac{m}{A} \quad \left[\because \Delta\theta, t, \sigma, (T^4-T_0^4) \text{ are constant}\right]$

$\Rightarrow \quad t \propto \frac{m}{A} \propto \frac{\text{Volume}}{\text{Area}} \propto \frac{a^3}{a^2}$

$\Rightarrow \quad t \propto a$

$$\Rightarrow \quad \frac{t_1}{t_2} = \frac{a_1}{a_2} \Rightarrow \frac{100}{t_2} = \frac{1}{2} \Rightarrow t_2 = 200 \text{ sec.}$$

36. (3) $\frac{Q}{t} = \frac{KA\Delta\theta}{l} \Rightarrow \frac{mL}{t} = \frac{K(\pi r^2)\Delta\theta}{l}$

$\Rightarrow$ Rate of melting of ice, $\left(\frac{m}{t}\right) \propto \frac{Kr^2}{l}$

Since for second rod, K becomes $\frac{1}{4}$th, r becomes double and length becomes half, so rate of melting will be twice.

$$\therefore \quad \left(\frac{m}{t}\right)_2 = 2\left(\frac{m}{t}\right)_1 = 2\times 0.1 = 0.2 \text{ g/sec.}$$

37. (3) Heat transferred in one minute is utilised in melting the ice so,

$$\frac{KA(\theta_1-\theta_2)t}{l} = m\times L$$

$$\Rightarrow \quad m = \frac{10^{-3}\times 92\times(100-0)\times 60}{1\times 8\times 10^4} = 6.9\times 10^{-3} \text{ kg}$$

38. (4) $\frac{dQ}{dt} = \frac{KA}{l}d\theta = \frac{0.01\times 1}{0.05}\times 30 = 6$ J/sec

Heat transferred in on day (or 86400 sec),

$\theta = 6\times 86400 = 518400$ J

Now, $Q = mL$

$$\Rightarrow \quad m = \frac{Q}{L} = \frac{518400}{334\times 10^3} = 1.552 \text{ kg} = 1552 \text{ g}$$

39. (2) For no current flow between C and D,

$$\left(\frac{Q}{t}\right)_{AC} = \left(\frac{Q}{t}\right)_{CB}$$

$$\Rightarrow \quad \frac{K_1A(\theta_A-\theta_D)}{l} = \frac{K_2A(\theta_D-\theta_B)}{l}$$

$$\Rightarrow \quad \frac{\theta_A-\theta_C}{\theta_C-\theta_B} = \frac{K_2}{K_1} \quad ...(i)$$

Also, $\left(\frac{Q}{t}\right)_{AD} = \left(\frac{Q}{t}\right)_{DB}$

$$\Rightarrow \quad \frac{K_3A(\theta_A-\theta_D)}{l} = \frac{K_4A(\theta_D-\theta_B)}{l}$$

$$\Rightarrow \quad \frac{\theta_A-\theta_D}{\theta_D-\theta_B} = \frac{K_4}{K_3} \quad ...(ii)$$

It is given that $\theta_C = \theta_D$, hence from equation (i) and (ii), we get

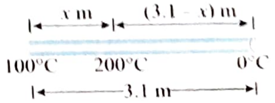
$$\frac{K_2}{K_1} = \frac{K_4}{K_3} \Rightarrow K_1K_4 = K_2K_3$$

40. (1) Rate of flow of heat is given by $\frac{dQ}{dt} = \frac{\Delta\theta}{l/KA}$

Also, $\frac{dQ}{dt} = L\frac{dm}{dt}$ (where L = Latent heat).

$$\Rightarrow \quad \frac{dm}{dt} = \frac{KA}{l}\left(\frac{\Delta\theta}{L}\right).$$

Let the desire point is at a distance x from water at 100°C.



∵ Rate at which ice is melting = Rate at which steam is being produced

$$\Rightarrow \left(\frac{dm}{dt}\right)_{\text{Steam}} = \left(\frac{dm}{dt}\right)_{\text{Ice}} \Rightarrow \left(\frac{\Delta\theta}{Ll}\right)_{\text{Steam}} = \left(\frac{\Delta\theta}{Ll}\right)_{\text{Ice}}$$

$$\Rightarrow \frac{(200-100)}{540\times x} = \frac{(200-0)}{80(3.1-x)} \Rightarrow x = 0.4 \text{ m} = 40 \text{ cm}$$

41. (3) $Q = \sigma A t(T^4 - T_0^4)$

If T, T_0, σ and t are same for both bodies, then

$$\frac{Q_{\text{sphere}}}{Q_{\text{cube}}} = \frac{A_{\text{sphere}}}{A_{\text{cube}}} = \frac{4\pi r^2}{6a^2} \qquad \text{...(i)}$$

But according to problem, Volume of sphere = Volume of cube

$$\Rightarrow \frac{4}{3}\pi r^3 = a^3 \Rightarrow a = r\left(\frac{4}{3}\pi\right)^{1/3}$$

Substituting the value of a in equation (i), we get

$$\frac{Q_{\text{sphere}}}{Q_{\text{cube}}} = \frac{4\pi r^2}{6a^2} = \frac{4\pi r^2}{6\left\{\left(\frac{4}{3}\pi\right)^{1/3} r\right\}^2}$$

$$= \frac{4\pi r^2}{6\left(\frac{4}{3}\pi\right)^{2/3} r^2} = \left(\frac{\pi}{6}\right)^{1/3} : 1$$

42. (3) According to Kirchhoff's law, the ratio of emissive power to absorptive power is same for all bodies and is equal to the emissive power of a perfectly black body i.e.,

$$\left(\frac{e}{a}\right)_{\text{body}} = E_{\text{Black body}} \quad \text{(for a particular wavelength)}$$

$$\left(\frac{e_\lambda}{a_\lambda}\right)_{\text{body}} = (E_\lambda)_{\text{Black body}} \Rightarrow e_\lambda = a_\lambda E_\lambda$$

43. (1) According to Wien's law, $\lambda_m T$ = constant

$$\Rightarrow \lambda_{m_1} T_1 = \lambda_{m_2} T_2 \Rightarrow T_2 = \frac{\lambda_{m_1}}{\lambda_{m_2}} T_1 = \frac{\lambda_0}{3\lambda_0/4} \times T_1 = \frac{4}{3} T_1$$

$$\text{Now, } P \propto T^4 \Rightarrow \frac{P_2}{P_1} = \left(\frac{T_2}{T_1}\right)^4 \Rightarrow \frac{P_2}{P_1} = \left(\frac{4/3\, T_1}{T_1}\right)^4 = \frac{256}{81}$$

44. (2) According to Newton's law of cooling

$$\frac{\theta_1 - \theta_2}{t} = K\left[\frac{\theta_1 + \theta_2}{2} - \theta_0\right]$$

In the first case,

$$\frac{(60-50)}{10} = K\left[\frac{60+50}{2} - \theta_0\right] \Rightarrow 1 = K(55-\theta) \qquad \text{...(i)}$$

In the second case,

$$\frac{(50-42)}{10} = K\left[\frac{50+42}{2} - \theta_0\right] \Rightarrow 0.8 = K(46-\theta_0) \qquad \text{...(ii)}$$

Dividing (i) by (ii), we get $\frac{1}{0.8} = \frac{55-\theta_0}{46-\theta_0}$

$$\Rightarrow 46 - \theta_0 = 44 - 0.8\,\theta_0$$

$$\Rightarrow \theta_0 = 10°\text{C}$$

45. (3) According to Newton's law of cooling, Rate of cooling ∝ Mean temperature difference

$$\Rightarrow \frac{\text{Fall in temperature}}{\text{Time}} \propto \left(\frac{\theta_1 + \theta_2}{2} - \theta_0\right)$$

$$\because \left(\frac{\theta_1+\theta_2}{2}\right)_1 > \left(\frac{\theta_1+\theta_2}{2}\right)_2 > \left(\frac{\theta_1+\theta_2}{2}\right)_3$$

$$\Rightarrow T_1 < T_2 < T_3$$

46. (3) $S_l = \frac{1}{m_l}\left[\frac{t_l}{t_W}(m_W C_W + W) - W\right]$

$$= \frac{1}{300}\left[\frac{95}{3\times 60}(350\times 1 + 10) - 10\right] = 0.6 \text{ Cal/g} \times °\text{C}$$

47. (4) Rate of loss of heat $\left(\frac{\Delta Q}{t}\right) \propto$ temperature difference $\Delta\theta$

$$\frac{\left(\frac{\Delta Q}{t}\right)_1}{\left(\frac{\Delta Q}{t}\right)_2} = \frac{\Delta\theta_2}{\Delta\theta_1} \Rightarrow \frac{60}{\left(\frac{\Delta Q}{t}\right)_2} = \frac{80-60}{40-20} \Rightarrow \left(\frac{\Delta Q}{t}\right)_2 = \frac{20\text{ cal}}{\text{sec}}$$

48. (3) According to Newton's law of cooling, Rate of cooling ∝ mean temperature difference

Initially, mean temperature difference

$$= \left(\frac{70+60}{2} - \theta_0\right) = (65 - \theta_0)$$

Finally, mean temperature difference

$$= \left(\frac{60+50}{2} - \theta_0\right) = (55 - \theta_0)$$

In second case, mean temperature difference decreases, so rate of fall of temperature decreases. Thus, it takes more time to cool through the same range.

49. (3) $\frac{\theta_1 - \theta_2}{t} = K\left[\frac{\theta_1 + \theta_2}{2} - \theta_0\right]$

In the first 10 minute, $\frac{62-50}{10} = K\left[\frac{62+50}{2} - \theta_0\right]$

$$\Rightarrow 1.2 = K[56 - \theta_0]. \qquad \text{...(i)}$$

$$\frac{50-42}{10} = K\left[\frac{50+42}{2} - \theta_0\right] \Rightarrow 0.8 = K[46 - \theta_0]$$

$$\Rightarrow 0.8 = K[56 - \theta_0]. \qquad \text{...(ii)}$$

From equations (i) and (ii), we get

$$\frac{1.2}{0.8} = \frac{(56-\theta_0)}{(46-\theta_0)} \Rightarrow \theta_0 = 26°\text{C}$$

50. (3) Rate of cooling, $R_C = \frac{A\varepsilon\sigma(T^4 - T_0^4)}{mc} = \frac{A\varepsilon\sigma(T^4 - T_0^4)}{V\rho C}$

$$\Rightarrow R_C \propto \frac{A}{V} \propto \frac{1}{r} \propto \frac{1}{(\text{Diameter})} \qquad (\because m = \rho V)$$

Since diameter of A is half that of B, its rate of cooling will be twice that of B.

51. (2) The rate of heat loss by a thin hollow sphere of thickness Δr, mean radius r and made of density ρ is given by

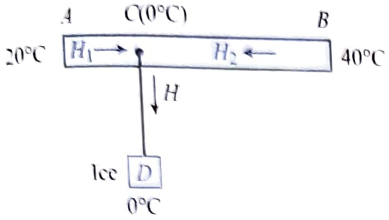
$$mS\frac{dT}{dt} = -\varepsilon\sigma A(T^4 - T_0^4)$$

$$(\rho 4\pi r^2 \Delta r)S\frac{dT}{dt} = -\varepsilon\sigma 4\pi r^2(T^4 - T_0^4)$$

$$\frac{dT}{dt} = -\frac{\varepsilon\sigma(T^4 - T_0^4)}{S\Delta r}$$ is independent of radius

Hence the rate of cooling is same for both spheres.

(4) Thermal resistance of $AC = \frac{L}{KA} = \frac{0.1}{336 \times 10^{-4}} = \frac{10^3}{336} = R$ (say).



Thermal resistance of $BC = \frac{0.2}{336 \times 10^{-4}} = 2\text{ R}$

Heat flow rates are $H_1 = \frac{20}{R}; H_2 = \frac{40}{2R} = \frac{20}{R}$

$$H = H_1 + H_2 = \frac{40}{R} = \frac{40 \times 336}{10^3} = \frac{13440}{10^3} = 13.44\text{ W}$$

Rate of melting of ice $= \frac{H}{L_f} = \frac{13.44/4.2}{80}$ g/s = 40 mg/s

(4) Heat current: $i = -kA\frac{dT}{dx}$

$$i dx = -kA\, dT$$

$$i\int_0^l dx = -A\alpha\int_{T_1}^{T_2} T\, dT$$

$$il = -A\alpha\frac{(T_2^2 - T_1^2)}{2} \Rightarrow i = \frac{A\alpha(T_1^2 - T_2^2)}{2l}$$

(2) Consider a time interval Δt while the door is closed. Then the rate of heat into the box is

$$\frac{Q}{\Delta t} = kA\left(\frac{\Delta T}{\Delta x}\right) = (0.05)(6)\left(\frac{22}{0.09}\right) = 73.3\text{ W}$$

To remove heat at this rate, while the motor runs only for a time $(0.15)\Delta t$, it must cause heat to leave at the rate,

$$H = \frac{73.3}{0.15} = 500\text{ W}$$

(3) According to Wien's law $\lambda_0 T_0 = \lambda T$

According to Stefan's law $\frac{P_0}{P} = \left(\frac{T_0}{T}\right)^4$

As $P = \frac{256}{81}P_0 \Rightarrow \lambda = \frac{3}{4}\lambda_0$

$\therefore$ Wavelength shift $\Delta\lambda = \lambda - \lambda_0 = -\frac{\lambda_0}{4}$

(1) Rate of heat loss per unit area due to radiation i.e., emissive power, $e = \varepsilon\sigma(T^4 - T_0^4)$.

$$= 0.6 \times \frac{17}{3} \times 10^{-8} \times [(400)^4 - (300)^4]$$

$$= 3.4 \times 10^{-8} \times (175 \times 10^8) = 3.4 \times 175 = 595\text{ J/m}^2 \times \text{sec}$$

57. **(4)** Let θ be the temperate of B

$$\frac{2KA(\theta - 100)}{l} + \frac{\left(\frac{K}{2}\right)A(\theta - 0)}{l} = 200$$

Substituting values, we get $\theta = 880°\text{C}$

Also from $B \to D$, $\frac{\frac{K}{2}A(\theta - 0)t}{l} = 440 \times 80$

$t = 800$ s

58. (3) At 70°C, the system attains steady state.

i.e., Rate of heat generated = Rate of heat loss

or, 10 W = k (70 – 30)°C (From Newton's law of cooling)

or, $k = (1/4)$ W/°C

At 50°C, rate of heat loss should be k (50 – 30)°C = 5 W.

But rate of heat loss = (heat capacity) × (rate of cooling)

i.e., $\frac{-dQ}{dt} = c\left(\frac{-dT}{dt}\right) - 5 = c\left[\frac{49.9 - 50}{60}\right]$ °C/s

or the heat capacity is 3000 J/°C

59. (1) According to Stefan's law, $E = \sigma T^4$

Total surface area of the sun $= 4\pi R_s^2$.

Therefore, the total energy radiated per second by the sun per unit solid angle

$$= -\frac{\sigma T^4 \times 4\pi R_s^2}{4\pi} = \sigma T^4 R_s^2$$

Let R_{es} be the distance of earth from the sun. Hence, intensity of radiation of earth

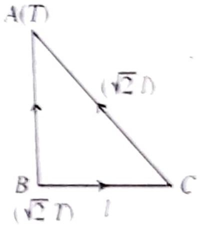
$$I = \sigma T^4 R_s^2 / R_{es}^2$$

$\therefore$

$$1400 = (5.6 \times 10^{-8})T^4\left[\frac{7.0 \times 10^8}{1.5 \times 10^{11}}\right]^2$$

$$T = 5801\text{ K}$$

60. (1) As $T_B > T_A$, heat flows from B to A through both paths BA and BCA.



Rate of heat flow in BC = Rate of heat flow in CA

$$\frac{KA(\sqrt{2}T - T_c)}{l} = \frac{KA(T_c - T)}{\sqrt{2}l}$$

Solving this, we get $T_c = \frac{3T}{\sqrt{2} + 1}$

61. (3) $\frac{\Delta Q}{\Delta t} = \frac{KA\Delta T}{\Delta x} \Rightarrow \Delta Q = KA\left(\frac{\Delta T}{\Delta x}\right)\Delta t$

Assuming the thickness of the spheres to be small, we have

For smaller sphere: (rate of heat flow) (time) = (volume of ice melted) (ρL)

i.e., $K_1(4\pi r^2)\frac{\Delta\theta}{d} \cdot 16 = \frac{4}{3}\pi r^3\rho L$ ...(i)

For larger sphere: $K_2[4\pi(2r)^2]\frac{\Delta\theta}{d/4} \cdot 25 = \frac{4\pi}{3}(2r)^3\rho L$ ...(ii)

Dividing Eq. (ii) by Eq. (i), $K_2/K_1 = 8/25$

62. (2) If θ is the temperature of outside, heat passing per second through the glass window,

$$\frac{dQ}{dt} = KA\frac{(\theta_1 - \theta_2)}{L} = \frac{0.2 \times 1 \times (20 - \theta)\text{ cal}}{0.2 \times 10^{-2}} = 100(20 - \theta)$$

And heat produced per second by the heater in the room

$$P = \frac{V^2}{R}\frac{J}{s} = \frac{V^2}{RJ}\frac{\text{cal}}{s} = \frac{200 \times 200}{20 \times 4.2} = 476.2\frac{\text{cal}}{s}$$

Now as the temperature of the room is constant, the heat produced per second by heater must be equal to the heat conducted through the glass window.

$100(20 - \theta) = 476.2$; $\theta = 15.24$°C

63. (4) Suppose the temperatures of junctions B, C, D are θ_1, θ_2 and θ_3, respectively. Let Q_1, Q_2, Q_3, Q_4, Q_5 and Q_6 be the amounts of heat flowing from A to B, B to C, B to D, C to D to E and C to E per second, respectively. Temperature of junctions A and E are 60°C and 10°C, respectively.

$$\lambda_1 T_1 = \lambda_2 T_2$$

$$\therefore \quad \frac{\lambda_1}{\lambda_2} = \frac{T_2}{T_1} = \frac{0.26}{0.13} = 2$$

$$\therefore \quad T_2 = 2T_1$$

By Stefan's law, emissive power $E = \sigma T^4$

$$E_1 = \sigma T_1^4; \quad E_2 = \sigma T_2^4$$

$$\therefore \quad \frac{E_1}{E_2} = \frac{\sigma T_1^4}{\sigma T_2^4} = \frac{T_1^4}{(2T_1)^4} = \frac{1}{16}$$

64. (3) According to Stefan–Boltzmann law, the energy radiated per second through the surface of area A is given by

$$E = \sigma A T^4$$

$$\therefore \quad \frac{E_1}{E_2} = \frac{A_1}{A_2}\left(\frac{T_1}{T_2}\right)^4 \quad \text{or} \quad 10000 = \frac{r_1^2}{r_2^2}\left(\frac{2000}{6000}\right)^4$$

$$\text{or} \quad \frac{r_1^2}{r_2^2} = (30)^4, \quad \text{or} \quad r_1 : r_2 = 900:1$$

65. (3) Rate of loss of energy by unit area of the planet $= \sigma T^4$, where σ is the Stefan's constant. Let Q be the total energy emitted by the sun every second. If d is the distance of the planet from sun, then Q falls uniformly over the inner surface of the sphere of radius d.

Rate of gain of heat by unit area of planet $= \dfrac{Q}{4\pi d^2}$

For steady temperature of planet, $\sigma T^4 = \dfrac{Q}{4\pi d^2}$

$$\text{or} \quad T^4 = \frac{Q}{4\pi\sigma d^2} \quad \text{or} \quad T = \left(\frac{Q}{4\pi\sigma d^2}\right)^{1/4}$$

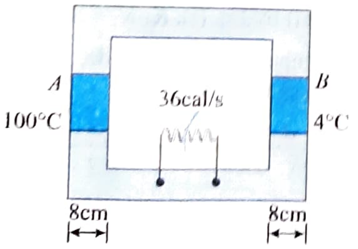
$$\text{or} \quad T \propto \frac{1}{\sqrt{d}}$$

66. (4) From Wien's law, $\lambda_m T =$ constant, where T is the temperature of black body and λ_m is the wavelength corresponding to maximum energy of emission. Energy distribution of black body radiation is given below:

(i) U_1 and U_2 are not zero because a black body emits radiations of nearly all wavelengths.

(ii) Since U_1 corresponding to lower wavelength, U_3 corresponds to higher wavelength and U_2 corresponds to medium wave length, hence $U_2 > U_1$.

67. (3)



$$36 + \left(\frac{T-100}{0.08}\right)kA = \left(\frac{T-4}{0.08}\right)kA$$

$$\Rightarrow \quad K = 0.5 \text{ cal/°C/cm}$$

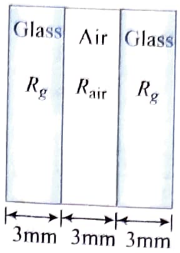
$$A = 12 \text{ cm}^2$$

68. (4) In case of double glass pane window, the equivalent thermal resistance

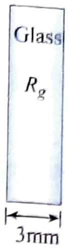
$$R_{eq} = 2R_g + R_{air} = \frac{2(3\text{mm})}{k_g A} + \frac{(3\text{mm})}{k_{air} A}$$

In case of single glass pane window, the thermal resistance

$$R = \frac{3\text{mm}}{k_g \cdot A}$$



Double pane window



Single pane window

The ratio of heat flow from double pane to single pane

$$\frac{i_A}{i_B} = \frac{\Delta T / R_{eq}}{\Delta T / R}$$

$$\Rightarrow \quad \frac{i_A}{i_B} = \frac{R}{R_{eq}} = \frac{\dfrac{3\text{mm}}{K_g \cdot A}}{\dfrac{2(3\text{mm})}{K_g\ A} + \left(\dfrac{3\text{mm}}{K_{air}\ A}\right)}$$

$$= \frac{\dfrac{1}{K_g}}{\dfrac{2K_{air} + K_g}{K_g \cdot K_{air}}} = \frac{K_{air}}{K_g + 2K_{air}} = \frac{1}{[K_g/K_{air}]+2} = \frac{1}{48+2} = \frac{1}{50}$$

Multiple Correct Answers Type

1. (3), (4)

This situation arises during melting (alternative (3)) or freezing (alternative (4)) of the mass.

2. (3), (4)

From metallic piece, more heat is conducted into the body than from a wooden piece. So, alternative (3) is correct. Wooden piece will act as a black body. It will absorb more heat as compared to a polished metallic piece when placed in open in bright sun. Now, according to Kirchhoff's law, as a good absorber is also a good emitter, hence wooden piece will lose heat at a faster rate than the metallic piece.

3. (3), (4)

Statement (1) is incorrect. According to Kirchhoff's law, the ratio of the emissive power e and absorptive power a is constant for all substances at any given temperature and for radiation of the same wavelength,

$$\text{i.e.,} \quad \frac{e}{a} = \text{constant}$$

Thus, if e is large, a must also be large, i.e., if a body is a good emitter of a radiation of a particular wavelength, it is also a good absorber of that radiation. Conversely, if a body is a poor emitter of radiation, it is also a poor absorber (and hence, a good reflector) of the radiation.

Statement (2) is also incorrect. The latent heat of steam is very high (2.25×10^6 J/kg). This means that 1 kg of steam at 100°C gives out 2.25×10^6 J of heat energy to convert into 1 kg of boiling water at 100°C. Hence, the burns caused by steam are more severe than those caused by boiling water. For the same reason, heating systems based on circulation of steam are more efficient in warming a house than those based on circulation of hot water.

Statement (3) is correct. The thermal radiation from the sun warms the earth during the day. Since, air is a poor conductor of heat, the atmosphere acts as a blanket for the earth and keeps the earth warm during the night. Moon is very cold because it has no atmosphere.

Statement (4) is also correct. The efficiency of an ideal heat engine is given by

$$\eta = 1 - \frac{T_2}{T_1}$$

where T_2 is the temperature of the sink. To have an efficiency $\eta = 1$ (or 100%), $T_2 = 0$ K. Since, absolute zero cannot be achieved, even an ideal heat engine cannot have a 100% efficiency.

4. (1), (2)

According to Stefan's law

$$E = eA\sigma T^4 \Rightarrow E_A = e_A A\sigma T_A^4 \text{ and } E_B = e_B A\sigma T_B^4$$

$$\because \quad E_A = E_B \quad \therefore \quad e_A T_A^4 = e_B T_B^4$$

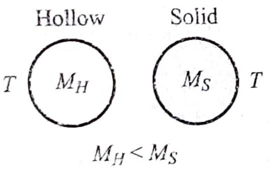
$$\Rightarrow \quad T_B = \left(\frac{e_A}{e_B} T_A^4\right)^{\frac{1}{4}} = \left(\frac{1}{81} \times (5802)^4\right)^{\frac{1}{4}} \Rightarrow T_B = 1934 \text{ K}$$

And, from Wien's law $\lambda_A \times T_A = \lambda_B \times T_B$

$$\Rightarrow \quad \frac{\lambda_A}{\lambda_B} = \frac{T_B}{T_A} \Rightarrow \frac{\lambda_B - \lambda_A}{\lambda_B} = \frac{T_A - T_B}{T_A}$$

$$\Rightarrow \quad \frac{1}{\lambda_B} = \frac{5802 - 1934}{5802} = \frac{3968}{5802} \Rightarrow \lambda_B = 1.5\ \mu\text{m}$$

5. (1), (2)



$P_{\text{emitted}} = \sigma e A T^4$

Since $T_1 = T_2$

$P_{\text{absorb}} = \sigma e A T_s^4$

So $P_1 = P_2$ at $t = 0$

$$\text{cooling rate} \left(-\frac{dT}{dt}\right) = \frac{\sigma e A}{mS}[T^4 - T_S^4]$$

Since $M_H < M_S$, so cooling rate will be different. As cooling rate is not same, so both will not have same temp. at any instant t (except $t = 0$).

6. (1), (3)

According to Wien's law, $\lambda_m T = \text{constant}$

$$\text{or} \quad \frac{T}{v_m} = \text{constant}$$

So, if T is doubled, v_m is also doubled

Further, according to Stefan's law, $E \propto T^4$

When T is doubled, E increases by a factor of 16.

7. (1), (4)

$$H = \text{rate of heat flow} = \frac{900}{\dfrac{l_i}{K_i A} + \dfrac{l_0}{K_0 A}}$$

$$\text{Now, } 1000 - \theta = \frac{H l_i}{K_i A}$$

$$\text{or,} \quad \theta = 1000 - \left[\frac{900}{\dfrac{l_i}{K_i A} + \dfrac{l_0}{K_0 A}}\right]\frac{l_i}{K_i A} = 100 - \frac{900}{1 + \dfrac{l_0}{K_0}\dfrac{K_i}{l_i}}$$

Now, we can see that θ can be decreased by increasing thermal conductivity of outer layer (K_0) and thickness of inner layer (l_i).

8. (1), (2), (4)

Statement (1) is correct because the coefficient of volume expansion is very nearly three times the coefficient of linear expansion.

Statement (2) is also correct. When a solid is heated at constant pressure, it does expand a little and some heat is required for doing the mechanical work associated with this expansion. The difference between the two specific heats (at constant pressure and at constant volume) is small enough to be generally neglected for solids and liquids. This difference is large for gases.

Statement (3) is incorrect. The thermal conductivity of air is less than that of wool. Yet we prefer wool which traps pockets of air between its fibres. There air pockets cannot carry heat by convection either.

Statement (4) is correct because of the expansion of steel tape on a hot day, the markings on the tape would be farther apart than on a cold day.

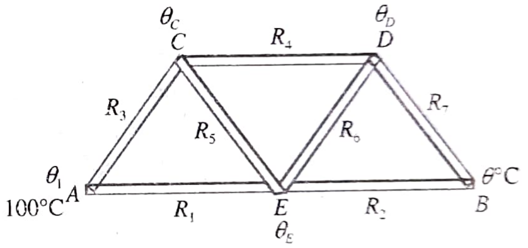
9. (2), (4)

Every object emits and absorbs the radiation simultaneously. If energy emitted is more than energy absorbed, temperature falls and vice versa.

10. (1), (3)

This problem can be solved like electric current problem.

Let R_1, R_2, R_3, R_4, R_5, R_6 and R_7 be the rates of heat flow through AE, EB, AC, CD, CE, ED and DB, respectively.



$$\text{Since } R_1 = R_2, \quad \theta_E = 50°\text{C} \qquad \text{...(i)}$$

$$R_5 = R_6, \quad R_3 = R_4 + R_5 = R_7 \qquad \text{...(ii)}$$

$$R_4 + R_6 = R_7$$

$$\frac{kA(\theta_c - 50)}{l} = \frac{kA}{l}(50 - \theta_D)$$

$$\frac{kA}{l}(100 - \theta_c) = \frac{kA}{l}(\theta_c - 50) + \frac{kA}{l}(\theta_c - \theta_D) = \frac{kA}{l}\theta_D$$

$$\theta_C + \theta_D = 100$$

$$2\theta_C - 2\theta_D = 50 \Rightarrow \theta_C = 62.5°\text{C}$$

$$\theta_D = 37.5°\text{C}$$

$$\therefore \quad \theta_C > \theta_E > \theta_D$$

11. (2), (3), (4)

The flow of heat will always be in the direction of the temperature gradient from higher to lower temperature. Hence Q_1 in rod AB, Q_2 in rod BC will both be in clockwise sense while Q_3 in CA will be in anti-clockwise sense. Also, we have if L is the length of each rod and A its area of cross-section,

$$Q_1 = \frac{\alpha_1 A(100 - 50)}{L} = (50\alpha_1)\frac{A}{L}$$

$$Q_2 = \frac{\alpha_2 A(50 - 0)}{L} = (50\alpha_2)\frac{A}{L}$$

$$Q_3 = \frac{\alpha_3 A(100 - 0)}{L} = (100\alpha_3)\frac{A}{L}$$

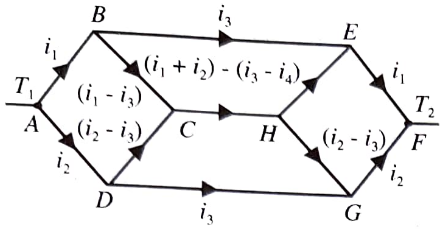
Hence $Q_1 : Q_2 : Q_3 :: \alpha_1 : \alpha_2 : 2\alpha_3$

Also,

$$\frac{Q_1}{\alpha_1}+\frac{Q_2}{\alpha_2}=\left(50\frac{A}{L}\right)+\left(50\frac{A}{L}\right)=\frac{100A}{L}=\frac{Q_3}{\alpha_3}$$

12. (2), (3), (4)

Resistance of each rod $R=\frac{l}{kA}$



In steady state $T_B=T_D$

$T_E=T_G$

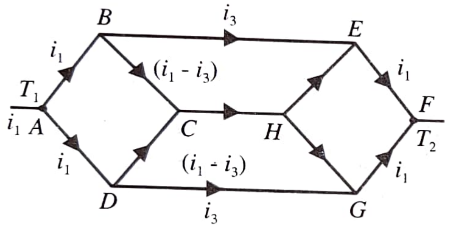
Thermal current $\left(\frac{dQ}{dt}\right)=i$

$$i_1R+i_3R+i_1R=(T_1-T_2)$$

$$2i_1+i_3=\frac{(T_1-T_2)}{R} \qquad \text{...(i)}$$

$$2i_1+i_3=\frac{T_1-T_2}{R} \qquad \text{...(ii)}$$

$$i_1=i_2 \qquad \text{...(iii)}$$



For the path *ABCHGF*

$$i_1R+(i_1-i_3)R+2(i_1-i_3)R+(i_1-i_3)R+i_1R=(T_1-T_2)$$

$$6i_i-4i_3=\frac{(T_1-T_2)}{R}=2i_1+i_3 \qquad \text{...(iv)}$$

$$4i_i=5i_3 \Rightarrow i_3=\frac{4}{5}i_i \qquad \text{...(v)}$$

From Eqs. (ii) and (v)

$$2i_i+\frac{4}{5}i_1=\frac{(T_1-T_2)}{R}$$

$$\frac{14i_i}{5}=\frac{(T_1-T_2)}{R}$$

Equivalent thermal resistance $\frac{(T_1-T_2)}{2i_1}=\frac{7}{5}R$

$$\left(\frac{dQ}{dt}\right)_{AB}=i_1=\frac{5(T_1-T_2)KA}{14l}$$

$$\left(\frac{dQ}{dt}\right)_{BC}=\frac{(T_1-T_2)KA}{14l}$$

$$\left(\frac{dQ}{dt}\right)_{CH}=\frac{(T_1-T_2)KA}{7l}$$

13. (3), (4)

Rate of cooling = Radiation rate/heat capacity. As the heat capacity of both bodies are constant and radiation rate is same due to same surface area and same temperature, the ratio of rate of cooling of both will be a constant.

14. (1), (3)

As the two ends of the rod are at constant temperature that means the rod is in steady state of thermal conduction hence throughout the length the temperature gradient will remain constant and in steady state of thermal conduction the rate of heat flow (heat current) is given by the product of thermal conductivity, cross sectional area and the temperature gradient.

15. (1), (2)

As the two ends of the rod are at constant temperature that means the rod is in steady state of thermal conduction and in steady state of thermal conduction the rate of heat flow (heat current) is given by the product of thermal conductivity, cross sectional area and the temperature gradient.

16. (1), (2)

At 45°C we use $\left.\frac{dQ}{dt}\right|_{\text{less}}=k(45-15)=12$

$$\Rightarrow \qquad k=\frac{12}{30}=0.4$$

At 20°C $\left.\frac{dQ}{dt}\right|_{\text{less}}=k(20-15)=0.4\times5=2\text{ W}$

hence option (1) and (2) are correct.

17. (1), (4)

By wein's displacement law we use $\lambda_m T=\text{constant}$

$$\Rightarrow \qquad \frac{\lambda_{m_A}}{\lambda_{m_B}}=\frac{T_B}{T_A}$$

$$\Rightarrow \qquad \frac{T_A}{T_B}=\frac{\lambda_{m_B}}{\lambda_{m_A}}=\frac{800}{400}=2$$

By Stefan's law, we use $\frac{E_A}{E_B}=\frac{A_AT_A^4}{A_BT_B^4}=\frac{4\pi r_A^2}{4\pi r_B^2}\left(\frac{T_A}{T_B}\right)^4=4$

Linked Comprehension Type

For Problems 1–3

1. (4) Under steady state conditions, the temperatures at all sections in the system remain constant and maintain a constant temperature gradient for a given material. The temperature gradient in copper, aluminium and brass will not be same however, the rate of heat conducted across all sections whether in copper or aluminium or brass will be the same.

2. (1) Since heat transmitted per second in the steady state is $Q=\frac{KA(T_2-T_1)}{L}$ and the dimensions of copper, aluminium and brass rods are identical, we must have

$$K_1(100-T_{\text{ca}})=K_2(T_{\text{ca}}-T_{\text{ab}})=K_3(T_{\text{ab}}-0)$$

where K_1, K_2 and K_3 are thermal conductivities of copper, aluminium and brass, respectively, and T_{ca} and T_{ab} are the steady state temperatures of copper–aluminium and aluminium–brass junctions, respectively.

Now $K_1=2K_2=4K_3$ (given)

Hence, $4(100-T_{\text{ca}})=2(T_{\text{ca}}-T_{\text{ab}})=T_{\text{ab}}$

Solving for T_{ab} and T_{ca} we obtain, $14\,T_{\text{ca}}=1200$

$$\Rightarrow \qquad T_{\text{ca}}=\frac{1200}{14}=85.7=86°\text{C}$$

Also $T_{\text{ab}}=\frac{2}{3}T_{\text{ca}}=\frac{2\times86}{3}=57°\text{C}$.

3. (1) See solution above.

for Problems 4–6

4. (2) We consider the leaf to be a black body. The rate of energy radiated at any instant.

$$\left(\frac{dQ}{dt}\right)_e = \sigma e A T_0^4 \qquad \text{...(i)}$$

If m is the mass of the body and C is its specific heat then the heat gained

$$\frac{dQ}{dt} = mc\frac{dT}{dt} \qquad \text{...(ii)}$$

Since intensity of sun beam is uniform, power incident on the body is SA heat absorbed $\left(\frac{dQ}{dt}\right)_{\text{ab}} = SA\,e$

Rate of increase of temperature

$$= \frac{(\text{net power absorbed})}{\text{thermal capacity of body}} = \frac{(SAe - \sigma AeT^4)}{mc}$$

Given $\sigma = 5.67 \times 10^{-8}\,\text{J/s m}^2\,\text{K}^4$

$A = 0.8 \times 10^{-2}\,\text{m}^2$; $\alpha = e = 0.8$

$S = 1.4 \times 10^3\,\text{W/m}^2$

$T = 300\,\text{K}$

$m = 5 \times 10^{-4}\,\text{kg}$

$C = 0.8\,\text{kcal/kg K} = 0.8 \times 4.2\,\text{kJ/kg K}$

$$\frac{dT}{dt} = \frac{[1.4 \times 10^3 \times 0.8 \times 10^{-2} \times 0.8 - 5.67 \times 10^{-8} \times 0.8 \times 10^{-2} \times 0.8 \times (300)^4]}{5 \times 10^{-4} \times (0.8 \times 4.2 \times 10^3)}$$

$$= \frac{(8.96 - 2.9)}{1.68} = 3.6\,°\text{C/s} = 3.6\,\text{K/s}$$

5. (1) The temperature becomes maximum when net rate of absorption of energy becomes equal to zero, i.e., at any instant $P = (S - \sigma T^4)\,eA = 0$.

$$T_{\max} = \left(\frac{S}{\sigma}\right)^{1/4} = \left(\frac{1.4 \times 10^3}{5.67 \times 10^{-8}}\right)^{1/4} = 396\,\text{K}$$

6. (3) At maximum temperature, $\frac{dT}{dt} = 0$

Initially, $\frac{dT}{dt} = 3.6\,°\text{C/s}$

$$\left(\frac{dT}{dt}\right)_{\text{av}} = \frac{3.6}{2}\,°\text{C/s}$$

$$\text{Rate of loss of heat} = ms\left(\frac{dT}{dt}\right)_{\text{av}} = \sigma A e\left(T^4 - T_0^4\right)$$

$$T^4 = \frac{\left(5 \times 10^{-4} \times 0.8 \times 4.2 \times 10^3 \times \frac{3.6}{2}\right)}{5.67 \times 10^{-8} \times 0.8 \times 10^{-2} \times 0.8} + 300^4$$

$T = 358\,\text{K} = 85°\text{C}$

or Problems 7–9

7. (2) Resistance of lead bar

$$R_{\text{Pb}} = \frac{l_{\text{Pb}}}{K_{\text{Pb}}A_{\text{Pb}}} = \frac{5 \times 10^{-2}}{350 \times 6 \times 10^{-4}} = \frac{10}{21} = \frac{5}{21}\,\text{K/W}$$

Thermal current through lead bar

$$I_{\text{Pb}} = \frac{\Delta T}{R_{\text{Pb}}} = \frac{100 \times 21}{5} = 420\,\text{W}$$

8. (4) Resistance of silver bar

$$R_{\text{Ag}} = \frac{l_{\text{Ag}}}{K_{\text{Ag}}A_{\text{Ag}}} = \frac{5 \times 10^{-2}}{425 \times 6 \times 10^{-4}} = \frac{100}{85 \times 6} = \frac{20}{17 \times 6} = \frac{10}{51}\,\text{K/W}$$

Thermal current through silver bar

$$I_{\text{Ag}} = \frac{\Delta T}{R_{\text{Ag}}} = \frac{100 \times 51}{10} = 510\,\text{W}$$

Total thermal current $I_{\text{Pb}} + I_{\text{Ag}} = 930\,\text{W}$

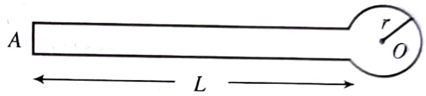
9. (1) $$\frac{1}{R_{\text{eq}}} = \frac{1}{R_{\text{Pb}}} + \frac{1}{R_{\text{Ag}}} = \frac{21}{5} + \frac{51}{10} = \frac{42 + 51}{10} = \frac{93}{10}$$

$$R_{\text{eq}} = \frac{10}{93} = 0.107\,\text{W} \simeq 0.1\,\text{K/W}$$

For Problems 10–12

10. (4) The distance from the end A of the copper rod (where power is supplied) and centre O of the metal sphere is $(L + r)$. Hence, in the steady state conditions,

$$P = \frac{KA(T - T_S)}{L + r}$$



Which gives $T_S = T - \frac{P(L + r)}{KA}$

(The heat is transmitted up to centre of the spherical end, and the sphere loses energy by radiation out of its spherical surface.)

11. (1) Under steady state conditions, all temperatures remain constant including T_S, the temperature at centre of sphere. Hence no thermal energy accumulates inside the sphere.

Hence $P_S = P$ = energy received from source.

12. (2) The temperature T_S will change to a new value. The transmitted thermal energy will be the same.

For Problems 13–15

13. (2) $\rho_{300} = \pi^2 \times 10^{-8}$ ohm m

$$\rho_{900} = \frac{\rho_{300}(1 + 900\alpha)}{(1 + 300\alpha)} = \frac{\pi^2(10^{-8})(1 + 900 \times 7.8 \times 10^{-3})}{(1 + 300 \times 7.8 \times 10^{-3})}$$

$$= \pi^2 \times 10^{-8} \times \frac{8.02}{3.34} = 2.4\,\pi^2 \times 10^{-8}$$

$$\simeq 2.4 \times 10^{-7}\,\text{ohm m} \quad (\pi^2 \simeq 10)$$

14. (2) Heat radiated per second, $E = \sigma A(T^4 - T_0^4)$

$= \sigma(2\pi rl)\,(T^4 - T_0^4)$

$= 5.68 \times 10^{-8} \times 2\pi \times 10^{-3} \times 1\,[(900^4 - 300^4)]$

$= 11.36\,\pi\,(6480) \times 10^{-3} \approx 230\,\text{W}$

15. (3) Heat dissipated per second $= I^2R$

$I^2R = E$

$$I = \sqrt{\frac{E}{R}} = \sqrt{\frac{E\pi r^2}{\rho_{900}l}} = \sqrt{\frac{231 \times .14 \times 10^{-6}}{2.4 \times 10^{-7} \times 1}} \simeq 55\,\text{A}$$

For Problems 16–18

16. (2) We have $\theta - \theta_s = (\theta_0 - \theta_s)e^{-kt}$,

where θ_0 = initial temperature of body = 40°C

θ = temperature of body after time t.

Since body cools from 40 to 38 in 10 min, we have

$$38-30=(40-30)\,e^{\angle 10k} \quad \text{...(i)}$$

Let after 10 min, the body temperature be θ

$$\theta-30^\circ=(38-30)\,e^{\angle 10k} \quad \text{...(ii)}$$

$$\frac{\text{Eq. (i)}}{\text{Eq. (ii)}} \text{ gives } \frac{8}{\theta-30}=\frac{10}{8}, \quad \theta-30=6.4$$

$$\theta=36.4\ ^\circ\text{C}$$

17. (1) Self-explanatory.

18. (3) During heating process from 38°C to 40°C in 10 min, the body will lose heat in the surrounding which will be exactly equal to the heat lost when it cooled from 40°C to 38°C in 10 min, which is equal to $ms\,\Delta\theta=2\times2=4$ J.

During heating process heat required by the body

$$ms\,\Delta\theta=4 \text{ J}.$$

$\therefore$ Total heat required = 8 J.

Matrix Match Type

1. i → b.; ii → c.; iii → a.

i. Rate at which heat is radiated from the body

$$= Q_r \text{ J/s}$$

$$= e\sigma A T_1^4 = 0.55\times5.67\times10^{-8}\times1.5\times(323)^4 \text{ J/s}$$

$$= 509 \text{ W}$$

ii. Rate at which heat radiation is absorbed by the body

$$= Q_a \text{ J/s}$$

$$= e\sigma A T_2^4 = 0.55\times5.67\times10^{-8}\times1.5\times(295)^4 \text{ J/s}$$

$$= 354 \text{ W}$$

iii. Rate at which net radiation is emitted by the body

$$= Q_n \text{ J/s} = Q_r - Q_a = (509-354)\text{ W} = 155 \text{ W}$$

2. i. → a., c.; ii. → d.; iii. → b.; iv. → d.

Initially temperature greatest at A will be more and also area is more at A. So the rate of flow of heat is maximum at A.

At steady state, rate of flow of heat is constant at all sections as all sections are connected in series.

$$\text{As} \quad \left(\frac{dQ}{dt}\right)=KA\left(-\frac{dT}{dx}\right)$$

$$\left|\frac{dT}{dx}\right| \propto \frac{1}{A}$$

At steady state, the temperature at each at every section is constant.

Hence $\dfrac{dT}{dt}=0$ at each section.

3. i. → c.; ii. → d.; iii. → b.; iv. → a.

We have for the four sections, AB, BC, CD and DE with (dQ/dt) as the steady state thermal, energy transmitted per second (A being the area of cross section).

$$\frac{dQ}{dt}=\frac{KA(100-T_C)}{L}=\frac{A(0.8)K(T_C-T_D)}{(1.2)L}$$

$$=\frac{(1.2)KA(T_D-T_E)}{(1.5)L}=\frac{(1.5)KAT_E}{(0.6)L}$$

These give

$$(100-T_C)=\left(\frac{0.8}{1.2}\right)(T_C-T_D)=\left(\frac{1.2}{1.5}\right)(T_D-T_E)=\left(\frac{1.5}{0.6}\right)T_E$$

$$6(100-T_C)=4(T_C-T_D)=(4.8)(T_D-T_E)=15T_E$$

Solving for the differences $(100-T_C)$, (T_C-T_D), (T_D-T_E) and T_E remaining that the sum of these differences is 100, we obtain

$$(T_A-T_C)=24.1,\ (T_C-T_D)=36.2$$

$$(T_D-T_E)=30.1 \text{ and } (T_E-T_B)=T_E=9.6$$

Numerical Value Type

1. (1800)

Power radiated by black body is $P=\sigma e(4\pi r^2)T^4$

$$\Rightarrow \quad P\propto r^2T^4 \quad \Rightarrow \quad \frac{P_1}{P_2}=\frac{r_1^2T_1^4}{r_2^2T_2^4}$$

As $T_2=2T_1$ and $r_2=\dfrac{r_1}{2}$ $\Rightarrow$ $P_2=P_1\times4=450\times4=1800$ W

2. (0.3)

If emissivity of body is e we use

Power required = Radiation rate $P=\sigma eA(T^4-T_S^4)$ ...(i)

For a black body $P_B=\sigma A(T^4-T_S^4)$ ...(ii)

$$\frac{(i)}{(ii)} \text{ gives } \frac{P}{P_B}=e \quad \Rightarrow \quad e=\frac{210}{700}=0.3$$

3. (800)

Using average form of Newton's law of cooling, we use

$$\text{For water} \quad \frac{40-35}{5}=\frac{k}{0.1\times4200}\left(\frac{40+35}{2}-T_s\right) \quad \text{...(i)}$$

$$\text{For liquid} \quad \frac{40-35}{5}=\frac{k}{m\times2100}\left(\frac{40+35}{2}-T_s\right) \quad \text{...(ii)}$$

$$\frac{(ii)}{(i)} \text{ gives } \frac{2}{5}=\frac{m\times2100}{0.1\times4200}$$

$$\Rightarrow \quad m=\frac{2\times420}{5\times2100}=0.08 \text{ kg}=80 \text{ g}$$

As volume of liquid is same that of water 100 cm³, then density of liquid is $P=\dfrac{m}{V}=\dfrac{80\times10^{-3}}{100\times10^{-6}}=800$ kg/m³

4. (3025)

By Wein's displacement law, we use $\lambda_m T=$ constant

$$\Rightarrow \quad \lambda_{m_1}T_1=\lambda_{m_2}T_2$$

$$\Rightarrow \quad 4753\times6050=9506\times T_2$$

$$\Rightarrow \quad T_2=3025 \text{ K}$$

5. (8.0)

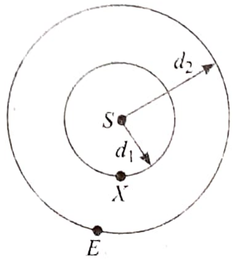
d_1 = Distance between sun and cloud

d_2 = distance between sun and earth

$$P_{\text{emission}}=P_{\text{absorption}}$$

$$\Rightarrow \quad \sigma T^4\cdot4\pi d_1^2=I\cdot4\pi d_2^2$$

$$d_1=\frac{d_2}{T^2}\sqrt{\frac{I}{\sigma}}$$



$$d_1 = \frac{d_2}{T^2}\sqrt{\frac{2.4\times10^3}{6.0\times10^{-8}}} \Rightarrow d_1 = \frac{1.6\times10^{11}}{4\times10^6}\times2.0\times10^5$$

$$d_1 = 8.0\times10^9$$

(4875)

If $\frac{\Delta Q}{\Delta t}$ is the rate of heat conduction through the rod from the end at the furnace to the exposed one, then $\frac{\Delta Q}{\Delta t} = KA\left(\frac{-\Delta T}{l}\right)$ where A is the cross-sectional area and $\frac{\Delta T}{l}$ the temperature gradient.

$$\left(\frac{\Delta Q}{\Delta t}\right)\frac{1}{A} = K\left(\frac{-\Delta T}{l}\right)$$

But $\left(\frac{\Delta Q}{\Delta t}\right)\frac{1}{A}$ is the emissive power (E) of the free end.

$$E = K\left(\frac{-\Delta T}{l}\right) = 65\frac{\text{J}}{\text{m-s-°C}}\times\frac{75\text{°C}}{\text{m}} = 4875\ \text{W/m}^2$$

(1.24)

Assuming that heat loss from the sides of the slab is negligible, the amount of heat flowing through the slab is

$$Q = \frac{kA(T_1 - T_2)t}{d} \quad \text{...(i)}$$

If m is the mass of ice and L the latent heat of fusion, then

$$Q = mL \quad \text{...(ii)}$$

From Eqs. (i) and (ii), we have $mL = \frac{kA(T_1 - T_2)t}{d}$

or $$k = \frac{mLd}{A(T_1 - T_2)t} \quad \text{...(iii)}$$

Given $m = 4.8$ kg, $d = 10$ cm $= 0.1$ m, $A = 3600\ \text{cm}^2 = 0.36\ \text{m}^2$

$T_1 = 100$°C, $T_2 = 0$°C and $t = 1$ h $= (60 \times 60)$ s

We know that $L = 80$ cal/g $= 80000$ cal/kg

$= 80000 \times 4.2$ J/kg $= 3.36 \times 10^5$ J/kg

Substituting these values in Eq. (iii) and solving, we get

$k = 1.24$ J/s-m-°C or 1.24 W/m-K

8. (12.5)

At steady state, rate of doing work by gravity on block M = Rate of energy produced

or $$\frac{d}{dt}\left[Mgh\right] = \frac{d}{dt}\left[Q + \frac{1}{2}Mv^2\right]$$

or $$Mg\left(\frac{dh}{dt}\right) = \frac{dQ}{dt} + \frac{M}{2}\times 2v\frac{dv}{dt} \quad \text{...(i)}$$

As block goes down with constant velocity,

$$\frac{dh}{dt} = v \quad \text{and} \quad \frac{dv}{dt} = 0$$

$\therefore$ $$Mgv = \frac{dQ}{dt}$$

Here $$\frac{dQ}{dt} = KA\frac{\Delta T}{L}$$

$\therefore$ $$Mgv = KA\frac{\Delta T}{L}$$

or $$M\times10\times(0.1) = 0.50\times0.50\frac{(1.0-0)}{2\times10^{-3}}$$

or $$M = 12.5\ \text{kg}$$

9. (300)

Power absorbed by earth = power emitted by earth.

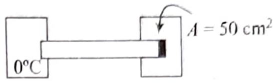
$$\frac{e\sigma(4\pi R_S^2)T_S^4}{4\pi r^2}\times\pi R_e^2 = e\sigma(4\pi R_e^2)T_e^4$$

$$T_e = T_S\sqrt{\frac{R_S}{2r}} = T_S\sqrt{\frac{R_S}{2\times200R_S}}$$

$$T_e = \frac{T_s}{20} = 300\ \text{K}$$

10. (1.8)

Blackened portion will absorb heat energy by radiation and deliver 0°C chamber by conduction process.



$$\frac{17-0}{R} = \frac{(17)}{0.5}kA = \sigma eA\{(300)^4 - (290)^4\}$$

$e = 1$ for blackened object.

On solving, we get $k = 1.8$ W/m-°C

Archives

JEE Main

Single Correct Answer Type

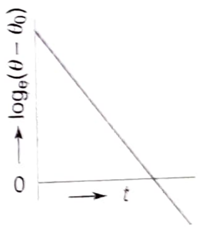
1. (2) We know that $\frac{dQ}{dt} = KA\frac{d\theta}{dx}$

In steady state flow of heat, $d\theta = \frac{d\theta}{dt}\times\frac{1}{KA}\times dx$

$$\Rightarrow \quad \theta_H - \theta = K'x \Rightarrow \theta = \theta_H - K'x$$

The equation $\theta = \theta_H - K'x$ represents a straight line.

2. (1) $$\frac{d\theta}{dt} = -k(\theta - \theta_0)$$



$$\int_{\theta_0}^{\theta}\frac{d\theta}{\theta - \theta_0} = -k\int_0^t dt$$

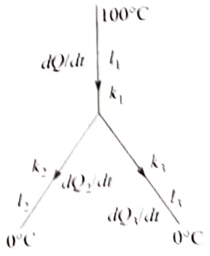
$$(\theta - \theta_0) = -kt + \chi$$

So graph is straight line.

3. (2) The temperature goes on decreasing with time (non-linearly). The rate of decrease will be more initially which is depicted in the second graph.

4. (1) $$\frac{dQ_1}{dt} = \frac{dQ_2}{dt} + \frac{dQ_3}{dt}$$

$$\Rightarrow \quad \frac{0.92(100-T)}{46} = \frac{0.26(T-0)}{13} + \frac{0.12(T-0)}{12}$$



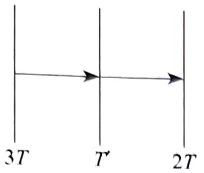
$$\Rightarrow \quad T = 40\text{°C}$$

$$\frac{dQ_1}{dt} = \frac{0.92\times4(100-40)}{46} = 4.8\ \text{cal/s}$$

JEE Advanced

Single Correct Answer Type

1. (3)



In steady state energy absorbed by middle plate is equal to energy released by middle plate.

$$\sigma A(3T)^4 - \sigma A(T')^4 = \sigma A(T')^4 - \sigma A(2T)^4$$

$$(3T)^4 - (T')^4 = (T')^4 - (2T)^4$$

$$(2T')^4 = (16+81)T^4$$

$$T' = \left(\frac{97}{2}\right)^{1/4} T$$

2. (1) $R_1 = \dfrac{L}{kA} + \dfrac{L}{2kA} = \dfrac{3L}{2kA}$

$$\frac{1}{R_2} = \frac{1}{\left(\frac{L}{kA}\right)} + \frac{1}{\left(\frac{L}{2kA}\right)} = \frac{3kA}{L}$$

$$R_2 = \frac{1}{3kA}$$

$$\Delta Q_1 = \Delta Q_2$$

$$\frac{\Delta T_1}{R_1} t_1 = \frac{\Delta T_2}{R_2} t_2 \quad \Rightarrow \quad t_2 = \frac{R_2}{R_1} t_1 = 2\,\text{s}$$

3. (1) Rate of radiation energy lost by the sphere = rate of radiation energy incident

$$= \sigma(4\pi r^2)\,[T^4 - (300)^4] = 912 \times \pi r^2$$

Now, $\sigma \times T^4 = \sigma(300)^4 + \dfrac{912}{4}$

$$T^4 = (300)^4 + \frac{912}{(4 \times 5.7 \times 10^{-8})}$$

$$= (300)^4 + \frac{912}{22.8} \times 10^8$$

$$= (300)^4 + (40 \times 10^8) = (81 + 40) \times 10^8$$

$$= 121 \times 10^8 \quad \therefore T = 330\ \text{K}$$

Multiple Correct Answers Type

1. (1), (3), (4)

(1) At steady state, heat flow through A and E are same. So option (1) is correct and option (2) is incorrect.

(3) $\Delta T = H \times R$, where H is heat current.
'H' is same for A and E but R is smallest for E. So temperature difference across slab E is smallest. So option (3) is correct.

(4) $H_B = \dfrac{\Delta T}{R_B}$, $H_C = \dfrac{\Delta T}{R_C}$ and $H_D = \dfrac{\Delta T}{R_D}$,

From this option if, $H_C = H_B + H_D$

$$\Rightarrow \quad \frac{1}{R_C} = \frac{1}{R_B} + \frac{1}{R_D}$$

$$\Rightarrow \quad \frac{4K(2Lb)}{4L} = \frac{3K(Lb)}{4L} + \frac{5K(Lb)}{4L}$$

which is true. Here b is the width of slab.

Hence option (4) is also correct.

2. (3), (4)

Because of non-uniform evaporation at different section, area of cross-section would be different at different sections.

Region of highest evaporation rate would have rapidly reduced area and would become break up cross-section.

Resistance of the wire as a whole increases with time.

Overall resistance increases hence power decreases. At break up junction temperature would be highest, thus light of highest band frequency would be emitted at those cross-section.

3. (1), (3)

Body metabolic system will have to produce additional energy in order to maintain the temperature.

Let due to internal metabolism, power produced is P_{net}.

Net power radiated $P_{\text{net}} = \sigma A(T^4 - T_0^4)$

A = surface area of human body

For small temperature difference, $\Delta T = T - T_0$

$$P_{\text{net}} = \sigma A\left[(T_0 + \Delta T)^4 - T_0^4\right]$$

$$= \sigma A T_0^4 \left[\left(1 + \frac{\Delta T}{T_0}\right)^4 - 1\right] = \sigma A T_0^4 \left[\frac{4\Delta T}{T_0}\right]$$

$$\therefore \quad P(\text{Net}) = 4\sigma A T_0^3 (T - T_0) = 4\sigma A T_0^3 \Delta T = 4A\sigma T_0^4 \frac{\Delta T}{T_0}$$

(1) From the above result, P_{net} decreases with A, hence (1) will be correct.

(2) Peak shifts to shorter wave lengths for rise in temperature (Wien's displacement law of radiation)

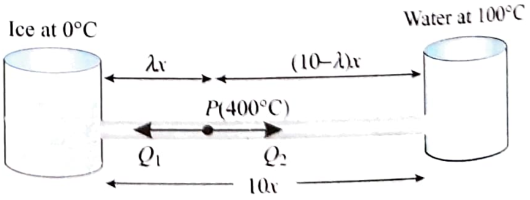
(3) $P_{\text{net}} = 4 \times 1 \times 460 \times \dfrac{10}{300} \approx 60\ \text{W}$

Hence (3) is correct.

(4) Energy radiated by a body is dependent only on its own temperature, not the temperature of surroundings. Hence (4) is incorrect. If P_{net} is taken, then from the calculation shown above, (4) will also be correct.

Numerical Value Type

1. (9) $\dfrac{dm_{\text{ice}}}{dt} = \dfrac{dm_{\text{vapour}}}{dt}$



$$Q_1 = \frac{KA400}{t} = m \times 80$$

$$Q_1 = \frac{KA(400-100)t}{(10-\lambda)x} t = m \times 540$$

Dividing both, $\lambda = 9$

2. (9) $\lambda_m T$ = constant. $\lambda_A T_A = \lambda_B T_B$

Now rate of total energy radiated $\propto AT^4$

3. (2) Let A be the surface area of smaller body then,

$$\Rightarrow \quad P_A = \sigma \cdot (16 \times 10^4) A T_A^4 \text{ and } P_B = \sigma A T_B^4$$

$$\therefore \quad \frac{16 \times 10^4 T_A^4}{T_B^4} = 10^4 \Rightarrow \left(\frac{T_A}{T_B}\right)^4 = \frac{1}{16}$$

$$\Rightarrow \quad \frac{T_A}{T_B} = \frac{1}{2}$$

By Wien's law, $\lambda T = \text{const} \therefore$ ratio $\frac{\lambda_A}{\lambda_B} = 2$

4. (9) $P = eA\sigma T^4$ where T is in kelvin

$$\log_2 \frac{eA\sigma(487+273)^4}{P_0} = 1 \qquad \ldots\text{(i)}$$

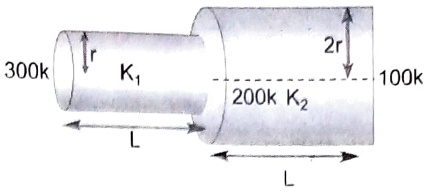
$$\log_2 \frac{eA\sigma(2767+273)^4}{P_0} = x \qquad \ldots\text{(ii)}$$

(ii) – (i)

$$\log_2 \left(\frac{3040}{460}\right)^4 = x - 1$$

$\therefore x = 9$

5. **(4)** We have in steady state, thermal current should be equal



$$\left(\frac{\frac{200-300}{L}}{k_1 \pi r^2}\right) = \left(\frac{\frac{200-100}{L}}{k_2 \pi (2r)^2}\right)$$

$$\Rightarrow \quad \frac{k_1 \pi r^2 \times 100}{L} = \frac{100 k_2 \pi \times 4r^2}{L}$$

$$\Rightarrow \quad \frac{k_1}{k_2} = 4$$

Chapter 3

Concept Application Exercises

Exercise 3.1

1. $P = nkT$

$$\Rightarrow \quad n = \frac{p}{kT} = \frac{10^{-11} \times 13.6 \times 1000 \times 9.8}{1.38 \times 10^{-23} \times 273} = 3.5 \times 10^{14}$$

2. $\frac{1}{2}Mv^2 = C_v\Delta T = \frac{R}{\gamma - 1}\Delta T$

$$\left(C_v = \frac{R}{\gamma - 1}\right) \Rightarrow \Delta T = \frac{(\gamma - 1)Mv^2}{2R}$$

Now $\quad pV = RT \Rightarrow p = \frac{RT}{V} \quad \Rightarrow \Delta p = \frac{R}{V}\Delta T$

$$\therefore \quad \frac{\Delta p}{p} = \frac{\left(\frac{7}{5} - 1\right) \times 28 \times 10^{-3} \times 100^2}{2 \times 8.3 \times 300} = 0.0225 = 2.25\%$$

3. $p = \frac{1}{3}\rho v_{\mathrm{rms}}^2 \quad \Rightarrow v_{\mathrm{rms}} = \sqrt{\frac{3p}{\rho}}$

$$\therefore \quad v_{\mathrm{rms}} = \sqrt{\frac{3 \times 1.00 \times 10^{-2} \times 1.013 \times 10^5}{1.24 \times 10^{-2}}} = 495 \text{ m/s}$$

Again, $v_{\mathrm{rms}} = \sqrt{\frac{3RT}{M}}$

or $\quad M = \frac{3RT}{v_{\mathrm{rms}}^2}$

$$\therefore \quad M = \frac{3 \times 8.3 \times 400}{(495)^2} = 40.6 \text{ g/mol}$$

This is the molecular weight of argon and so the gas is argon.

4. The degrees of freedom of rotational motion = 2

$$\langle \varepsilon \rangle_{\mathrm{rot}} = 2 \times \frac{1}{2}kT \text{ (by equipartition principle)}$$

$$= \left(2 \times \frac{1}{2}I\omega^2\right)$$

$$\Rightarrow \quad kT = I\langle \omega^2 \rangle \Rightarrow \omega_{\mathrm{rms}} = \sqrt{\frac{kT}{I}}$$

Therefore, $\omega_{\mathrm{rms}} = \sqrt{\frac{1.38 \times 10^{-23} \times 300}{2 \times 10^{-40}}} = 4.5 \times 10^9$ rad/s

5. Let m be the mass of the neon gas, then mass of argon will be $28 - m$.

Number of moles of neon, $n_1 = \frac{m}{20}$

Number of moles of argon, $n_2 = \frac{28 - m}{40}$

Now by ideal gas equation, we have $PV = nRT$

Here $n = n_1 + n_2 = \frac{m}{20} + \frac{28 - m}{40}$,

$P = 1.0 \times 10^5 \text{ N/m}^2$

$T = 273 + 27 = 300 \text{ K}, V = 0.02 \text{ m}^3$

$$\therefore \quad (1.0 \times 10^5) \times 0.02 = \left[\frac{m}{20} + \frac{(28 - m)}{40}\right] \times 8.314 \times 300$$

After solving, we get $m = 4.07$ g

$\therefore \quad$ Mass of neon gas = 4.07 g

Mass of argon gas = $28 - m = 28 - 4.07 = 23.93$ g

6. From gas law, we have for a constant volume container $\frac{P_1}{T_1} = \frac{P_2}{T_2}$

or $\quad T_2 = \left(\frac{P_2}{P_1}\right)T_1$

Given that $P_1 = 8 \times 10^5$ Pa, $T_1 = 300$ K, $P_2 = 10^6$ Pa, then we have

$$T_2 = \frac{10^6}{8 \times 10^5} \times 300 = 375 \text{ K}$$

7. The pressure exerted by a gas is given by

$$P = \frac{2}{3} \text{ (KE per unit volume)}$$

or $\quad \text{KE} = \frac{3}{2}P$

Here $\quad P = 10$ mm of mercury = 1 cm of mercury

$= 1 \times 13.6 \times 980 = 1.33 \times 10^4 \text{ dynes/cm}^2$

Thus, we have $\text{KE} = \frac{3}{2} \times 1.33 \times 10^4 \times 1 = 1.99 \times 10^4$ ergs

As mean kinetic energy per molecule is 4×10^{-14} ergs, the number of molecule will be

$$\frac{1.99 \times 10^4}{4 \times 10^{-14}} = 4.9 \times 10^{17} \because 5 \times 10^{17} \text{ molecules}$$

8. $P = \frac{2N}{3V}\bar{K}$ from the kinetic theory account for pressure.

$$N = \frac{3}{2}\frac{PV}{\bar{K}}$$

$$n = \frac{N}{N_A} = \frac{3}{2}\frac{PV}{\bar{K}N_A}$$

9. (a) From kinetic theory of the gases, $E_{\mathrm{trans}} = \frac{3}{2}\mathrm{PV}$

$$\Rightarrow \quad E_{\mathrm{trans}} = \frac{3}{2}(3.00 \times 1.0 \times 10^5 \text{ Pa})(5.00 \times 10^{-3}\text{m}^3)$$

$$= 2.28 \text{ kJ}$$

(b) The average kinetic energy per molecule,

$$\bar{K} = \frac{1}{2}m_0v^2 = \frac{3}{2}k_BT = \frac{3}{2}\frac{RT}{N_A}$$

$$= \frac{3(8.3 \text{ J/mol.K})(300 \text{ K})}{2(6.0 \times 10^{23})} = 6.22 \times 10^{-21} \text{ J}$$

10. (a) The volume occupied by this gas is

$V = 7.00 \text{ L}(103 \text{ cm}^3/1 \text{ L})(1 \text{ m}^3/10^6 \text{ cm}^3) = 7.00 \times 10^{-3} \text{ m}^3$

Then, the ideal gas law gives

$$T = \frac{PV}{nR} = \frac{(1.66 \times 10^6 \text{ Pa}) \times (7.00 \times 10^{-3}\text{m}^3)}{(3.50 \text{ mol})(8.3 \text{ J/mol.K})} = 400\text{K}$$

(b) The average kinetic energy per molecule in this gas is

$$\overline{KE}_{\text{molecule}} = \frac{3}{2}k_BT = \frac{3}{2}(1.38\times10^{-23}\ \text{J/K})(400\,\text{K})$$
$$= 8.28\times10^{-21}\ \text{J}$$

(c) You need to know the mass of the gas molecule to find its average speed, which in turn requires knowledge of the molecular mass of the gas.

11. We know that kinetic energy of a molecule is given by

$$\frac{1}{2}m_0\overline{v^2} = \frac{3}{2}k_BT \Rightarrow T = \frac{2}{3}\left(\frac{\frac{1}{2}m_0\overline{v^2}}{k_B}\right)$$

$$\Rightarrow \quad T = \frac{2}{3}\left(\frac{3.60\times10^{-22}}{1.38\times10^{-23}\ \text{J/K}}\right) = 17.4\ \text{K}$$

Now, $PV = nRT \Rightarrow n = \dfrac{PV}{RT}$

$$n = \frac{(1.20\times10^5\ \text{N/m}^2)(4.00\times10^{-3}\ \text{m}^3)}{(8.314\ \text{j/mol.K})(17.4\ \text{K})} = 3.32\ \text{mol}$$

ercise 3.2

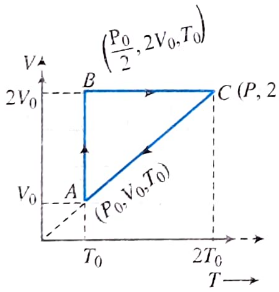
1. Ideal gas equation, in terms of density

$$\frac{P_1}{\rho_1T_1} = \frac{P_2}{\rho_2T_2} = \text{constant} \quad \therefore \quad \frac{\rho_1}{\rho_2} = \frac{P_1}{P_2}\times\frac{T_2}{T_1}$$

$$\therefore \quad \frac{\rho_{\text{Top}}}{\rho_{\text{Bottom}}} = \frac{P_{\text{Top}}}{P_{\text{Bottom}}}\times\frac{T_{\text{Bottom}}}{T_{\text{Top}}} = \frac{70}{76}\times\frac{300}{280} = \frac{75}{76}$$

2. For $A \to B$, temperature is constant (isothermal)

$$P_1V_1 = P_2V_2$$
$$2P_0V_0 = 2P_0V_2$$



V
2V0
V0
B
A
$\left(\frac{P_0}{2}, 2V_0, T_0\right)$
C (P, 2V0, 2T0)
(P0, V0, T0)
T0
2T0
T

$$\Rightarrow \quad P_2 = \frac{P_0}{2}$$

For $B \to C$, volume is constant (Isochoric)

$$\frac{P_B}{T_B} = \frac{P_C}{T_C} \Rightarrow \frac{P_0}{2T_0} = \frac{P_C}{2T_0} \Rightarrow P_C = P_0$$

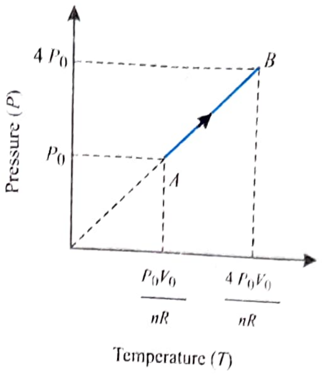
3. From the equation of state for an ideal gas, $PV = nRT$.

(a) $P = \dfrac{nRT}{V}$ i.e., $P \propto T$ (V = constant)

At $P = P_0$, and $V = V_0$, $T = \dfrac{P_0V_0}{nR}$

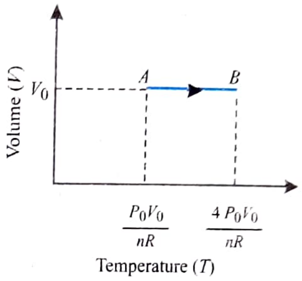
And at $P = 4P_0$, $V = V_0$, $T = \dfrac{4P_0V_0}{nR}$

The graph is a straight line (passing through the origin, when produced) as shown in figure.



4P0
B
P0
A
Pressure (P)
$\frac{P_0V_0}{nR}$
$\frac{4P_0V_0}{nR}$
Temperature (T)

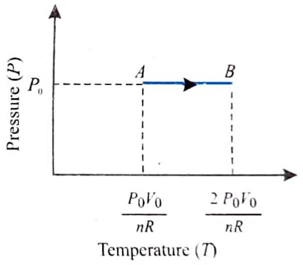
(b) Since temperature increases from $\dfrac{P_0V_0}{nR}$ to $\dfrac{4P_0V_0}{nR}$, (volume remaining constant), so, the V-T graph will be as shown in figure.



Volume (V)
V0
A
B
$\frac{P_0V_0}{nR}$
$\frac{4P_0V_0}{nR}$
Temperature (T)

4. From the equation state $PV = nRT$,

We have, $V = \left(\dfrac{nR}{P_0}\right)T$ or $V \propto T$

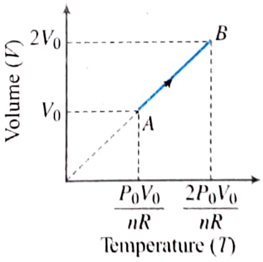


Pressure (P)
P0
A
B
$\frac{P_0V_0}{nR}$
$\frac{2P_0V_0}{nR}$
Temperature (T)

At $V = V_0$, $T_1 = \dfrac{P_0V_0}{nR}$

And at $V = 2V_0$, $T_2 = \dfrac{2P_0V_0}{nR}$

For the graph of P, versus T, the variation is a straight line normal to the pressure axis; the temperature is varying from T_1 to T_2 as shown in figure.



Volume (V)
2V0
V0
A
B
$\frac{P_0V_0}{nR}$
$\frac{2P_0V_0}{nR}$
Temperature (T)

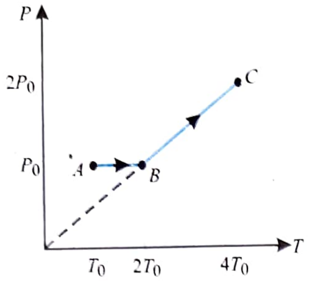
For the graph of V versus T, the equation $V = \left(\dfrac{nR}{P_0}\right)T$ or $V = kT$ shows that the volume varies directly as the temperature (Charles's law). So, the graph is a straight line inclined to the (V-T) axes, and passing through the origin (when produced) as shown in the above figure.

5. The process $A \rightarrow B$ is isobaric.

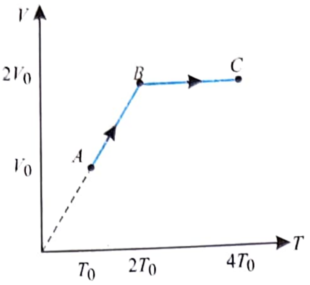
$$\therefore \quad \frac{V_0}{T_0} = \frac{2V_0}{T_B} \Rightarrow T_B = 2T_0$$

The process $B \rightarrow C$ is isochoric.

$$\therefore \quad \frac{P_1}{T_1} = \frac{P_2}{T_2} \quad \Rightarrow \quad \frac{P_0}{2T_0} = \frac{2P_0}{T_2} \quad \Rightarrow \quad T_2 = 4T_0$$



P–T graph



V–T graph

6. On combining the two vessels, the total number of moles remains constant, i.e. $n = n_1 + n_2$

Using gas equation, we can write $n_1 = \dfrac{P_1V}{RT_1}$; $n_2 = \dfrac{P_2V}{RT_2}$

And $\quad n = \dfrac{P(2V)}{RT}$

Where V is the volume of each vessel.

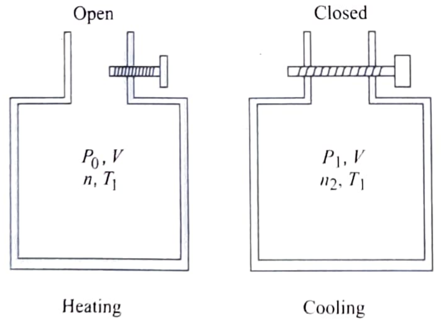
$$\text{Thus,} \quad \frac{P(2V)}{RT} = \frac{P_1V}{RT_1} + \frac{P_2V}{RT_2} \quad \text{or} \quad \frac{P}{T} = \frac{1}{2}\left[\frac{P_1}{T_1} + \frac{P_2}{T_2}\right]$$

7. As size of the flask is not changing hence the volume of the air in the flask remains constant.

Let $V = 2\text{L}$ (remains constant). When the flask is heated open to the atmosphere, its pressure remains the same at P atmosphere. Hence, using $PV = nRT$, we apply

$n_1T_1 = n_2T_2$ for the heating process , $n_1(300) = n_2(400)$

$\Rightarrow \quad 3n_1 = 4n_2$



For the cooling process (taking place at constant volume and given moles)

$$\frac{P_2}{T_2} = \frac{P_3}{T_3} \quad \Rightarrow \quad P_3 = \frac{P_2}{T_2}T_3$$

$$\Rightarrow \quad P_3 = \frac{P_{at}}{400}(300) = \frac{3}{4}\text{ atm}$$

$$\text{Loss of grams of } O_2 = (n_1 - n_2) \times 32 = \left(n_1 - \frac{3}{4}n_1\right)32$$

$$8n_1 = 8\left(\frac{P_{at}V}{RT_1}\right) = \frac{8(1 \times 2)}{0.082 \times 300} = 0.6504\text{ g}$$

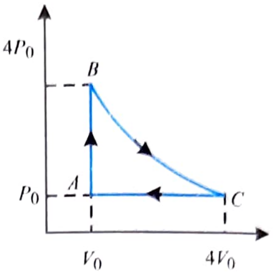
8. As the separator is conducting, the temperatures in the two parts will be the same. Suppose that the common temperature is T when the separator is in the middle. Let n_1 and n_2 be the numbers of moles of the gas in the left part and the right part, respectively. Using ideal gas equation, $p'V_1 = n_1RT$

And we get $p'V_2 = n_2RT$, $\dfrac{V_1}{V_2} = \dfrac{n_1}{n_2} = \dfrac{p_1}{p_2}$ [using (i)]

Also, $\quad V_1 + V_2 = V_0$

Thus, $\quad V_1 = \dfrac{p_1V_0}{p_1 + p_2}$ and $V_2 = \dfrac{p_2V_0}{p_1 + p_2}$

9. (a) The process is shown in a *p-V* diagram in the figure. The process starts from A and goes through $ABCA$.



(b) Applying $pV = nRT$ at A and B,

At A, $p_0V_0 = nRT_0$ ...(i)

And at B, $(4p_0)V_0 = nRT_B$ ...(ii)

From (i) and (ii)

Thus, $T_B = 4T_0$

This is the temperature in the isothermal part BC.

(c) As the process BC is isothermal, $T_C = T_B = 4T_0$

Applying $pV = nRT$ at A and C, at A, $p_0V_0 = nRT_0$

and at C, $p_0V_0 = nR(4T_0)$

or $V_C = 2V_0$

10. According to the given problem, VP^2 = constant. So the gas equation $PV = nRT$ in the light of above (eliminating P) yields.

$$\left(\frac{K}{\sqrt{V}}\right)V = nRT \Rightarrow \sqrt{V} = \frac{nR}{K}T$$

$$\text{So} \quad \sqrt{\frac{V_1}{V_2}} = \left(\frac{T_1}{T_2}\right), \text{ i.e., } \sqrt{\frac{V}{2V}} = \frac{T}{T'}, \text{ or } T' = (\sqrt{2})T$$

11. Suppose that there are n_1 moles of hydrogen and n_2 moles of oxygen in the mixture. The pressure of the mixture will be

$$p = \frac{n_1RT}{V} + \frac{n_2RT}{V} = (n_1 + n_2)\frac{RT}{V}$$

$$\text{or} \quad 100 \times 10^{-3}\text{ Pa} = (n_1 + n_2)\frac{(8.3\text{ J/mol-K})400\text{ K}}{1660 \times 10^{-6}\text{m}^3}$$

$$\text{or} \quad n_1 + n_2 = 0.1\text{ mol} \quad \text{...(i)}$$

$$\text{The mass of the mixture} = n_2 \times \frac{2\text{g}}{\text{mol}} + n_2 \times 32\frac{\text{g}}{\text{mol}} = 2.0\text{ g} \quad \text{...(ii)}$$

$$\text{or} \quad n_1 = 16n_2 = 1.0\text{ mol}$$

From (i) and (ii), $n_1 = 0.06$ mol and $n_2 = 0.04$ mol.

The mass of hydrogen is $0.06 \times 2\text{ g} = 0.12\text{ g}$ and the mass of oxygen $0.04 \times 32\text{ g} = 1.28\text{ g}$.

12. The number of moles of hydrogen $(14\text{ g})/(2\text{ g}) = 7$ and the number of moles of oxygen $= 96\text{ g}/32\text{ g} = 3$.

The total number of moles in the vessel $= 7 + 3 = 10$.

When electric spark is passed, hydrogen reacts with oxygen to form water (H_2O). Each gram of hydrogen reacts with eight grams of oxygen. Thus, 96 g of oxygen will be totally consumed together with 12 g of hydrogen. The gas left in the vessel will be 2 g of hydrogen which is $n' = 1$ mole.

Neglecting the volume of the water formed, $p'V = n'RT$

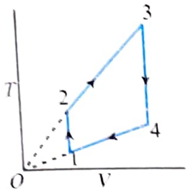
From equation, $\dfrac{p'}{p} = \dfrac{n'}{n} = \dfrac{1}{10}$

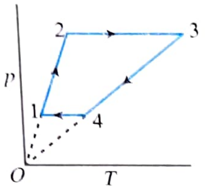
or $\quad p' = p \times 0.10 = 0.10\text{ atm}$

The plot is shown in figure.

Explanation: Since from 1–2, V remains constant, the plot of T against V is a straight line parallel to the T-axis. In the process 2–3, pressure remains constant.

For an ideal gas $pV = RT$. So when p remains constant, $V \propto T$. Hence the plot of T against V is a straight line passing through the origin. The same is repeated from 3–4 and 4–1. The plot of p against T is shown in the figure and can be explained similarly.

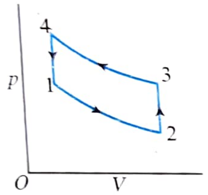


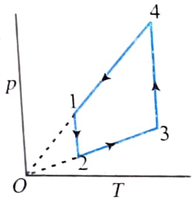


The plot p against V is shown in figure.

Explanation: Since from 1–2 temperature remains constant, the plot p against V is a curve (hyperbola) because pV = constant. From 2–3 volume remains constant, so the plot of p against V is parallel to the p-axis.

The plot of p against T is shown figure.





Exercises

Single Correct Answer Type

1. (4) Process CD is isochoric as volume is constant, process DA is isothermal as temperature is constant and process AB is isobaric as pressure is constant.

2. (2) For an isothermal process, PV = constant and for the given process PV^2 = constant.

   Therefore, the gas is cooled because volume expands by a greater exponent than in an isothermal process.

3. (1) AB is isobaric process, BC is isothermal process, CD is isochoric process and DA is isothermal process.

   These processes are correctly represented by graph (1).

4. (3) From the given V–T diagram, we can see that in process AB, $V \propto T$. Therefore pressure is constant (as quantity of the gas remains same).

   In process BC, V = constant and in process CA, T = constant.

   Therefore, these processes are correctly represented on P–V diagram by graph (3).

5. (4) Since $U\rho$ = constant, $\frac{P}{\rho} = \frac{RT}{M}$

   P = constant since ρ is increasing, therefore V is decreasing.

6. (2) $P = \dfrac{P_0}{1 + (V/V_0)^3} = \dfrac{P_0}{2}$

   $T = \dfrac{P_0 V_0}{2R}$

   Therefore translational kinetic energy is equal to

   $$\frac{3}{2}RT = \frac{3R}{2}\frac{P_0V_0}{2R} = \frac{3P_0V_0}{4}$$

7. (4) If x and y axes are represented by volume and pressure respectively, and since $P \propto \frac{1}{V}$, then parameter temperature (Z) cannot be represented by a straight line passing through origin. Option (1) is wrong. If pressure is constant, $V \propto T$ and a graph between V and T is represented by a straight line passing through origin. Further $\frac{V}{T} = \frac{nR}{P}$ or slope $\left(\frac{V}{T}\right) \propto \frac{1}{P}$. But (slope of Z_1Z_2) < (slope of Z_3Z_4). So (pressure) Z_2 > (pressure Z_3Z_4). So (pressure) Z_2 > (pressure) Z_3 and $Z_1 = Z_2$ and $Z_3 = Z_4$. So, option (2) is correct.

   For similar arguments, option (3) is also correct.

8. (4) BC is isochoric. $V_B > V_A$, $V_B = V_C$, $V_D > V_C$

9. (2) As $\lambda = \dfrac{1}{\sqrt{2}}\dfrac{kT}{\pi d^2 P}$

   $\therefore \quad P \propto \dfrac{1}{\lambda}$

   i.e., by increasing λ two times, pressure will become half.

10. (4) For process 2, $\Delta U = 0$. It will be isothermal process.

    In options (1) and (2) temperature is indicated on x-axis. If temperature is shown on x-axis, isothermal process cannot be shown as otherwise shown in the question for process 2. Hence options (1) and (2) are not correct.

    Obviously, option (3) is not correct.

11. (1) Root mean square velocity,

    $$v_{\text{rms}} = \sqrt{\frac{3RT}{M}} = 1930 \text{ m/s} \quad \text{(given)}$$

    $$\therefore \quad M = \frac{3RT}{(1930)^2} = \frac{3 \times 8.31 \times 300}{1930 \times 1930} = 2 \times 10^{-3} \text{ kg} = 2 \text{ g}$$

    i.e. the gas is hydrogen.

12. (2) The pressure in the vessel is because of the relative motion that is when collision takes place, it is the relative velocity which changes.

    The motion of the vessel as a whole does not affect the relative motion of the gas molecules with respect to the walls of the vessel, hence pressure of the gas inside the vessel, as observed by us, on the ground remains the same.

13. (4) When a molecule of an ideal gas collides elastically with a wall, the momentum transferred to each molecule will be twice the magnitude of its normal momentum.

    For the face $EFGH$, it transfers only half of that.

14. (2) At lower pressure,we can assume that given gas behaves as ideal gas so $\frac{PV}{RT}$ = constant. But, when pressure increases, the decrease in volume will not take place in same proportion.

    So, $\frac{PV}{RT}$ will increases.

15. (1) As $v_{\text{rms}} = \sqrt{\dfrac{3RT}{M}}$

    $$\therefore \quad \frac{v_A}{v_B} = \sqrt{\frac{T_A/T_B}{M_A/M_B}} = \sqrt{4} = 2 \left[\text{As } \frac{T_A}{T_B} = 4\frac{M_A}{M_B} \text{ is given}\right]$$

16. (1) From Dalton's law,

    Pressure of mixture $= P_1 + P_2 = P + P = 2P$

    Similarly, mass also will become double i.e., $2M$.

17. (3) Root mean square velocity does not depend upon the quantity of gas. For a given gas and at constant temperature, it always remains the same.

18. (2) $P = \dfrac{1}{3}\dfrac{mN}{V}v_{\text{rms}}^2 \quad \therefore \quad P \propto mv_{\text{rms}}^2$

so $$\frac{P_2}{P_1} = \frac{m_2}{m_1} \times \left(\frac{v_2}{v_1}\right)^2 = \frac{m_1/2}{m_1}\left(\frac{2v_1}{v_1}\right)^2 = 2$$

$\Rightarrow$ $P_2 = 2P_1 = 2P_0$

19. (3) $v_{rms} = \sqrt{\frac{3RT}{M}}$. According to question, T will become $2T$ and M will become $M/2$ so the value of V_{rms} will increase by $\sqrt{4} = 2$ times i.e., new root mean square velocity will be $2v$.

20. (2) At constant temperature for the given volume of gas,

$$\frac{P_1}{P_2} = \frac{m_1}{m_2}$$

$$\therefore \quad \frac{10^7}{2.5 \times 10^6} = \frac{10}{m_2} \Rightarrow m_2 = \frac{2.5 \times 10^6 \times 10}{10^7} = 2.5\,\text{kg}$$

$\therefore$ Quantity of gas taken out of the cylinder = 10 – 2.5 = 7.5 kg.

21. (4) $PV \propto$ Mass of gas × Temperature

In this problem pressure and volume remain constant,

So $M_1T_1 = M_2T_2 =$ constant

$$\therefore \quad \frac{M_2}{M_1} = \frac{T_1}{T_2} = \frac{(27+273)}{(52+273)} = \frac{300}{325} = \frac{12}{13}$$

$$M_2 = M_1 \times \frac{12}{13} = 13 \times \frac{12}{13}\,\text{g} = 12\,\text{g}$$

i.e., the mass of gas released from the flask = 13 g – 12 g = 1 g.

22. (4) $M_1 = M$. $T_1 = 60 + 273 = 333$ K

$M_2 = M - \frac{M}{4} = \frac{3M}{4}$ (as 1/4th part of air escapes)

If pressure and volume of the gas remain constant, then

$MT =$ constant

$$\therefore \quad \frac{T_2}{T_1} = \frac{M_1}{M_2} = \left(\frac{M}{3M/4}\right) = \frac{4}{3}$$

$$T_2 = \frac{4}{3} \times T_1 = \frac{4}{3} \times 333 = 444\,\text{K} = 171°\text{C}$$

23. (4) $PV \propto MT$ or $\frac{V}{T} \propto \frac{M}{P}$

Here (M/P) represents the slope of curve drawn on volume and temperature axis.

For the first condition slope (M/P) graph is D (given in the problem)

For the second condition slope $\frac{2M}{P/2} = 4\left(\frac{M}{P}\right)$

i.e., slope becomes four times so graph A is correct in this condition.

24. (2) As the quantity of gas remains constant, $\mu_A + \mu_B = \mu$

$$\frac{P_A V_A}{RT} + \frac{P_B V_B}{RT} = \frac{P(V_A + V_B)}{RT}$$

$$P = \frac{P_A V_A + P_B V_B}{V_A + V_B} = \frac{1.4 \times 0.1 + 0.7 \times 0.15}{0.1 + 0.15}$$

$\Rightarrow$ $P = 0.98$ MPa

25. (4) From Dalton's law, final pressure of the mixture of nitrogen and oxygen is

$$P = P_1 + P_2 = \frac{\mu_1 RT}{V} + \frac{\mu_2 RT}{V} = \frac{m_1}{M_1}\frac{RT}{V} + \frac{m_2}{M_2}\frac{RT}{V}$$

$$\frac{8}{32}\frac{RT}{V} + \frac{7}{28}\frac{RT}{V} = \frac{RT}{2V} \Rightarrow 10 = \frac{RT}{2V} \qquad \text{...(i)}$$

When oxygen is absorbed then for nitrogen let pressure is

$$P = \frac{7}{28}\frac{RT}{V}$$

$$\Rightarrow \quad P = \frac{RT}{4V} \qquad \text{...(ii)}$$

From Eqs. (i) and (ii), we get pressure of the nitrogen $P = 5$ atm

26. (2) Energy of 1 mol of gas $= \frac{f}{2}RT = \frac{f}{2}pV$

where f = degree of freedom

Monatomic or diatomic gases possess equal degree of freedom for translational motion and that is equal to 3, i.e., $f = 3$

$$\therefore \quad E = \frac{3}{2}PV$$

27. (4) Total internal energy of system

$$= U_{\text{oxygen}} + U_{\text{argon}} = \mu_1 \frac{f_1}{2}RT + \mu_2 \frac{f_2}{2}RT$$

$$= 2 \times \frac{5}{2}RT + 4 \times \frac{3}{2}RT = 5RT + 6RT = 11\,RT$$

[As $f_1 = 5$ (for oxygen) and $f_2 = 3$ (for argon)]

28. (3) $$\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}$$

$$\frac{(1.38 \times 10^7\,\text{Pa})\,(16\,\text{L})}{300\,\text{K}} = \frac{(10^5\,\text{Pa})\left(2.4\,\frac{\text{L}}{\text{m}}\,t\right)}{300\,\text{K}}$$

$$t = \frac{1.38 \times 10^7 \times 16}{10^5 \times 2.4} = 920\,\text{min} = \frac{920}{60}\,\text{h} = 15\,\text{h (approx.)}$$

29. (3) Internal energy of n moles of an ideal gas at temperature T is given by

$U = \frac{f}{2}nRT$ (f = degrees of freedom)

$U_1 = U_2$

$f_1n_1T_1 = f_2n_2T_2$

$$\therefore \quad \frac{n_1}{n_2} = \frac{f_2T_2}{f_1T_1} = \frac{3 \times 2}{5 \times 1} = \frac{6}{5}$$

Here, f_2 = degrees of freedom of He = 3

and, f_1 = degrees of freedom of H_2 = 5

30. (1) Let T be the temperature of the mixture.

Then $U = U_1 + U_2$

or $$\frac{f}{2}(n_1 + n_2)RT = \frac{f}{2}(n_1)(R)(T_0) + \frac{f}{2}(n)(R)(2T_0)$$

or $(2 + 4)T = 2T_0 + 8T_0$

$(\because n_1 = 2, \quad n_2 = 4)$

$$\therefore \quad T = \frac{5}{3}T_0$$

31. (1) Atmospheric pressure = P

Tension before heating = $P_0A - PA$

Tension after heating = $P_2A - PA = \frac{5}{4}P_0A - PA$

Increase in tension = $P_0A/4$

$$\%\ \text{Increase} = \frac{(1/4)P_0A \times 100}{P_0A - P.A} = \frac{25}{(1 - P/P_0)}$$

(1) The rms speed depend upon the molecular mass i.e., $v_{\text{rms}} \propto \frac{1}{\sqrt{M}}$ but kinetic energy does not depend on it i.e., $E \propto M^0$

In the $m_1 > m_2 > m_3$

$$\therefore \quad (v_{\text{rms}})_1 < (v_{\text{rms}})_2 < (v_{\text{rms}})_3 \text{ but } (\bar{K}_1) = (\bar{K}_2) = (\bar{K}_3)$$

(2) Kinetic energy of N molecules of gas, $E = \frac{3}{2}NkT$

Initially, $E_1 = \frac{3}{2}N_1kT_1$ and finally, $E_2 = \frac{3}{2}N_2kT_2$

But according to $E_1 = E_2$ and $N_2 = 2N_1$

$$\therefore \quad \frac{3}{2}N_1kT_1 = \frac{3}{2}(2N_1)kT_2 \Rightarrow T_2 = \frac{T_1}{2}$$

Since, the kinetic energy is constant,

$$\frac{3}{2}N_1kT_1 = \frac{3}{2}N_2kT_2$$

$$\Rightarrow \quad N_1T_1 = N_2T_2 \therefore NT = \text{constant}$$

From ideal gas equation of N molecule, $PV = NkT$

$$\Rightarrow \quad P_1V_1 = P_2V_2 \therefore P_1 = P_2$$

[As $V_1 = V_2$ and NT = constant]

34. (3) Mean K.E. of gas molecules, $E = \frac{3}{2}kT = \frac{3}{2}k(t + 273)$, where T is temperature is in kelvin and t is in centigrade.

$$\therefore \quad E = \frac{3}{2}kt + \frac{3}{2} \times 273\,k \text{ where } k = \text{Boltzmann's constant}$$

By comparing this equation with standard equation of straight line $y = mx + c$,

We get, $m = \frac{3}{2}k$ and $c = \frac{3}{2}273\,k$. So the graph between E and t will be straight line with positive intercept on E-axis and positive slope with t-axis.

35. (4) Average kinetic energy per unit molecule, $E = \frac{3}{2}kT$

$$\therefore \quad k = \frac{2E}{3T}$$

But, Avogadro number $= N_A = \frac{R}{k} = \frac{R}{(2E/3T)}$

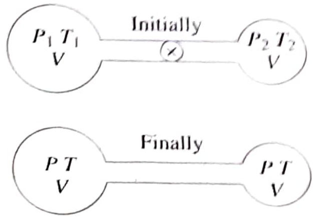
$$\therefore \quad N_A = \frac{3RT}{2E}$$

36. (1) Number of molecules in 22.4 litres gas at N.T.P. = 6.023×10^{23} or number of molecule in 22.4×10^3 cm³ gas = 6.023×10^{23} [As 22.4 litre = 22.4×10^3 cm³]

$\therefore$ Number of molecules in 1cm³ gas $= \frac{6.023 \times 10^{23}}{22400}$.

37. (4) Number of moles in the first vessel $\mu_1 = \frac{P_1V}{RT_1}$

Number of moles in the second vessel $\mu_2 = \frac{P_2V}{RT_2}$



If both vessels are joined together, then quantity of gas remains same, i.e., $\mu = \mu_1 + \mu_2$

$$\frac{P(2V)}{RT} = \frac{P_1V}{RT_1} + \frac{P_2V}{RT_2}$$

$$\Rightarrow \quad \frac{P}{T} = \frac{P_1}{2T_1} + \frac{P_2}{2T_2}$$

38. (3) From ideal gas equation

$$PV = RT \qquad \text{...(i)}$$

$$P\Delta V = R\Delta T \qquad \text{...(ii)}$$

Dividing Eq. (ii) by Eq. (i), we get

$$\frac{\Delta V}{V} = \frac{\Delta T}{T} \Rightarrow \frac{\Delta V}{V\Delta T} = \frac{1}{T} = \delta \qquad \text{(given)}$$

$$\therefore \quad \delta = \frac{1}{T}$$

So the graph between δ and T will be a rectangular hyperbola.

39. (1) $P_1 = P$, $T_1 = T$, $P_2 = P + (0.4\%$ of $P)$

$$\Rightarrow P_2 = P + \frac{0.4}{10}P = P + \frac{P}{250} \text{ and } T_2 = T + 1$$

From Gay-Lussac's law $\frac{P_1}{P_2} = \frac{T_1}{T_2}$

$$\frac{P}{P + \frac{P}{250}} = \frac{T}{T+1} \qquad \text{(as } V = \text{constant for closed vessel)}$$

By solving, we get $T = 250$ K

40. (1) From ideal gas equation, $PV = \mu RT$

$$\therefore \quad P = \frac{\mu R}{V}T$$

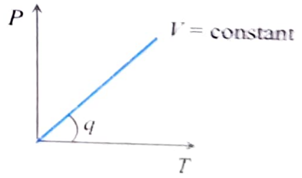
Comparing, this equation with $y = mx$

Slope of line, $\tan\theta = m = \mu R/V$

i.e., $V \propto \frac{1}{\tan\theta}$

It means line of smaller slope represents greater volume of gas.

For the given problem figure



Points 1 and 2 are on the same line, so they will represent same volume, i.e., $V_1 = V_2$

Similarly points 3 and 4 are on the same line, so they will represent same volume, i.e., $V_3 = V_4$

But $V_1 > V_3 (= V_4)$ or $V_2 > V_3 (= V_4)$ as slope of line 1–2 is less than that of 3–4.

41. (1) By Dalton's law

$$P = P_1 + P_2 + P_3 = \frac{\mu_1 RT}{V} + \frac{\mu_2 RT}{V} + \frac{\mu_3 RT}{V}$$

$$= \frac{RT}{V}[\mu_1 + \mu_2 + \mu_3] = \frac{RT}{V}\left[\frac{m_1}{M_1} + \frac{m_2}{M_2} + \frac{m_3}{M_3}\right]$$

$$= \frac{8.31 \times 300}{3 \times 10^{-3}}\left[\frac{6}{32} + \frac{8}{28} + \frac{5}{44}\right]$$

$$= 498 \times 10^3 \approx 500 \times 10^3 = 5 \times 10^5 \text{ N m}^{-2}$$

42. (4) Suppose there are n_1 moles of hydrogen and n_2 moles of helium in the given mixture. Then the pressure of the mixture will be

$$P = \frac{n_1 RT}{V} + \frac{n_2 RT}{V} = (n_1 + n_2)\frac{RT}{V} \qquad \text{...(i)}$$

$$2 \times 101.3 \times 10^3 = (n_1 + n_2)\frac{(8.3)(300)}{20 \times 10^{-3}}$$

$$(n_1 + n_2) = \frac{2 \times 101.3 \times 10^3 \times 20 \times 10^{-3}}{(8.3)(300)} = 1.62 \qquad \text{...(ii)}$$

The mass of the mixture is (in grams): $n_1 \times 2 + n_2 \times 4 = 5$

$(n_1 + 2n_2) = 2.5$

Solving Eqs. (i) and (ii), $n_1 = 0.74$, $n_2 = 0.88$

Hence $\frac{m_{\text{H}}}{m_{\text{He}}} = \frac{0.74 \times 2}{0.88 \times 4} = \frac{1.48}{3.52} \rightarrow 2:5$

43. (3) For the gas in container A

$$\Delta P = (P_A)_{\text{final}} - (P_A)_{\text{initial}} = \frac{n_A RT}{2V} - \frac{n_A RT}{V}$$

$$\Delta P = -\frac{n_A RT}{2V} \qquad \text{...(i)}$$

For gas in container B

$$1.5\,\Delta P = (P_B)_{\text{final}} - (P_A)_{\text{initial}} = \frac{n_B RT}{2V} - \frac{n_B RT}{V}$$

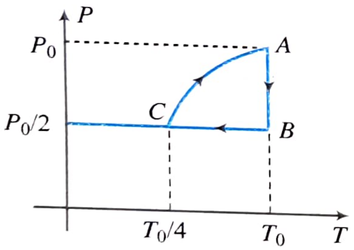
$$1.5\,\Delta P = -\frac{n_B RT}{2V} \qquad \text{...(ii)}$$

From Eqs. (i) and (ii), we get $n_B = 1.5\, n_A$

$\Rightarrow \quad 2n_B = 3n_A$

$\Rightarrow \quad 2m_B = 3m_A$

44. (3) Process AB is isothermal expansion, BC is isobaric compression and in process CA



$$P \propto \frac{nRT}{P} \Rightarrow P^2 \propto T$$

45. (2) Density of mixture, $\rho_{\text{mix}} = \frac{PM_{\text{mix}}}{RT}$

Mass of nitrogen gas, $m_N = 7 \text{ g} = 7 \times 10^{-3}$ kg

Mass of carbon dioxide, $m_{CO_2} = 11 \text{ g} = 11 \times 10^{-3}$ kg

Molecular weight of nitrogen gas, $M_N = 28 \times 10^{-3}$ kg

Molecular weight of carbon dioxide, $M_{CO_2} = 44 \times 10^{-3}$ kg

$$M_{\text{mix}} = \frac{m_N M_N}{n_N + n_{CO_2}} = \frac{\frac{m_N}{M_N} M_N + \frac{m_{CO_2}}{M_{CO_2}} M_{CO_2}}{\frac{m_N}{M_N} + \frac{m_{CO_2}}{M_{CO_2}}}$$

$$= \frac{m_N + m_{CO_2}}{\frac{n_N}{M_N} + \frac{m_{CO_2}}{M_{CO_2}}} = \frac{(7 + 11) \times 10^{-3}}{\left(\frac{7}{28} + \frac{11}{44}\right)}$$

$$= \frac{18 \times 10^{-3}}{\frac{1}{4} + \frac{1}{4}} = 36 \times 10^{-3} \text{ kg}$$

$$\rho = \frac{(1 \times 10^5)(36 \times 10^{-3})}{\frac{25}{3} \times 300} = 1.44 \text{ kg/m}^3$$

46. (4) Pressure is directly proportional to square of the diameter of the balloon.

$$\frac{P_2}{P_1} = \frac{D_2^2}{D_1^2} \qquad \text{...(i)}$$

When, $V_2 = 2V_1 \Rightarrow D_2^3 = 2D_1^3$

$$\Rightarrow \quad \frac{D_2}{D_1} = \left(\frac{V_2}{V_1}\right)^{1/3} \qquad \text{...(ii)}$$

From Eqs. (i) and (ii).

$$\Rightarrow \quad \frac{P_2}{P_1} = \left(\frac{V_2}{V_1}\right)^{2/3} \quad \Rightarrow PV^{-2/3} = \text{constant}$$

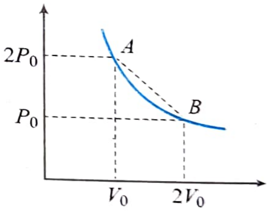
47. (1) Given $\frac{P_1}{P_2} = \frac{n_1 T_2}{n_2 T_2}$

$P \propto nT$. It implies that volume of both containers is same. After mixing,

$$P(2V) = (n_1 + n_2)RT = \left(\frac{P_1 V}{RT_1} + \frac{P_2 V}{RT_2}\right)RT$$

$$P/T = \frac{1}{2}\left(\frac{P_1}{T_1} + \frac{P_2}{T_2}\right) = \frac{1}{2}\left(\frac{P_1 T_2 + P_2 T_1}{T_1 T_2}\right)$$

48. (3) $P = -\left(\frac{P_0}{V_0}\right)V + 3P_0$



$$PV = -\frac{P_0}{V_0}V^2 + 3P_0 V \qquad \text{...(i)}$$

or $$T = \frac{1}{nR}\left[-\frac{P_0}{V_0}V^2 + 3P_0 V\right] \qquad \text{...(ii)}$$

For $T_{\text{max}}, \frac{dT}{dV} = 0$

or $+3P_0 - \frac{2P_0 V}{V_0} = 0$ or $V = \frac{3}{2}V_0 \qquad \text{...(iii)}$

From Eq. (i), $P = +\frac{3}{2}P_0$

49. (4) From the last problem, from Eq. (ii), at point $A(V = V_0)$

$$T = \frac{2P_0 V_0}{nR}$$

This is the temperature which is constant along isothermal process

T_{max} is for $V = \frac{3}{2}V_0$

$$T_{\text{max}} = \frac{9P_0 V_0}{4nR} \qquad \text{[from Eq. (iii)]}$$

$$T_{\max} - T = \frac{P_0V_0}{4nR}$$

4) Initial volume $= 1 \times \left[\pi R^2 + \frac{\pi R^2}{4}\right] = \frac{5}{4}\pi R^2$

Piston moves towards left by 1 m on heating. Addi-tional volume available due to heating is

$$= 1 \times \pi R^2 - \frac{1 \times \pi R^2}{4} = \frac{3}{4}\pi R^2$$

Total volume after heating $= 2\pi R^2$

$$P_0 \times \frac{5}{4}\pi R^2 = nRT_0 \qquad \text{...(i)}$$

After heating, $P_2 \times 2\pi R^2 = nR \times 2T_0$ ...(ii)

$$\left(\frac{P_1}{P_2}\right) \times \frac{5}{4} \times \frac{1}{2} = \frac{1}{2}$$

$$P_2 = 5P_0/4$$

(4) $2m_A = 3m_B$

$$p_AV = \frac{m_A}{M}RT \qquad \text{...(i)}$$

$$p_BV = \frac{m_B}{m}RT = \frac{2}{3}\frac{m_A}{M}RT \qquad \text{...(ii)}$$

So, $\frac{p_A}{p_B} = \frac{3}{2}$

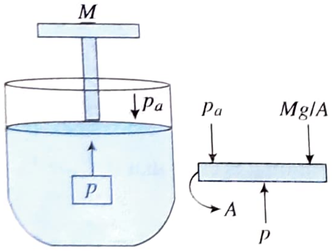
The expansion is isothermal, so pressure will reduce to half when volume is doubled.

$$p_A - \frac{p_A}{2} = \Delta p \quad \text{or} \quad p_A = 2\Delta p$$

$$p_B - \frac{p_B}{2} = \frac{p_B}{2} = \frac{1}{2}\cdot\frac{2}{3}p_A = \frac{1}{3}p_A = \frac{2}{3}\Delta p$$

So, $p_B = \frac{4}{3}\Delta p$

3) Consider the FBD of piston.



The pressure inside the gas will be $p = pa + \frac{Mg}{A}$

where, p_a = atmospheric pressure

A = area of cross-section of the piston

Mg = weight of piston

Hence, p = constant

When temperature increases as $pV = nRT$ thus, volume (V) increases at constant pressure.

3. (4) When the molecules break into atoms, the number of moles would become twice.

Now, by ideal gas equation

P = Pressure of gas, n = Number of moles

R = Gas constant, T = Temperature

We have $pV = nRT$

As volume (V) of the container is constant.

As gases break, number of moles becomes twice of initial number, so $n_2 = 2n_1$

So $\quad p \propto nT$

$$\Rightarrow \quad \frac{p_2}{p_1} = \frac{n_2T_2}{n_1T_1} = \frac{(2n_1)(3000)}{n_1(300)} = 20$$

$$\Rightarrow \quad p_2 = 20\,p_1$$

Hence, final pressure of the gas would be 20 times the pressure initially.

54. (4) Quantity of gas in these bulbs is constant i.e., initial number of moles in both bulbs = final number of moles

$$(n_1 + n_2)_{\text{initial}} = (n_1 + n_2)_{\text{final}}$$

$$\frac{PV}{R(273)} + \frac{PV}{R(273)} = \frac{1.5PV}{R(273)} + \frac{1.5PV}{R(T)}$$

$$\Rightarrow \quad \frac{2}{273} = \frac{1.5}{273} + \frac{1.5}{T} \Rightarrow T = 819\text{ K} = 546°\text{C}.$$

55. (4) Root mean square velocity of molecule in left part

$$v_{\text{rms}} = \sqrt{\frac{3KT}{m_L}}$$

Mean or average speed of molecule in right part

$$v_{\text{av}} = \sqrt{\frac{8\,KT}{\pi\,m_R}}$$

According to question, $\sqrt{\frac{3KT}{m_L}} = \sqrt{\frac{8\,KT}{\pi\,m_R}} \Rightarrow \frac{3}{m_L} = \frac{8}{\pi m_R}$

$$\Rightarrow \quad \frac{m_L}{m_R} = \frac{3\pi}{8}.$$

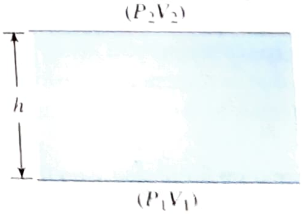
56. (3) If the gas is not ideal then its molecule will possess potential energy. Hence statement (1) is wrong.

rms speed of different gases at same temperature depends on its molecular weight $\left(v_{\text{rms}} \propto \frac{1}{\sqrt{M}}\right)$. Hence statement (2) is also wrong.

Kinetic energy of one gram gas depends on the molecular weight $\left(E_{\text{gm}} \propto \frac{1}{M}\right)$. Hence statement (3) is also wrong.

But K.E. of one mole of ideal gas does not depend on the molecular weight $\left(E = \frac{3}{2}RT\right)$. Hence (4) is correct.

57. (4) According to Boyle's law product of pressure and volume will remain constant at the bottom and top.



If P is the atmospheric pressure at the top of the lake and the volume of bubble is V then from $P_1V_1 = P_2V_2$

$$(P + h\rho g)V_0 = PV \Rightarrow V = \left(\frac{P + h\rho g}{P}\right)V_0$$

$$\therefore \quad V = V_0\left[1 + \frac{\rho gh}{P}\right]$$

58. (2) Mean free path λ = 0.8 × 10 [illegible] m

Number of molecules per unit volume, n = 2.7 × 10 [illegible] per m^3

Substituting these values in $\lambda = \frac{1}{\sqrt{2}\pi nd^2}$, we get

$$d = \sqrt{1.04 \times 10^{-19}} = 3.2 \times 10^{-10}\text{ m} = 3.2\text{ Å}$$

59. **(1)** Molar specific heat = Molecular weight × Gram specific heat

$C_v = M \times c_v$

$$\Rightarrow 2.98 \frac{\text{calorie}}{\text{mole} \times \text{kelvin}} = M \times 0.075 \frac{\text{kcal}}{\text{kg-kelvin}}$$

$$= M \times \frac{0.075 \times 10^3}{10^3} \frac{\text{calorie}}{\text{gm} \times \text{kelvin}}$$

$\therefore$ molecular weight of argon, $M = \frac{2.98}{0.075} = 39.7\,\text{gm}$

i.e., mass of 6.023×10^{23} atom = 39.7 gm

$\therefore$ mass of single atom $= \frac{39.7}{6.023 \times 10^{23}} = 6.60 \times 10^{-23}\,\text{gm}$

60. **(1)** $n_1RT_1 + n_2RT_2 = (n_1 + n_2)RT$

$$T = \frac{1}{2} \times 300 + \frac{1}{2} \times 310 = 305\text{ K} = 32°\text{C}$$

61. **(1)** The elastic collisions of the molecules with a moving part of the systems increases their energy.

When the gas is compressed adiabatically, the total work done on the gas increases its internal energy which in turn increases the KE of gas molecules and hence, the collisions between molecules also increase.

Multiple Correct Answers Type

1. **(1), (3), (4)**

For all gases coefficient of expansion is $1/T$ so at a given temperature it is a constant hence option (1) is correct. In each degree of freedom always average energy is same and is equal to 0.5 kT hence, option (3) is correct. As pressure of gas decreases, density of gas also decreases hence, the intermolecular separation increases hence option (4) is correct.

2. **(1),(4)**

The rate of collisions of the molecules with per square meter of the wall is $\left(\frac{1}{6}\right) n_0 v$ where, n_0 is the molecular density and v is RMS speed of molecules and pressure exerted by the gas on wall is given by $\left(\frac{1}{6}\right) n_0 v \times 2m'v$ where m' is the mass of each molecule.

3. **(2), (4)**

As ions of hydrogen are present inside the cube, they are having charge. Now, due to presence of positive charge on the surface EFGH, hydrogen ions would experience forces of electrostatic nature.

The usual expression for pressure on the basis of kinetic theory will not be valid as ions would also experience electrostatic forces other than the forces due to collisions with the walls of the container. Due to presence of positive charge, the isotropy is also lost.

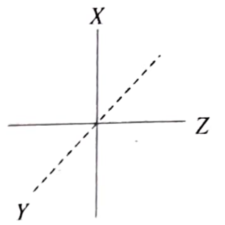
4. **(1), (4)**

Consider a diatomic molecule as shown in the diagram.

The total energy associated with the molecule is

$$E = \frac{1}{2}mv_x^2 + \frac{1}{2}mv_y^2 + \frac{1}{2}mv_z^2 + \frac{1}{2}I_x\omega_x^2 + \frac{1}{2}I_y\omega_y^2$$

This above expression contains translational kinetic energy $\left(\frac{1}{2}mv^2\right)$ corresponding to velocity in each x, y and z-directions as well as rotational KE $\left(\frac{1}{2}I\omega^2\right)$ associated with axis of rotations x and y.



The number of independent terms in the above expression is 5.

As we can predict velocities of molecules by Maxwell's distribution, hence the above expression also obeys Maxwell's distribution,

$\because$ 2 rotational and 3 translational energies are associated with each molecule.

$\therefore$ Rotational energy at a given temperature is $\left(\frac{2}{3}\right)^{\text{rd}}$ of translational KE of each molecule.

5. **(1), (3)**

We know that ideal gas equation is

$$pV = nRT \quad \text{...(i)}$$

When p = constant, From (i), volume $V \propto$ Temperature T

(2) When T = constant,

From (i), pV = constant

So, the graph is rectangular hyperbola.

(3) When V = constant,

From (i), $p \propto T$

So, the graph is straight line passing through the origin.

(4) From (i), $pV \propto T$

$\Rightarrow \frac{pV}{T}$ = constant

$\Rightarrow$ So, the graph passes through origin.

So, (4) is not correct.

6. **(2), (3)**

According to the problem, mass of gases is equal so number of moles will not be equal, i.e., $\mu_A \neq \mu_B$

From ideal gas equation, $PV = \mu RT$

$\therefore \frac{P_AV_A}{\mu_A} = \frac{P_BV_B}{\mu_B}$ (as temperatures of the containers are equal)

From this relation it is clear that if $P_A = P_B$; then $\frac{V_A}{V_B} = \frac{\mu_A}{\mu_B} \neq 1$

i.e., $V_A \neq V_B$

Similarly, if $V_A = V_B$ then $\frac{P_A}{P_B} = \frac{\mu_A}{\mu_B} \neq 1$

i.e., $P_A \neq P_B$

7. **(1), (4)**

$$V_{\text{rms}} = \sqrt{\frac{3kT}{m}}$$

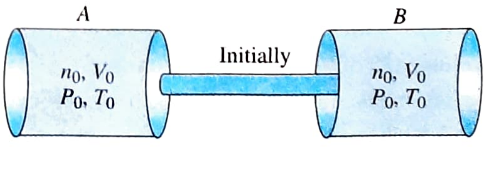
Since $PV = nRT$, therefore P and V both can change simultaneously keeping the temperature constant.

8. **(2), (3)**

Initially for container A, $P_0V_0 = n_0RT_0$

For container B, $P_0V_0 = n_0RT_0$

$$\therefore \quad n_0 = \frac{P_0V_0}{RT_0}$$



Total number of moles $= n_0 + n_0 = 2n_0$

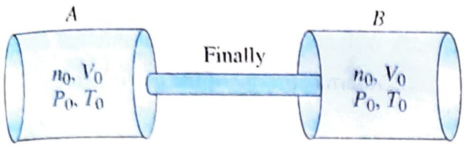
Since even on heating, the total number of moles is conserved,

We have $n_1 + n_2 = 2n_0$ ...(i)

Let P be the common pressure. Then for container A,

$$PV_0 = n_1 R\, 2T_0$$

$$\therefore \quad n_1 = \frac{PV_0}{2RT_0}$$



And for container B, $PV_0 = n_2 RT_0$

$$\therefore \quad n_2 = \frac{PV_0}{RT_0}$$

Substituting the values of n_0, n_1 and n_2 in equation (i), we get

$$\frac{PV_0}{2RT_0} + \frac{PV_0}{RT_0} = \frac{2.P_0V_0}{RT_0} \Rightarrow P = \frac{4}{3}P_0$$

Number of moles in container A (at temperature $2T_0$)

$$= n_1 = \frac{PV_0}{2RT_0} = \left(\frac{4}{3}P_0\right)\frac{V_0}{2RT_0} = \frac{2}{3}\frac{P_0V_0}{RT_0}\left[\text{As } P = \frac{4}{3}P_0\right]$$

(3)

$$\frac{P^2}{\rho} = k \quad \Rightarrow \quad \frac{P^2RT}{PM} = k$$

$$PT = \left(\frac{kM}{R}\right) \Rightarrow P \propto \frac{1}{T} \qquad \text{...(i)}$$

$$\frac{P^2}{\rho} = \frac{P'^2}{\rho/2} \quad \Rightarrow \quad P' = \frac{P}{\sqrt{2}}$$

Hence, from Eq. (i) $T' = T\sqrt{2}$

$PT =$ constant, hence P–T curve is a parabola.

(1), (2), (3), (4)

Since both the gases are contained in the same vessel, temperature of both the gases is same.

Average KE per molecule of a diatomic gas is 5/2 KT. Hence, average KE per molecule of both the gases is same. Therefore, option (1) is correct.

$$v_{rms} = \sqrt{\frac{3RT}{M}}$$

$$\text{Hence, } \frac{(v_{rms})_2}{(v_{rms})_1} = \sqrt{\frac{M_1}{M_2}} = \sqrt{16} = 4$$

Hence, option (2) is correct.

Let molar mass of B be M, then that of A will be equal to $16M$.

Let mass of gas B in the vessel be m; then that of A will be $2m$.

The number of moles of a gas, in the vessel will be $n = m/M$.

Hence, number of moles of gases A and B will be

$$n_1 = \frac{2m}{16M} \quad \text{and} \quad n_2 = \frac{m}{M}$$

$$\text{Hence, } \frac{n_1}{n_2} = \frac{1}{8}$$

Hence, option (4) is correct.

Partial pressure exerted by a gas is $P = \dfrac{nRT}{V}$

$$\text{Hence, } \frac{P_2}{P_1} = \frac{n_2}{n_1} = 8$$

Therefore, option (3) is also correct.

11. (1), (3)

Process AB in the given figure is an isobaric process. During this process, $V \propto T$

But $\quad PV = nRT$

or $\quad PV \propto T$

Therefore, during this process, pressure P remains constant. Process BC is an isochoric cooling. During this process, volume of the gas remains constant but the temperature decreases.

Process CA is an isothermal process. During this process, volume decreases and temperature remains constant. Hence, pressure increases during this process. Hence, on P–V diagram, process AB will be a straight line parallel to the V-axis, process BC will be a straight line parallel to the P-axis and CA will be a rectangular hyperbola. Hence, option (3) is correct, while (4) is wrong.

On P–V diagram, process AB will be a straight line parallel to the P-axis, during which temperature increases. Process BC will be a straight line passing through origin, during which temperature and pressure both decrease and process CA will be a straight line parallel to the P-axis during which pressure increases. Hence, option (1) is correct while (2) is wrong.

12. (2), (3), (4)

Statement (1) is incorrect. It is true only if the two bodies have the same thermal capacity.

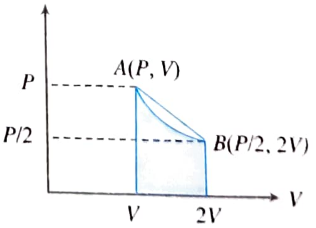
Statement (2) is correct. The coolant is used to prevent the engine or the nuclear plant from becoming too hot. The heat absorbed by a substance per unit mass is directly proportional to its specific heat. Therefore, if a coolant has a high specific heat, it will remove a large amount of heat from the engine or the nuclear plant.

Statement (3) is correct. If the volume of vapour is decreased, at a constant temperature a part of the vapour will condense into liquid such that the vapour pressure remains unchanged. In other words, the pressure does not increase but the volume of vapour decreases.

Statement (4) is also correct. Since the vessels are of the same capacity, the volume occupied by the gas is doubled, hence the pressure reduces to half.

13. (1), (2), (4)

Figure shows the straight line path along with the corresponding isothermal path. Since, the work done by the gas is equal to area under the curve (such as shown in the figure by the shaded portion for the isothermal path), it is obvious that the gas does more work along the straight line path as compared with that for the isothermal path.



As the volume is increased from V to $2V$, the difference of pressure between the straight line path and isothermal path initially increases and then decreases after attaining a maximum value. The same trend is observed in the case of temperature ($P \propto T$, $\therefore$ V is constant)

Now, the slope of straight line path is $m = \dfrac{P - P/2}{V - 2V} = -\dfrac{P}{2V}$

or $\quad P = -2V\,m$

Putting this in the ideal gas equation, $PV = nRT$

or $\quad [-2V\,m]\,V = nRT$

$$V^2 = \frac{nR}{2m}T$$

$$V^2 = kT$$

Which is the equation of a parabola.

Similarly, eliminating V from ideal gas equation, we get

$$P\left[\frac{P}{2m}\right] = nRT$$

or $\quad P^2 = (\text{constant})T$

Which is again an equation of a parabola.

14. (1), (2), (3), (4)

In the equilibrium position, the net force on the partition will be zero.

Hence pressure on both sides is same.

Hence, (1) is correct.

$$n_1 = \frac{P_1V_1}{RT_1} = \frac{PV}{RT}$$

$$n_2 = \frac{(2P)(2V)}{RT} = \frac{4PV}{RT} \Rightarrow n_2 = 4n_1$$

Moles remain conserved.

Finally, pressure becomes equal in both the parts.

Using, $P_1V_1 = n_1RT_1$

$$P_2V_2 = n_2RT_2$$

$$P_1 = P_2 \text{ and } T_1 = T_2$$

$$\frac{V_1}{V_2} = \frac{n_1}{n_2} = \frac{1}{4}$$

$$V_2 = 4V_1$$

Also. $\quad V_1 + V_2 = 3V \Rightarrow V_1 + 4V_1 = 3V$

$$V_1 = \frac{3}{5}V \text{ and } V_2 = \frac{12}{5}V$$

Hence, options (2) and (3) are correct.

In compartment I: $P_1'V_1 = n_1RT_1$

$$P_1'\left(\frac{3V}{5}\right) = \left(\frac{PV}{RT}\right)R(T)$$

$$P_1' = \frac{5PV}{3V} = \frac{5}{3}P$$

Hence, option (4) is also correct.

15. (1), (3)

Initial state is same for all the three processes (say initial internal energy = E_0).

In the final state, $V_A = V_B = V_C$ and $P_A > P_B > P_C$

$$P_AV_A > P_BV_B > P_CV_C$$

$$E_A > E_B > E_C$$

If $T_1 < T_2$, then $E_0 > E_f$ for all the three processes and hence

$$(E_0 - E_A) < (E_0 - E_B) < (E_0 - E_C)$$

$$|\Delta E_A| < |\Delta E_B| < |\Delta E_C|$$

If $T_1 < T_2$, then $E_0 < E_f$ for all the three processes and hence

$$(E_A - E_0) > (E_B - E_0) > (E_C - E_0)$$

$$|\Delta E_A| > |\Delta E_B| > |\Delta E_C|$$

16. (1), (2), (3), (4)

Equilibrium of piston gives $PS = Kx_0$

$$P = \frac{Kx_0}{S}$$

Since the chamber is thermally insulated, $\Delta Q = 0$

∴ Elastic PE of spring = Work done by gas

or, Work done by gas $= \frac{1}{2}Kx_0^2$

This work is done at the expense of internal energy of the ga

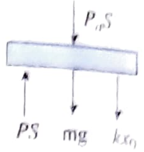
Therefore, internal energy of the gas is decreased by $(1/2)Kx$

17. (1), (3)

Equilibrium of piston gives

$$PS = P_aS + mg + Kx_0$$

$$P = P_a + \frac{mg}{S} + \frac{Kx_0}{S}$$



(P = final pressure of gas)

Work done by the gas = work done against atmospheric pressu + elastic potential energy stored in the spring + increase gravitational potential energy of the piston

$$= P_a\Delta V + \frac{1}{2}Kx_0^2 + mgx_0 = P_aSx_0 + \frac{1}{2}Kx_0^2 + mgx_0$$

There occurs decrease in internal energy of the gas, becaus the gas is thermally insulated and hence, work is done at th expense of internal energy of the gas.

Linked Comprehension Type

For Problems 1–2

Since volumes and temperatures for the two vessels A and B ar the same, so $P \propto n$,

But $n \propto N$

∴ $\quad P \propto N$

where N be the total number of molecules.

If N_1 and N_2 be the total number of molecules in the two vessel A and B, then

$$\frac{N_2}{N_1} = \frac{P_B}{P_A} = \frac{1.5 \times 10^5 \text{ Pa}}{1.0 \times 10^5 \text{ Pa}} = \frac{3}{2} \qquad \text{...(i}$$

Now, if x is the number of molecules transferred from vesse B to vessel A, until the pressure difference between the tw becomes $\Delta P = 0.25 \times 10^5$ Pa, when the valve stops enterin the gas from B to A, then, the pressures in A and B will be respectively,

$$P_A' = \frac{(N_1 + x)}{N_1} \times 10^5 \text{ Pa}$$

and $\quad P_B' = \dfrac{(N_2 - x) \times 1.5 \times 10^5 \text{ Pa}}{N_2}$

∵ $\quad P_B' - P_A' = \Delta P$

So $\quad \dfrac{(N_2 - x)}{N_2} \times 1.5 \times 10^5 \text{ Pa} - \dfrac{(N_1 + x)}{N_1} \times 10^5 \text{ Pa}$

$$= \frac{1}{4} \times 10^5 \text{ Pa}$$

or $\quad x\left(\dfrac{6}{N_2} + \dfrac{4}{N_1}\right) = 1$ or $\dfrac{x}{N_2}\left(6 + \dfrac{4N_2}{N_1}\right) = 1$

Substituting $\frac{N_2}{N_1} = \frac{3}{2}$ from Eq. (i), we get

$$\frac{x'}{N_2} = \frac{1}{12} \text{ and } \frac{x}{N_1} = \frac{1}{8}$$

Thus, the fraction of molecules escaped from vessel B is $\frac{1}{12}$ and the fraction of molecules entered into vessel A is $\frac{1}{8}$.

roblems 3–5

1) The rms speed of oxygen molecules in a gas is related to the temperature by $v_{\text{rms}} = \sqrt{\frac{3kT}{m_{O_2}}}$ which can be rearranged and solved numerically for the temperature, $T = \frac{m_{O_2} v_{\text{rms}}^2}{3k}$

Here m is the mass of the gas molecule.

The mass of oxygen molecule,

$$m_{O_2} = 32 \times \text{amu} = 32\ (1.66 \times 10^{-27}\ \text{kg})$$

$$T = \frac{32(1.66 \times 10^{-27}\ \text{kg})(535\ \text{m/s})^2}{3(1.38 \times 10^{-23}\ \text{J/k})} = 367\ \text{K}$$

1) The rms speed is inversely related to the mass of the gas molecule (the mass is in the denominator of the square-root function above). The rms speed of nitrogen would be higher because the molar mass of nitrogen is less than that of oxygen.

2) The rms speed of N_2, $v_{\text{rms}} = \sqrt{\frac{3\ kT}{m_{N_2}}}$

Here mass of N_2 molecule = 28 × amu

$$\Rightarrow \quad m_{N_2} = 28\ (1.66 \times 10^{-27}\ \text{kg})$$

$$v_{\text{rms}} = \sqrt{\frac{3(1.38 \times 10^{-23}\ \text{J/K})(367\ \text{K})}{28(1.66 \times 10^{-27}\ \text{kg})}} = 572\ \text{m/s}$$

roblems 6 and 7

3) $P_A V_0 = nRT_0$

$$\Rightarrow \quad n = \frac{P_A V_0}{RT_0}$$

Number of atoms $N = nA = \left(\frac{P_A V_0}{RT_0}\right) A = \frac{P_A V_0}{kT_0}$

[A = Avogadro number]

(2) Internal energy of n moles of gas $U = n \cdot \frac{1}{2} fRT$

Here f is degree of freedom

$$\Rightarrow \quad U = n\frac{3}{2}RT = \frac{3}{2}nRT = \frac{3}{2}P_A V_0$$

Problems 8–9

(2) Total internal energy before opening the valve and after opening the valve should be the same.

$$U_{\text{initial}} = n_1 \frac{f}{2} RT_1 + n_2 \frac{f}{2} RT_2$$

Here f is the degree of freedom. Oxygen is diatomic hence here f is equal to 5.

$$U_{\text{initial}} = \frac{5}{2}\left[n_1 RT_1 + n_2 RT_2\right] = \frac{5}{2}\left[P_0 V_0 + \left(\frac{P_0}{3}\right)(3V_0)\right]$$

$$U_{\text{initial}} = 5\ P_0 V_0 \qquad \ldots\text{(i)}$$

When valve is opened,

$$U_{\text{final}} = (n_1 + n_2)RT_f = \left[\frac{P_0 V_0}{RT_0} + \frac{(P_0/3)(3V_0)}{R(2T_0)}\right] RT_f$$

$$U_{\text{final}} = \left[\frac{3}{2}\frac{P_0 V_0}{T_0}\right] T_f \qquad \ldots\text{(ii)}$$

As energy should be the same, from Eqs. (i) and (ii)

$$\left[\frac{3}{2}\frac{P_0 V_0}{T_0}\right] T_f = 5 P_0 V_0 \Rightarrow T_f = \frac{10}{3} T_0$$

9. (3) The pressure of the whole sample in its final state is

$$P_f = (n_1 + n_2)\frac{R}{V_f} T_f$$

$$= \left(\frac{P_1 V_1}{RT_1} + \frac{P_2 V_2}{RT_2}\right)\left(\frac{R}{V_1 + V_2}\right)\frac{P_1 V_1 + P_2 V_2}{\left(\frac{P_1 V_1}{T_1} + \frac{P_2 V_2}{T_2}\right)}$$

$$P_f = \left(\frac{P_1 V_1 + P_2 V_2}{V_1 + V_2}\right) = \frac{P_0}{2}$$

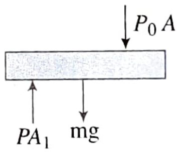
$$P_f = \frac{P_0 V_0 + (P_0/3)(3V_0)}{(V_0 + 3V_0)} = \frac{P_0}{2}$$

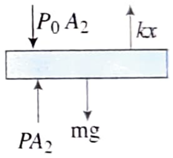
For Problems 10–12

10. (3) $PA_1 = mg + P_0 A_1$

$PA_2 + kx = P_0 A_2 + mg$

Here P_0 = atmospheric pressure and x = initial expression of spring II





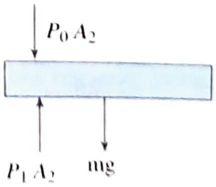
$$\Rightarrow \quad PA_2 + kx = \left(P - \frac{mg}{A_1}\right) A_2 + mg$$

$$PA_2 + kx = PA_2 + mg\left(1 - \frac{A_2}{A_1}\right)$$

$$x = \frac{mg}{K}\left(1 - \frac{A_2}{A_1}\right)$$

11. (2) Considering FBD of piston II, $P_1 = \frac{mg}{A_2} + P_0$

$$P_1 = \frac{mg}{A_2} + P - \frac{mg}{A_1} = P + mg\left(\frac{A_1 - A_2}{A_1 A_2}\right)$$

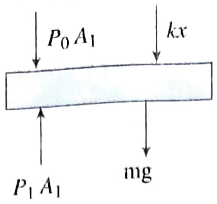


12. (2) $V_f = V + A_1 x_1 + A_2 x$

$$V_f = V + A_1 x_1 + A_2\left(\frac{mg}{k}\left(1 - \frac{A_2}{A_1}\right)\right)$$

From FBD of piston I,

$$\Rightarrow \quad P_1 = P_0 + \frac{kx}{A_1} + \frac{mg}{A_1}$$



$$\frac{mg}{A_2} + P_0 = P_0 + \frac{kx_1}{A_1} + \frac{mg}{A_1}$$

$$\Rightarrow \quad \frac{kx_1}{A_1} = mg\left[\frac{1}{A_2} - \frac{1}{A_1}\right]$$

$$\Rightarrow \quad x_1 = \frac{mg}{k}\frac{(A_1 - A_2)}{A_2}$$

$$\Rightarrow \quad V_f = V + A_1\left[\frac{mg}{kA_2}(A_1 - A_2)\right] + \frac{A_2 mg}{kA_1}(A_1 - A_2)$$

$$= V + \frac{mg}{k}(A_1 - A_2)\left[\frac{A_1}{A_2} + \frac{A_2}{A_1}\right]$$

$$= V + \frac{mg}{k}(A_1 - A_2)\left(\frac{A_1^2 + A_2^2}{A_2 A_1}\right)$$

$$V_f = V + \frac{mg}{k}\frac{(A_1 - A_2)(A_1^2 + A_2^2)}{A_1 A_2}$$

Numerical Value Type

1. (0.5)

By gas law we use $PV = nRT$

$$2 \times 20 = \left(\frac{m_1}{2} + \frac{m_2}{4}\right)(0.082)(293)$$

$$\Rightarrow \quad 2m_1 + m_2 = 6.66 \text{ and } m_1 + m_2 = 5$$

On Solving we get, $m_1 = 1.66\,g$, $m_2 = 3.34\,g$

$$\Rightarrow \quad \frac{m_1}{m_2} = \frac{1}{2}$$

2. (100)

If initial pressure and temperature are P and T respectively then

$$\frac{P}{T} = \frac{1.01P}{T+1}$$

$$\Rightarrow \quad T + 1 = 1.01T \quad \Rightarrow 0.01T = 1 \quad \Rightarrow T = 100 \text{ K}$$

3. (2) Using $P = \dfrac{nRT}{V}$, we get $\dfrac{P_{He}}{P_{O_2}} = \dfrac{2T}{V}\dfrac{V}{T} = 2$

4. (1) Both gases will have same number of rotational degree of freedom (equal to 2). Since both are diatomic and are at the same temperature. Average rotational kinetic energy per molecule for both the gases $= 2\left(\dfrac{1}{2}KT\right)$.

Required ratio = 1

5. (3) Since $PV = nRT = \dfrac{m}{M}RT$ we have $P = \dfrac{RT}{V}\left[\dfrac{m_1}{M_1} + \dfrac{m_2}{M_2} + \dfrac{m_3}{M_3}\right]$

where m's are mass of given gas and M is its molecular mass

$$P = \frac{8.3 \times 300}{10 \times 10^{-3}}\left[\frac{8}{32} + \frac{14}{28} + \frac{22}{44}\right] = 3.11 \times 10^5 \text{ N/m}^2$$

In 10^5 N/m², $P = 3$

6. (5.0)

Kinetic energy $E = 1.5 \times 10^5$ J,

Volume $V = 20$ litres $= 20 \times 10^{-3}$ m³

$$\text{Pressure} = \frac{2}{3}\frac{E}{V} = \frac{2}{3}\left(\frac{1.5 \times 10^5}{20 \times 10^{-3}}\right) = 5 \times 10^6 \text{ N/m}^2$$

7. (3) No change of temperature

So, $(n_1 + n_2)_i = (n_1 + n_2)_f$

$$\frac{P_1V_1}{RT} + \frac{P_2V_2}{RT} = \frac{PV_1}{RT} + \frac{PV_2}{RT}$$

$$\text{So,} \quad P = \frac{P_1V_1 + P_2V_2}{V_1 + V_2} = \frac{(4P_0)(3V) + (1.5P_0)(2V)}{5V}$$

$$= 3P_0 = IP_0 \quad \text{(given)}$$

So, $I = 3$

8. (0) $P_0V_0 = nRT_a$

$(P_0 + \Delta P)(V_0 + \Delta V) = nRT_b$

$(P_0 + \Delta P)(V_0 - \Delta V) = nRT_b$

$\Delta V = 0$

Pressure depends on number of moles and is independent of nature of gas.

9. (1) As the gas leaks out, the volume and the temperature of the remaining gas do not change. The number of moles of the gas in the vessel is given by $n = \dfrac{pV}{RT}$. The number of moles in the vessel before the leakage is $n_1 = \dfrac{p_1V}{RT}$ ...(i)

And that after the leakage is $n_2 = \dfrac{p_2V}{RT}$ ...(ii)

Thus, the amount leaked is, $n_1 - n_2 = \dfrac{(p_1 - p_2)V}{RT}$

$$= \frac{(200 - 125) \times 10^3 \text{ N/m}^2 \times 33.2 \times 10^{-3} \text{ m}^3}{(8.3 \text{ J/mol-K}) \times 300 \text{ K}} = 1 \text{ mol}$$

10. (8) Initially, the spring is in its relaxed state. So, the pressure of the gas equals the atmospheric pressure.

Initial pressure $= p_1 = 1.0 \times 10^6$ N/m²

$$\text{Final pressure} = p_2 = p_1 + \frac{kx}{A}$$

$$= 1.0 \times 10^6 \text{ N/m}^2 + \frac{(8000 \text{ N/m})(0.1 \text{ m})}{8.0 \times 10^{-3} \text{m}^2}$$

$$= 2.0 \times 10^5 \text{ N/m}^2$$

Final volume $= V_2 = V_1 + Ax = 2.4 \times 10^{-3}$ m³ $+ 8.0\ 10^{-3}$ m² × 0.1 m

$= 3.2 \times 10^{-3}$ m³

$$\text{Now,} \quad \frac{p_1V_1}{T_1} = \frac{p_2V_2}{T_2} \quad \Rightarrow \quad T_2 = \frac{p_2V_2}{p_1T_1}T_1$$

$$\Rightarrow \quad T_2 = \frac{(2.0 \times 10^5 \text{ N/m}^2)(3.2 \times 10^{-3} \text{ m}^3)}{(1.0 \times 10^5 \text{ N/m})(2.4 \times 10^{-3}\ E^3)} \times 300 \text{ K} = 800 \text{ K}$$

11. (5) Due to the heating, pressure inside is not changed. Let pressure inside be ρ. Then for equilibrium of the system

$$P(V_1 - V_2) = [P_0(A_1 - A_2) + (m_1 + m_2)g]l$$

$$P\Delta V = (\rho_0 \Delta A + mg)\,l$$

l is displacement of the piston.

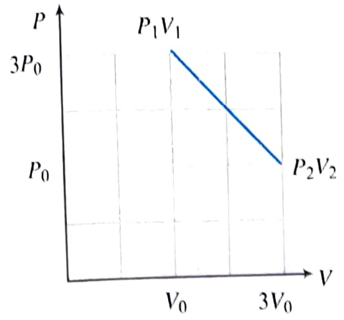
$$P \cdot \Delta V = nR\Delta T$$

$$\Delta T = \frac{P\Delta V}{nR} = \frac{(\rho_0 \Delta A + mg)l}{nR}$$

$$= \frac{(10^5 P_a \times 10^{-3} \text{ m}^2 + 5 \times 10)(50 \times 10^{-2})}{1 \times R}$$

$$\Delta T = \frac{75}{R} \text{ K}$$

(d) $$\frac{P-P_1}{P_1-P_2}=\frac{V-V_1}{V_1-V_2}$$



$$(P-P_1)(V_1-V_2)=(V-V_1)(P_1-P_2)$$

$$(P-3P_0)(V_0-3V_0)=(V-V_0)(3P_0-P_0)$$

$$(P-3P_0)(-2V_0)=(V-V_0)(2P_0)$$

$$-2V_0P+6P_0V_0=2VP_0-2P_0V_0$$

$$2VP_0+2V_0P-8P_0V_0=0$$

$$VP_0+\frac{V_0nRT}{V}-4P_0V_0=0$$

$$V^2P_0-4P_0V_0+V_0nRT=0$$

$$T=\frac{P_0(-V^2+4VV_0)}{V_0nR}$$

For maximum or minimum value of T,

$$\frac{dT}{dV}=-2V+4V_0=0 \Rightarrow V=2V_0$$

$$\frac{d^2T}{dV^2}=-2$$

It is negative at $V=2V_0$

T is maximum at $V=2V_0$

$$T_{\max}=\frac{P_0\left(-4V_0^2+8V_0^2\right)}{V_0nR}=\frac{4P_0V_0}{nR}$$

13. (10)

It is given that initially the separator is in equilibrium; thus pressure on both sides of the gas is equal say, it is P_i. If A be the area of cross section of cylinder, number of moles of gas in left and right part, n_1, and n_2, can be given as

$$n_1=\frac{P_i(10A)}{R(100)} \text{ and } n_2=\frac{P_i(20A)}{R(400)}$$

Finally if separator is displaced to right by a distance x, we have

$$n_1=\frac{P_f(10+x)A}{RT_f} \text{ and } n_2=\frac{P_f(20+x)A}{RT_f}$$

Here P_f and T_f are the final pressure and temperature on both sides after a long time.

Now if we equate the ratio of moles n_1/n_2 in initial and final states, we get

$$\frac{n_1}{n_2}=\frac{(10A/100)}{(20A/400)}=\frac{(10+x)A}{(20-x)A}$$

$$2(20-x)=10+x$$

$$x=10 \text{ cm}$$

Thus in final state when gases in both parts are in thermal equilibrium, the piston is displaced 10 cm rightward from its initial position.

14. (45)

$$P_{at}=10^5 \text{ Pa}$$

Initial pressure of gas, $P_1=P_{at}+P_{\text{spring}}+P_{\text{piston}}$

P_{spring}: pressure due to spring

P_{piston}: pressure due to piston

Initially, the spring is in relaxed position, $P_{\text{spring}}=0$

$$P_1=150 \text{ kPa}=1.5\times10^5 \text{ Pa}$$

$$\therefore \quad 1.5\times10^5=10^5+0+P_{\text{piston}}$$

Pressure due to piston $= mg/A = 0.5\times10^5$ N/m^2

In the final condition, $P'=P_0+P'_{\text{spring}}+P'_{\text{piston}}$ $(P_{\text{piston}}=P'_{\text{piston}})$

$$P'_{\text{spring}}=6\times10^5-1\times10^5-0.5\times10^5=4.5\times10^5 \text{ N/m}^2$$

Initial load in the spring, $F_1=0$

Final load in the spring, $F_2=P'_{\text{spring}}A=(4.5\times10^5A)$ N

Work done on the spring = (Average force) (distance moved)

$$=\frac{(F_1+F_2)}{2}\frac{(V_2-V_1)}{A}=\left[\left(\frac{0+4.5\times10^5}{2}\right)A\times\left(\frac{0.5-0.3}{A}\right)\right]$$

$$=0.45\times10^5 \text{ J}=45 \text{ kJ}$$

15. (0.16)

When jar is closed: Given, $V_1=1\times10^{-3}$ m^3, $P_1=1.5\times10^5$ Pa and $T_1=400$ K

Let n_1 be the initial number of moles of gas in the jar.

We know, $P_1V_1=n_1RT_1$

$$n_1=\frac{P_1V_1}{RT_1}=\frac{1.5\times10^5\times1\times10^{-3}}{8.3\times400} \Rightarrow n_1=\frac{1.5}{8.3\times4}$$

Let m_1 be the initial mass of the gas in the jar.

$$m_1=n_1M$$

$$\Rightarrow \quad m_1=\frac{1.5\times M}{8.3\times4}=\frac{1.5\times32}{4\times8.3}=1.4457\approx1.446$$

When jar is leaking: Again, $P_2=1\times10^5$ Pa, $V_2=1\times10^{-3}$ m^3, and $T_2=300$ K

Again using, $P_2V_2=n_2RT_2$

Final number of moles of gas in the jar

$$n_2=\frac{P_2V_2}{RT_2}=\frac{10^5\times10^{-3}}{8.3\times300}=\frac{1}{3\times8.3}=0.040$$

Final mass of the gas in the jar, $m_2=n_2M$

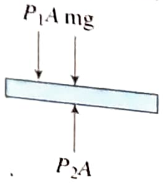
$$\Rightarrow \quad m_2=0.04\times32=1.285$$

Hence the mass of the gas that leaks out,

$$\Delta m=m_1-m_2=1.446-1.285=0.1608\approx0.16 \text{ g}$$

16. (41.5)

Let l_1 and l_2 be the lengths of the upper part and the lower part of the cylinder, respectively. Clearly, $l_1=60$ cm and $l_2=40$ cm. Let the pressures in the upper and lower parts be p_1 and p_2, respectively. Let the area of cross-section of the cylinder be A. The temperature in both parts is $T=300$ K.



Consider the equilibrium of the piston.

The forces acting on the piston are:

(a) its weight mg

(b) p_1A downwards, by the upper part of the gas and

(c) p_2A upward, by the lower part of the gas.

Thus, $p_2A = p_1A + mg$ ...(i)

Using $pV = nRT$ for the upper and the lower parts,

$$P_1l_1A = nRT \quad \text{...(ii)}$$

And $p_2l_2A = nRT$ ...(iii)

Putting p_1A and p_2A from (ii) and (iii) into (i), we get

$$\frac{nRT}{l_2} = \frac{nRT}{l_1} + mg$$

Thus, $m = \dfrac{nRT}{g}\left[\dfrac{1}{l_2} - \dfrac{1}{l_1}\right]$

$$= \frac{(0.2\,\text{mol})(8.3\ \text{J/mol-K})(300\ \text{K})}{10\ \text{m/s}^2}\left[\frac{1}{0.4\,\text{m}} - \frac{1}{0.6\,\text{m}}\right]$$

$= 41.5$ kg

Archives

JEE Main

Single Correct Answer Type

1. (2) Thermal energy corresponds to internal energy.

Mass = 1 kg

Density = 4 kg/m^3

Volume $= \dfrac{\text{Mass}}{\text{Density}} = \dfrac{1}{8}\text{m}^3$

Pressure = 8×10^4 N/m^2

Internal energy $= \dfrac{5}{2}P \times V = 5 \times 10^4$ J

2. (2) Let $T_3 > T_2 > T_1$ and final temperature is T such that $T_3 > T > T_2 > T_1$. Now heat gained by first two gases is equal to heat lost by third gas:

$$n_1C(T - T_1) + n_2C(T - T_2) = n_3C(T_3 - T)$$

$$\Rightarrow \quad T = \frac{n_1T_1 + n_2T_2 + n_3T_3}{n_1 + n_2 + n_3}$$

3. (3) $P = \dfrac{1}{3}\left(\dfrac{U}{V}\right) = \dfrac{1}{3}kT^4$ ...(i)

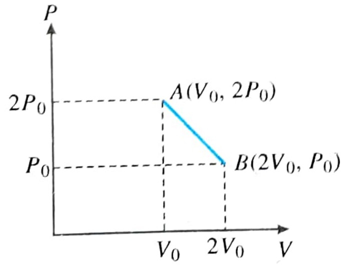
$PV = \mu RT$ ...(ii)

$$\frac{\mu RT}{V} = \frac{1}{3}kT^4$$

$$\Rightarrow \quad V \propto T^{-3}$$

$$R \propto \frac{1}{T}$$

4. (1)



$$P - 2P_0 = \frac{P_0 - 2P_0}{2V_0 - V_0}(V - V_0)$$

$$\Rightarrow \quad P = -\frac{P_0}{V_0}V + 3P_0$$

Hence, $\dfrac{nRT}{V} = -\left(\dfrac{P_0}{V_0}\right)V + 3P_0$

$$T = \frac{1}{nR}\left[-\frac{P_0}{V_0}V^2 + 3P_0V\right] \quad \text{...(i)}$$

For temperature to be maximum, $\dfrac{dT}{dV} = 0$

$$\frac{1}{nR}\left[-\frac{2P_0}{V_0}V + 3P_0\right] = 0,$$

we get $V = \dfrac{3}{2}V_0$ ...(ii)

From (i) and (ii)

$$\therefore \quad T_{\text{max}} = \frac{1}{nR}\left[-\frac{P_0}{V_0} \times \left(\frac{3}{2}V_0\right) + 3P_0 \times \frac{3}{2}V_0\right]$$

$$= \frac{1}{nR}\left[-\frac{9}{4}P_0V_0 + \frac{9}{2}P_0V_0\right] = \frac{9P_0V_0}{4nR}$$

5. (2) Using ideal gas equation, $PV = nRT$

Here n is number of moles.

$P_0V_0 = n_iR \times 290$ ...(i)

Initial temperature, $T_i = 273 + 17 = 290$ K

After heating $P_0V_0 = n_fR \times 300$ ...(ii)

Final temperature, $T_f = 273 + 27 = 300$ K

from Eqs. (i) and (ii)

$$n_f - n_i = \frac{P_0V_0}{R \times 300} - \frac{P_0V_0}{R \times 290}$$

difference in number of moles $-\dfrac{P_0V_0}{R}\left[\dfrac{10}{290 \times 300}\right]$

Hence $n_f - n_i$ is

$$= \frac{P_0V_0}{R} \times \left[\frac{10}{290 \times 300}\right] \times 6.023 \times 10^{23}$$

Putting $P_0 = 10^5$ P_A and $V_0 = 30$ m^3

Number of molecules $n_f - n_i = -2.5 \times 10^{25}$

JEE Advanced

Single Correct Answer Type

1. (4) Theory based

2. (4) $\dfrac{v_{\text{rms}_{\text{He}}}}{v_{\text{rms}_{\text{Ar}}}} = \dfrac{\sqrt{\dfrac{3RT}{m_{\text{He}}}}}{\sqrt{\dfrac{3RT}{m_{\text{Ar}}}}} = \sqrt{\dfrac{m_{\text{Ar}}}{m_{\text{He}}}} = \sqrt{\dfrac{40}{4}} = \sqrt{10} \approx 3.16$

3. (4) $PV = nRT = \dfrac{m}{M}RT$

$$\Rightarrow \quad PM = \rho RT$$

$$\frac{\rho_1}{\rho_2} = \frac{P_1M_1}{P_2M_2} = \left(\frac{P_1}{P_2}\right) \times \left(\frac{M_1}{M_2}\right) = \frac{4}{3} \times \frac{2}{3} = \frac{8}{9}$$

Here ρ_1 and ρ_2 are the densities of gases in the vessel contain the mixture.

le Correct Answers Type

, (2), (4)

For the mixture equivalent degrees of freedom is calculated using the relation $F_{eq} = \dfrac{n_1 f_1 + n_2 f_2}{n_1 + n_2}$

$$f_{eq} = \frac{5+3}{2} = 4$$

$$U = \frac{f}{2} nRT$$

For mixture, $U = \dfrac{4}{2} nRT$

$\therefore$ Energy per mole: $\dfrac{U}{n} = 2RT$

For mixture, $\gamma = 1 + \dfrac{2}{f} = 1 + \dfrac{1}{2} = \dfrac{3}{2}$

$$M_{\text{mix}} = \frac{1 \times 2 + 1 \times 4}{2} = 3$$

$$\gamma_{\text{He}} = 1 + \frac{2}{3} = \frac{5}{3}$$

Ratio of speed of sound $= \sqrt{\dfrac{\gamma_{\text{mix}}}{M_{\text{mix}}} \dfrac{M_{\text{He}}}{\gamma_{\text{He}}}} = \sqrt{\dfrac{6}{5}}$

$V_{\text{rms}} = \sqrt{\dfrac{3RT}{M}}$ ratio $= \sqrt{\dfrac{2}{4}} = \dfrac{1}{\sqrt{2}}$

Chapter 4

Concept Application Exercises

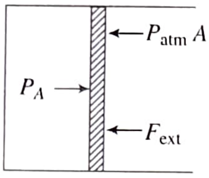
Exercise 4.1

1. **1st Method:** Work done by external agent is positive, because Fext and displacement are in the same direction. Since walls are conducting therefore temperature remains constant.

Applying equilibrium condition when pressure of the gas is P

$$PA + F_{ext} = P_{atm} A$$

$$F_{ext} = P_{atm} A - PA$$



$$W_{ext} = \int_0^d F_{ext}\, dx = \int_0^d P_{atm}\, A dx - \int_0^d PA\, dx$$

$$= P_{atm} A \int_0^d dx - \int_V^{2V} \frac{nRT}{V}\, dV$$

$$= P_{atm} Ad - nRT \ln 2 = P_{atm} \cdot V_0 - nRT \ln 2 = nRT (1 - \ln 2)$$

2nd Method: Applying work energy theorem on the piston

$$\Delta k = 0$$

$$W_{all} = \Delta k$$

$$W_{gas} + W_{atm} + W_{ext} = 0$$

$$nRT \ln \frac{V_f}{V_i} - nRT + W_{ext} = 0$$

$$W_{ext} = nRT (1 - \ln 2)$$

2. **(a)** $PV = nRT \quad \Rightarrow \quad \left(P_{atm} + \frac{mg}{A}\right) Ah = nRT$

$$\Rightarrow \quad n = \frac{\left(P_{atm} + \frac{mg}{A}\right) Ah}{RT}$$

(b) Applying Newton's law on the piston

$$mg + P_{atm} A + Kx = P_{gas} A$$

$$W_{gas} = \int_0^d P_g A\, dx = \int_0^d (mg + P_{atm} A + Kx)\, dx$$

$$\Rightarrow \quad W_{gas} = mgd + P_{atm} dA + \frac{1}{2} Kd^2$$

3. From the figure given in question

$$\frac{V_4 - V_3}{V_2 - V_1} = \frac{P_3 - P_0}{P_0 - P_1} \quad \Rightarrow \quad \frac{V_4 - V_3}{10} = \frac{4 \times 10^5 - 3 \times 10^5}{3 \times 10^5 - 10^5}$$

$$V_4 - V_3 = 5 \text{ L}$$

Now, work done

$$W = \left(\frac{1}{2} \times 10 \times 2 \times 10^5 - \frac{1}{2} \times 5 \times 1 \times 10^5\right) \times 10^{-3} = 750 \text{ J}$$

4. As cyclic process is in anticlockwise sense, hence work done by the gas should be negative and equal to area enclosed

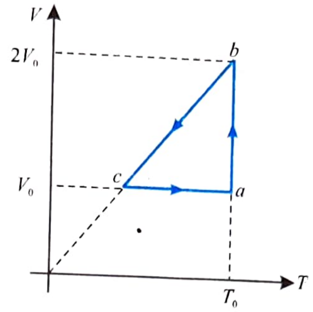
$$W = -\frac{\pi}{4}(P_2 - P_1)(V_2 - V_1) = -\frac{\pi}{4}(4 - 1) \times 10^5 \times (20 - 5)$$

$$= -\frac{\pi}{4} \times 3 \times 15 \times 10^5 = -\frac{225\pi}{2} \times 10^3 \text{ J} = -\frac{225\pi}{2} \text{ kJ}$$

5. $W = -\frac{\pi}{8}(P_2 - P_1)(V_2 - V_1)$

$$W = \frac{\pi}{8}(4 - 1) \times 10^5 \times (20 - 10) = \frac{\pi}{8} \times 3 \times 10 \times 10^5 = 375\pi \text{ kJ}$$

6.



(a) The V-T diagram for the process is shown in the figure. T initial state is represented by the point a.

In the first step, it is isothermally expanded to a volume 2 This is shown by ab. Then the pressure is kept constant a the gas is compressed to the volume V_0. From the ideal equation, V/T is constant at constant pressure. Hence, process is shown by a line bc which passes through the orig At point c, the volume is V_0. In the final step, the gas is heated constant volume to a temperature T_0. This is shown by ca. T final state is the same as the initial state.

(b) The process is cyclic so that the change in internal energy zero. The heat supplied is, therefore, equal to the work done the gas.

The work done during ab, $W_{ab} = nRT_0 \ln \frac{2V_0}{V_0}$

$$\Rightarrow \quad W_{ab} = nRT_0 \ln 2 = p_0 V_0 \ln 2$$

Also from the ideal gas equation, $p_a V_a = p_b V_b$

$$\text{or} \quad p_b = \frac{p_a V_a}{V_b} = \frac{p_0 V_0}{2V_0} = \frac{p_0}{2}$$

In the step bc, the pressure remains constant. Hence, the wo done is,

$$W_{bc} = \frac{p_0}{2}(V_0 - 2V_0) = -\frac{p_0 V_0}{2}$$

In the step ca, the volume remains constant and so the wo done is zero. The net work done by the gas in the cyclic proce is

$$W = W_1 + W_2 = p_0 V_0 \ln 2 - \frac{p_0 V_0}{2} + 0$$

$$W = \frac{p_0 V_0}{2}[2 \ln 2 - 1]$$

7. $6T_1 = 3T_2 = 2T_4 = T_3 = 1800 \text{ K}$

$T_1 = 300 \text{ K}, T_2 = 600 \text{ K}$

$T_4 = 900 \text{ K}, T_3 = 1800 \text{ K}$

1 → 4 and 2 → 3 are isochoric processes

Work done = 0

$W_{12} = P(V_2 - V_1)\, nR(T_2 - T_1) = 2 \times R(600 - 300) = 600R$

$W_{34} = P(V_4 - V_3) = nR(T_4 - T_3) = 2 \times R(900 - 1800) = 180($

$W_{total} = 600R - 1800R = -1200R$

8. $A \to B$ is an isobaric process $\quad \therefore V \propto T$

so, $W_{AB} = nR\Delta T = 2 \times R \times (750 - 250) = 1000\ R$

$B \to C$ is an isochoric process $\quad \therefore \quad W_{BC} = 0$

and $C \to D$ is an isothermal process,

$$\therefore \quad W_{CD} = nRT \ln\left(\frac{V_f}{V_i}\right) = 2 \times R \times 1000 \ \ln\left(\frac{20}{15}\right) = 2000R \ln(4/3)$$

Total work done $W = \Sigma \Delta W_{AB}$

9. $W = \Sigma \Delta W_{AB} = 1000R\left(1 + 2\ln\frac{4}{3}\right)$

Here $PT = P_1T_1 = P_2T_2 = C$ (constant)

$$\therefore \quad PT = C \Rightarrow P \cdot \frac{PV}{nR} = C$$

$$\Rightarrow \quad P^2V = nCR \Rightarrow P = \sqrt{\frac{nCR}{V}}$$

$$\therefore \quad \int PdV = \sqrt{nCR}\int_{V_1}^{V_2} \frac{1}{\sqrt{V}}dV = \sqrt{nCR}\left[2\left(\sqrt{V_2} - \sqrt{V_1}\right)\right]$$

$$= 2\left[\sqrt{nRP_2T_2V_2} - \sqrt{nRP_1T_1V_1}\right] = 2nR(T_2 - T_1) = 8RT_0$$

Exercise 4.2

1. For oxygen $C_p = 7.03$ cal/mol°C.

$$\Delta Q = nC_p\Delta T = 3 \times 7.03 \times (127 - 27) = 2109 \text{ cal } (n = 3)$$

Since $\gamma = C_p/C_v$ and $\gamma = 1.4$ for oxygen as it is diatomic,

$$\Delta U = nC_v\Delta T = 3 \times \frac{7.03}{1.4} \times 100 = \frac{2109}{1.4} \text{ cal} = 1506 \text{ cal}$$

By the first law of thermodynamics, $\Delta Q = \Delta U + \Delta W$

$$\therefore \quad \Delta W = \Delta Q - \Delta U = 2109 - 1506 = 603 \text{ cal}$$

$$= 603 \times 4.184 = 2523 \text{ J}$$

(a) Work done by the gas = 2523 J

(b) Change in internal energy = 1506 cal

(c) Heat added to the gas = 2109 cal

2. By the first law of thermodynamics, $\Delta Q = \Delta U + \Delta W$

For 1 mol of an ideal gas $\Delta Q = C_p\Delta T$ and $\Delta U = C_v\Delta T$

$$\therefore \quad \Delta W = (C_p - C_v)\ \Delta T = R\Delta T \qquad (\because C_p - C_v = R)$$

$$= 8.3 \times 72 = -597.6 \text{ J}$$

$$\Delta U = \Delta Q - \Delta W = 1.6 \times 1000 - 597.6 = 1002.4 \text{ J}$$

$$\Delta Q = C_p\Delta T$$

$$C_p = \frac{1.6 \times 1000}{72} = 22.2 \text{ J/kg–K}$$

$$\Delta U = C_v\Delta T$$

or $$C_v = \frac{\Delta U}{\Delta T} = \frac{1002.4}{72} = 13.9 \text{ J/kg–K}$$

$$\therefore \quad \gamma = \frac{C_p}{C_v} = \frac{22.2}{13.9} = 1.6$$

3. Process $1 \to 2$ is adiabatic, so $Q_{12} = \Delta U_{12} + W_{12}$

$$\Rightarrow \quad 0 = (U_2 - U_1) + W$$

$$\Rightarrow \quad U_2 - U_1 = -W$$

Process $2 \to 3$ is isochoric, so

$$Q_{23} = \Delta U_{23} + W_{23}$$

$$Q = U_3 - U_2$$

$$W = 0 \qquad \therefore V = \text{constant}$$

$$\Rightarrow \quad U_3 - U_2 = Q$$

Now $(U_3 - U_2) + (U_2 - U_1) = U_3 - U_1$

$$\Rightarrow \quad Q + (-W) = U_3 - U_1 \qquad \Rightarrow \quad U_3 = Q + U_1 - W$$

$$\Rightarrow \quad U_3 - U_1 = Q - W$$

4. First law of thermodynamics $Q = W + \Delta U$

$$Q = \frac{Q}{2} + \Delta U$$

$$\Rightarrow \quad \Delta U = \frac{Q}{2} = \frac{3}{2}nR\Delta T$$

$$\Rightarrow \quad Q = 3nR\Delta T \quad \Rightarrow \quad nC\Delta T = 3nR\Delta T \quad \Rightarrow \quad C = 3R$$

5. We define work done $W = \int Pdv = \int_{V_1}^{V_2} kVdV$

$$= \frac{kV_2^2 - kV_1^2}{2} = \frac{p_2V_2 - p_1V_1}{2} = \frac{nR\Delta T}{2}$$

$$\therefore \quad nC\Delta T = nC_V\Delta T + nR\frac{\Delta T}{2} \Rightarrow C = C_V + \frac{R}{2}$$

6. The adiabatic constant is independent of quantities of gases, hence value of this ratio after mixing will be same as that of initial value before mixing.

7. $(n_1 + n_2)C_V\Delta T = n_1C_{V_1}\Delta T + n_2C_{V_2}\Delta T$

$$C_V = \frac{n_1C_{V_1} + n_2C_{V_2}}{n_1 + n_2} = 2R$$

Similarly, $C_P = 3\ R$

$$\gamma = \frac{C_P}{C_V} = \frac{3R}{2R} = 1.5$$

8. $Q = \Delta U + W = n\frac{f}{2}R\Delta T + nR\Delta T$

$$Q = \left(1 + \frac{f}{2}\right)Rn\Delta T \Rightarrow \frac{2500}{100} = \left(\frac{1+f}{2}\right)R \Rightarrow f = 4$$

$$\gamma = 1 + \frac{2}{f} = \frac{3}{2}$$

9. $V = m\left(\frac{1}{T}\right)$; m = slope of the curve

$$VT = \text{Constant.}$$

$$TdV + VdT = 0 \qquad \text{...(i)}$$

$$\Rightarrow \quad dV = -\frac{V}{T}dT$$

Also, $Q = \Delta U + W$

$$CdT = C_vdT + PdV \qquad \text{...(ii)}$$

From Eqs. (i) and (ii),

$$CdT = C_VdT - \frac{P}{T}VdT \Rightarrow C = C_V - \frac{PV}{T}$$

For monatomic gas, $C_V = \frac{3}{2}R$

$$C = C_v - R = \frac{3}{2}R - R \Rightarrow C = \frac{R}{2}$$

10. We define molar heat capacity as $C = \frac{Q}{\Delta T}$

But $$\Delta U = \left(\frac{5}{2}R\right)\Delta T \quad \Rightarrow \quad \Delta T = \frac{2\Delta U}{5R}$$

Also, $$C = \frac{Q}{\Delta T} = \frac{5(Q)R}{2\Delta U} = \frac{10}{3}R \text{ (given)}$$

$$\Rightarrow \quad \frac{Q}{\Delta U} = \frac{4}{3} \qquad \text{...(i)}$$

From first law of thermodynamics,

$$\Delta U = Q - W \qquad \text{...(ii)}$$

From Eqs. (i) and (ii), $\dfrac{Q}{Q-W} = \dfrac{4}{3} \quad \Rightarrow \quad W = \dfrac{Q}{4}$

Exercise 4.3

1. $\Delta U = \int_T^{T+\Delta T} C_v dT = \dfrac{R}{\gamma - 1}\Delta T = \dfrac{8.3}{\frac{3}{5}-1}(-26) = -324 \text{ J}$

and $\quad \Delta W = \int_T^{T+\Delta T} pdV$

Here $\quad pV^{1.5} = \text{constant}; pV = RT$ (always)

$\therefore \quad V^{0.5} \propto \dfrac{1}{T} \quad \text{or} \quad V \propto \dfrac{1}{T^2}$

or $\quad V = \dfrac{a}{T^2}$, where a is a positive constant.

$\therefore \quad p = \dfrac{RT}{V} = \dfrac{RT}{a/T^2} = \dfrac{R}{a}T^3$

$\therefore \quad \Delta W = \int_T^{T+\Delta T} \dfrac{R}{a}T^3\left(-\dfrac{2a}{T^3}dT\right) = -\int_T^{T+\Delta T} 2RdT$

$\Rightarrow \quad \Delta W = -2R\,\Delta T = -2 \times 8.3 \times (-26) = +432 \text{ J}$

$\Delta Q = \Delta U + \Delta W = -324 + 432 = 108 \text{ J}$

2. $\Delta Q = \Delta U + \Delta W$

Since container is nonconducting, therefore $\Delta Q = 0 = \Delta U + \Delta W$

$\therefore \quad \Delta W = -\Delta U = -n\dfrac{f}{2}R\Delta T = -2 \times \dfrac{5}{2}R(400 - 300)$

$= -5 \times 8.314 \times 100 \text{ J} = -5 \times 831.4 \text{ J} = -4157 \text{ J}$

3. The change in internal energy of the gas is

$\Delta U = 1.5\, nR\, (\Delta T) = 1.5\, (1 \text{ mol})\, (8.3 \text{ J/mol-K})\, (2 \text{ K}) = 24.9 \text{ J}$

The heat given to the gas = 42 J

The work done by the gas is $\Delta W = \Delta Q - \Delta U = 42 \text{ J} - 24.9 \text{ J} = 17.1 \text{ J}$

If the distance moved by the piston is x, the work done is

$\Delta W = (100 \text{ kPa})\, (8.5 \text{ cm}^2)\, x$

Thus, $(10^5 \text{ N/m}^2)(8.5 \times 10^{-4} \text{ m}^2)x = 17.1 \text{ J}$

or, $\quad x = 0.2 \text{ m} = 20 \text{ cm}$

4. Suppose the sample contains n moles. Also suppose the volume changes from V_1 to V_2 and the temperature changes from T_1 to T_2.

The heat supplied is

$$\Delta Q = \Delta U + P\Delta V = \Delta U + nR\Delta T = \Delta U + \frac{2\Delta U}{f}$$

(a) The change in internal energy is

$\Delta U = n\dfrac{f}{2}R(T_2 - T_1) = \dfrac{f}{2}Rn(T_2 - T_1)$

$= \dfrac{f}{2+f}\Delta Q = \dfrac{140 \text{ J}}{1.4} = 100 \text{ J}$

(b) The work done by the gas is $\Delta W = \Delta Q - \Delta U = 140 \text{ J} - 100 \text{ J} = 40 \text{ J}$

5. From first law of thermodynamics $\Delta Q_{ABC} = \Delta U_{ABC} + \Delta W_{ABC}$

$\Delta W_{ABC} = \Delta W_{AB} + \Delta W_{BC} = 0 + nRT_B \ln\dfrac{V_C}{V_B} = nRT_B \ln\dfrac{2V_0}{V_0}$

$= nRT_B \ln 2 = 2P_0V_0 \ln 2$

$\Delta U = nC_V\Delta T = \dfrac{5}{2}(2P_0V_0 - P_0V_0)$

$\Rightarrow \quad \Delta Q_{ABC} = \dfrac{5}{2}P_0V_0 + 2P_0V_0 \ln 2$

6. Since the process is adiabatic, therefore

$P_0V^{\frac{5}{3}} = P_{\text{final}}\left(\dfrac{V}{8}\right)^{5/3} \quad \Rightarrow \quad \gamma = \dfrac{C_P}{C_V} = \dfrac{5R}{2}\Big/\dfrac{3R}{2} = \dfrac{5}{3}$

$P_{\text{final}} = 32\, P_0$

Since the process is adiabatic, therefore

$T_1V_1^{\gamma-1} = T_2V_2^{\gamma-1} \quad \Rightarrow \quad T_0V_0^{2/3} = T_{\text{final}}\left(\dfrac{V_0}{8}\right)^{2/3}$

$\Rightarrow \quad T = 4T_0$

7. From the first law thermodynamics $\Delta Q = \Delta U + \Delta W$

Since gas expands freely therefore $\Delta W = 0$, since no heat is given to gas $\Delta Q = 0$

$\Rightarrow \quad \Delta U = 0$ and temperature remains constant $T_{\text{final}} = T_0$

Since the process is isothermal, therefore

$P_0 \times V_0 = P_{\text{final}} \times 2V_0 \quad \Rightarrow \quad P_{\text{final}} = P_0/2$

8. (a) In the part $A \to B$, the volume remains constant. Thus, the work done by the gas is zero. It is given that the heat absorbed by the gas is 50 J.

From I law of thermodynamics, $Q = \Delta U + W$

$\Rightarrow \quad 50 = \Delta U + 0 \Rightarrow \Delta U = 50 \text{ J}$

or $\quad \Delta U = U_B - U_A = 50 \text{ J}$

The increase in internal energy from A to B is 50 J. The internal energy is 1500 J at A, then

$U_B - 1500 = 50 \text{ J} \Rightarrow U_B = 1550 \text{ J}$

In the part $B \to C$, the work done by the gas is $W = -40$ J and no heat is given to the system. Again using $Q = \Delta U + W$

$0 = \Delta U + (-40)$ or $\Delta U = U_C - U_B = 40 \text{ J}$

$U_C - 1550 = 40 \text{ J} \Rightarrow U_C = 1590 \text{ J}$

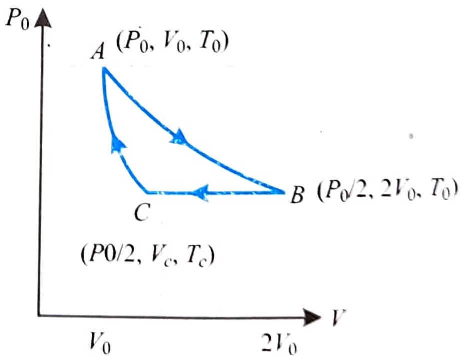
(b) The change in internal energy from $C \to A$.

$\Delta U = 1500 - 1580 = -90 \text{ J}$

The heat given to the system is $Q = -70$ J

Using $Q = \Delta U + W$, we get $W = Q - \Delta U = -70 \text{ J} + 80 \text{ J} = 20 \text{ J}$

9.



For process $A \to C$, $P_0V_0^{\gamma} = \dfrac{P_0}{2}V_C^{\gamma} \Rightarrow V_C = 2^{\frac{1}{\gamma}}V_0$

Also, $T_0V_0^{\gamma-1} = T_C(2^{\frac{1}{\gamma}}V_0)^{\gamma-1} \Rightarrow T_C = T_02^{\frac{1-\gamma}{\gamma}}$

Total work done $= W = W_{AB} + W_{BC} + W_{CA}$

The process $A \to B$ is an isothermal process, so $W_{AB} = nRT_0 \ln\dfrac{2V_0}{V_0}$

The process $B \to C$ is an isobaric process, so

$$W_{BC} = \frac{P_0}{2}\left(2^{\frac{1}{\gamma}}V_0 - 2V_0\right)$$

The process $C \rightarrow A$ is an adiabatic process, so

$$W_{CA} = -\frac{nR}{\gamma-1}(T_0 - T_C)$$

Hence, total work done in cyclic process,

$$W = nRT_0 \ln 2 + \frac{nRT_0}{2}\left(2^{\frac{1}{\gamma}} - 2\right) - \frac{nRT_0}{\gamma-1}\left(1 - 2^{-\frac{(\gamma-1)}{\gamma}}\right)$$

The heat is absorbed only in process $A \rightarrow B$.

Hence, $Q_{AB} = W_{AB}\left[\because \Delta U_{AB} = 0\right] = nRT_0 \ln 2$

$$\text{So } \eta = \frac{W}{Q_{AB}} = 1 - \frac{3\left(1 - \frac{1}{2^{\frac{1}{3}}}\right)}{\ln 2}$$

10. $PV^m = \text{constant}$

$$\Rightarrow \quad V^m dP + (mV^{m-1})\, P \cdot dV = 0$$

$$\Rightarrow \quad \frac{dP}{dV} = \frac{-mP}{V} = \tan(180° - 37°)$$

$$\Rightarrow \quad \frac{3}{4} = m\frac{2 \times 10^5}{4 \times 10^5} \Rightarrow m = \frac{3}{2}$$

11. Free expansion

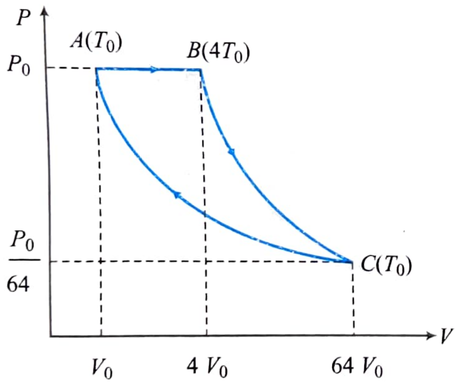
So $\Delta W = 0$ and $\Delta Q = 0$

So $\Delta U = 0$

$T = 300$ K

12. We are given the value of γ. We can calculate the molar heat capacity of the gas at constant pressure and constant volume.

$$C_P = \frac{\gamma R}{\gamma - 1} = 3R,\ C_V = \frac{R}{\gamma - 1} = 2R$$



$$P_A V_A = P_C V_C \quad \Rightarrow P_0 V_0 = P_C V_C \qquad \text{...(i)}$$

$$P_B V_B^{\gamma} = P_C V_C^{\gamma} \quad \Rightarrow P_0 (4V_0)^{1.5} = P_C V_C^{1.5} \qquad \text{...(ii)}$$

From Eqs. (i) and (ii), $\dfrac{(4V_0)^{1.5}}{V_0} = \dfrac{V_c^{1.5}}{V_c} \Rightarrow 8V_C^{1.5} = V_C^{0.5}$

$V_C = 64\ V_0 \quad \Rightarrow P_0 V_0 = P_C V_C$

Now, $P_C \times 64 V_0 = P_0 V_0$

$$P_C = \frac{P_0}{64}$$

The heat is absorbed only in AB and CA processes.

$$Q_{AB} = nC_p \Delta T = n \times 3R \times (3T_0) = 9\, P_0 V_0$$

$$Q_{CA} = \frac{nRT_0}{\ln 64} = \frac{nRT}{\ln 2} = \frac{6 P_0 V_0}{n}$$

$$\eta = 1 - \frac{2}{3}\ln 2 = \frac{3 - 2\ln 2}{3}$$

13. (a) $Q = Q_1 + Q_2 + Q_3 + Q_4 = 5960 - 5585 - 2980 + 3645 = 1040$ J

$W = W_1 + W_2 + W_3 + W_4 \quad = 2200 - 825 - 1100 + W_4 = 275 + W_4$

For a cyclic process, $\Delta U = 0$ (as $U_f = U_i$)

From first law of thermodynamics, $Q = \Delta U + W$

Or $\quad Q = W \quad$ or $\quad 1040 \text{ J} = 275 + W_4 \quad$ or $\quad W_4 = 765$ J

(b) Heat input $Q_{in} = Q_1 + Q_4$

$$\text{Thus, } \eta = \frac{W}{Q_{in}} = \frac{1040}{5960 + 3645} = 0.108 = 10.8\%$$

Exercise 4.4

1. (i) Work done for engine $A \propto (800 - T)$

Work done for engine $B \propto (T - 300)$

Since the work outputs of the two engines are equal,

$(800 - T) = (T - 300$ or $T = 550$ K

(ii) $\eta = 1 - \dfrac{800}{T},\ \eta' = 1 - \dfrac{T}{300}$

For $\eta = \eta',\ 1 - \dfrac{800}{T} = 1 - \dfrac{T}{300}$

or $\quad T = \sqrt{24 \times 10^4} = 490$ K

2. $\eta = \dfrac{W}{Q_1} = \dfrac{1}{6}$

$$\eta = 1 - \frac{T_2}{T_1} \quad \text{or} \quad \frac{1}{6} = 1 - \frac{T_2}{T_1} \quad \text{or} \quad \frac{T_2}{T_1} = \frac{5}{6}$$

When T_2 becomes $(T_2 - 62)$, η becomes (1/3).

$$\text{Thus, } \frac{1}{3} = 1 - \frac{(T_2 - 62)}{T_1}$$

$$\text{or} \quad \frac{T_2 - 62}{T_1} = \frac{2}{3}$$

$$\text{or} \quad \frac{(T_2 - 62)/T_1}{T_2/T_1} = \frac{2/3}{5/6} = \frac{4}{5}$$

$$\text{or} \quad T_2 = 310K,\ T_1 = 310 \times \left(\frac{6}{5}\right) = 372 \text{ K}$$

3. The amount of heat that must be extracted from the water to convert it into ice, i.e.,

$Q_2 = (10 \text{ kg})\ (80 \text{ kcal/kg}) = 800$ kcal

Thus, $T_1 = 22°\text{C} = 295$ K and $T_2 = 273$ K

$$\text{Thus, } \omega = \frac{T_2}{T_1 - T_2} = \frac{273}{25 - 273} = \frac{273}{22}$$

If W is the amount of work that must be done to make 10 kg of ice,

$$W = \frac{Q_2}{\omega} = \frac{800}{(273/22)} = 64.5 \text{ kcal}$$

$= 64.5 \times 10^3 \times 4 \times 4.186 \text{ J} = 2.7 \times 105$ J

Since this work must be done in (1/2) h, power required to run the refrigerator, i.e.,

$$\Rightarrow \quad P = \frac{W}{t} \quad \Rightarrow \quad P = \frac{2.7 \times 10^5 \text{ J}}{(30 \times 60) \text{ s}} = 150 \text{ W}$$

4. (a) $\omega = \dfrac{Q_2}{W}$ or $W = \dfrac{Q_2}{\omega} = \dfrac{250}{5} = 50$ J

(b) $Q_1 = Q_2 + W = (250 + 50)$ J $= 300$ J

5. Maximum efficiency, $\eta = \frac{T_1 - T_2}{T_1} = \frac{480 - 300}{480} = 37.5\%$

Actual efficiency, $\eta' = \frac{\text{energy output}}{\text{energy input}}$

$$= \frac{1.2\text{ kJ}}{1\text{ kcal}} = \frac{1.2\text{ J}}{1\text{ cal}} = \frac{1.2\text{ J}}{4.186\text{ J}} = 28.7\%$$

$$\frac{\eta'}{\eta} = \frac{28.7\%}{37.5\%} \approx \frac{3}{4}$$

Thus, $\eta' = \left(\frac{3}{4}\right)\eta$

6. $$\eta = \frac{T_1 - T_2}{T_1} = \frac{1}{6} \quad \text{...(i)}$$

$$\eta' = \frac{T_1 - (T_2 - 65)}{T_1} = \frac{1}{3} \quad \text{...(ii)}$$

From Eqs. (i) and (ii), $\frac{\eta'}{\eta} = \left(\frac{T_1 - T_2 + 65}{T_1}\right)\left(\frac{T_1}{T_1 - T_2}\right) = \frac{(1/3)}{(1/6)} = 2$

Or $\frac{T_1 - T_2 + 65}{T_1 - T_2} = 2$

Or $T_1 - T_2 = 65$ ...(iii)

From Eq. (i), $\frac{65}{T_1} = \frac{1}{6} \Rightarrow T_1 = 390$ K

And from Eq. (iii), $T_2 = T_1 - 65 = 325$ K

7. (a) As $\eta = \frac{T_1 - T_2}{T_1}$, when the temperature of the source is increased by ΔT, we get

$$\eta_1 = \frac{(T_1 + \Delta T) - T_2}{T_1 + \Delta T} = \frac{T_1 - T_2}{T_1 + \Delta T} + \frac{\Delta T}{T_1 + \Delta T}$$

(b) When the temperature of the sink is lowered by ΔT,

$$\eta_2 = \frac{T_1 - (T_2 - \Delta T)}{T_1} = \frac{T_1 - T_2}{T_1} + \frac{\Delta T}{T_1}$$

As $\frac{T_1 - T_2}{T_1} > \frac{T_1 - T_2}{T_1 + \Delta T}$

And $\frac{\Delta T}{T_1} > \frac{\Delta T}{T_1 + \Delta T}, \eta_2 > \eta_1$

Thus, efficiency of the Carnot cycle is higher in case (b).

8. $$\eta = 1 - \frac{T_2}{T_1}, \frac{T_1}{T_2} = \frac{1}{1-\eta}$$

$$\omega = \frac{T_2}{T_1 - T_2} = \frac{1}{(T_1/T_2) - 1} = \frac{1}{[1/(1-\eta)] - 1} = \frac{1-\eta}{\eta}$$

As $\eta = 10\% = 0.1$, we have $\omega = \frac{1 - 0.1}{0.1} = 9$

9. $\omega = \frac{T_2}{T_1 - T_2} = \frac{273}{293 - 273} = \frac{273}{20}$

Mass of ice melting per second $= \frac{2 \times 1000}{3600} = \left(\frac{5}{9}\right)$g

To prevent the melting of ice, heat to be driven out per second,

i.e., $Q_2 = \left(\frac{5}{9}\right) \times 80 = \frac{400}{9}$ cal $= 186$ J

As, $\omega = \frac{Q_2}{W}$, we have $W = \frac{Q_2}{\omega} = \frac{186}{(273/20)} = 13.6$ J

Power of the motor = 13.6 W

10. $\omega_A = \frac{263}{300 - 263} = 7.11$

$\omega_B = \frac{253}{298 - 263} = 6.84$

Since, $\omega_A > \omega_B$, refrigerator A is more efficient.

Exercises

Single Correct Answer Type

1. (1) Change in temperature in process 1 will be greater and in process 3 will be least.

2. (3) As we know that slopes of isothermal and adiabatic curves are always negative and slope of adiabatic curve is always greater than that of isothermal curve, in the given graph curve A and curve B represent adiabatic and isothermal changes, respectively.

3. (1) Area enclosed by curve 1 < Area enclosed by curve 2 < Area enclosed by curve 3

$\therefore \quad Q_1 < Q_2 < Q_3$ (As ΔU is same for all the curves)

4. (4) Heat given, $\Delta Q = 20$ cal $= 20 \times 4.2 = 84$ J

Work done, $\Delta W = -50$ J (as the process is anticlockwise)

By the first law of thermodynamics,

$$\Delta U = \Delta Q - \Delta W = 84 - (-50) = 134 \text{ J}$$

5. (3) $\Delta Q = \Delta U$

$$= \mu C_v \Delta T = \mu\left(\frac{R}{\gamma - 1}\right)\Delta T = 2 \times \frac{R}{\frac{5}{3} - 1}[373 - 273] = 300R$$

(as for monatomic gas $\gamma = 5/3$)

6. (3) In second part there is a vacuum, i.e., $P = 0$. So work done in expansion $= P\Delta V = 0$. Also, $\Delta Q = 0$. From the first law of thermodynamics, $\Delta U = 0$ i.e., temperature of an ideal gas remains same due to free expansion.

7. (3) For isothermal process

$$P_1 V = P_2' \frac{V}{2} \Rightarrow P_2' = 2P_1 \quad \text{...(i)}$$

For adiabatic process $P_1 V^\gamma = P_2\left(\frac{V}{2}\right)^\gamma$

$\Rightarrow \quad P_2 = 2^\gamma P_1$ ...(ii)

Since $\gamma > 1$, $P_2 > P_2'$

8. (4) $1 \rightarrow 2$: isothermal, $\Delta U_{12} = 0$

$2 \rightarrow 3$: isochoric, $\Delta W = 0$

$\Rightarrow \quad \Delta Q_{23} = \Delta U_{23} \quad \Rightarrow -40 = \Delta U_{23}$

For a cyclic process, $\Delta U = 0$

$$\Delta U_{12} + \Delta U_{23} + \Delta U_{31} = 0$$
$$0 + (-40) + \Delta U_{31} = 0$$
$$0 + (-40) + \Delta U_{31} = 0$$
$$\Delta U_{31} = +40 \text{ J}$$

9. (3) $W_{AB} = -P_0 V_0$

$W_{BC} = 0$ and $W_{CD} = 4P_0 V_0$

$\therefore \quad W_{ABCD} = -P_0 V_0 + 0 + 4P_0 V_0 = 3P_0 V_0$

10. (4) Heat absorbed by gas in three processes is given by

$$Q_{ACB} = \Delta U + W_{ACB}$$
$$Q_{ADB} = \Delta U$$
$$Q_{AEB} = \Delta U + W_{AEB}$$

The change in internal energy in all the three cases is same and W_{ACB} is positive, W_{AEB} is negative.

Hence $Q_{ACB} > Q_{ADB} > Q_{AEB}$

$$W_{ABC} = \frac{\pi r^2}{2} = \frac{\pi(6)^2}{2} = 18\pi$$

$$W_{DEF} = \frac{\pi \times 3^2}{2} + \frac{\pi \times 3^2}{4} + (15-12) \times 18 = 6.75\pi + 54$$

$W_{DEF} > W_{ABC}$

At constant pressure $(\Delta Q)_p = \mu C_p \Delta T = 1 \times C_p \times (30 - 20) = 40$

$\Rightarrow \quad C_p = 4$ cal/mol-L

$\therefore \quad C_v = C_p - R = 4 - 2 = 2$ cal/mol-K

Now $(\Delta Q)_v = \mu C_v \Delta T = 1 \times 2 \times (30 - 20) = 20$ cal

At constant volume the total energy will be utilized in increasing the temperature of gas

i.e., $\quad (\Delta Q)_v = \mu C_v \Delta T = \mu C_v (120 - 100) = 80$

$$\Rightarrow \quad \mu C_v = \frac{80}{20} = 4 \text{ J/K}$$

This is the heat capacity of 5 mol gas.

Fraction of energy supplied for increment in internal energy = $1/\gamma = 3/5$ (as $\gamma = 5/3$ for monatomic gas)

Therefore, percentage energy = 30/5 = 60%.

Fraction of energy supplied for external work done

$$= 1 - \frac{1}{\gamma} = \frac{\gamma - 1}{\gamma} = \frac{\frac{5}{3} - 1}{\frac{5}{3}} = \frac{2}{5}$$

$\therefore$ Percentage energy $= \frac{2}{5} \times 100\% = 40\%$

(1) Molar specific heat = molecular weight × gram specific heat

$C_v = M \times C_v$

2.98 cal/mol-K = $M \times 0.075$ kcal/kg-K

$$= M \times \frac{0.075 \times 10^3}{10^3} \text{cal/g-K}$$

$\therefore \quad$ Molecular weight of argon $M = \frac{2.98}{0.075} = 39.7$ g

i.e., mass of 6.023×10^{23} atom = 39.7 g

Therefore, mass of single atom $= \frac{39.7}{6.023 \times 10^{23}} = 6.60 \times 10^{-23}$ g

(3) For state A, $C_p - C_v = R$, i.e., the gas behaves as an ideal gas.

For state B, $C_p - C_v = 1.06\, R (\geq R)$, i.e., the gas does not behave like an ideal gas.

We know that at high temperature and at low pressure, nature of gas may be ideal.

So we can say that $P_A < P_B$ and $T_A > T_B$.

(1) Let t be the temperature of mixture.

Heat gained by CO_2 = Heat lost by O_2

$$\Rightarrow \quad \mu_1 C_{v_1} \Delta T_1 = \mu_2 C_{v_2} \Delta T_2$$

$$\Rightarrow \quad \frac{22}{44}(3R)(t - 27) = \frac{16}{32}\left(\frac{5}{2}R\right)(37 - t)$$

$$\Rightarrow \quad 3(t - 27) = \frac{5}{2}(37 - t)$$

By solving we get $t = 31.5$°C

18. (1) By the graph, $W_{AB} = 0$ and $W_{BC} = 8 \times 10^4 [5 - 2] \times 10^{-3} = 240$ J

$\therefore \quad W_{AC} = W_{AB} + W_{BC} = 0 + 240 = 240$ J

Now, $\Delta Q_{AC} = \Delta Q_{AB} + \Delta Q_{BC} = 600 + 200 = 800$ J

From the first law of thermodynamics, $\Delta Q_{AC} = \Delta U_{AC} + \Delta W_{AC}$

$\Rightarrow \quad 800 = \Delta U_{AC} + 240 \Rightarrow \Delta U_{AC} = 560$ J

19. (1) $\frac{\Delta W}{\Delta Q} = \frac{\Delta Q - \Delta U}{\Delta Q} = \frac{C_p - C_v}{C_p} = 1 - \frac{1}{\gamma} = 1 - \frac{1}{\frac{5}{3}} = \frac{2}{5}$

i.e., percentage energy utilized in doing external work

$$= \frac{2}{5} \times 100 = 40\%$$

20. (4) Process AB is isochoric; therefore $W_{AB} = P\Delta V = 0$

Process BC is isothermal; therefore $W_{BC} = RT_2 \ln\left(\frac{V_2}{V_1}\right)$

Process CA is isobaric; therefore

$$W_{CA} = P\Delta V = R\Delta T = R(T_2 - T_1)$$

21. (3) The work done = area of P–V diagram

$$a = \frac{V_2 - V_1}{2}, b = \frac{P_2 - P_1}{2}$$

$$W = -\pi\left(\frac{V_2 - V_1}{2}\right)\left(\frac{P_2 - P_1}{2}\right)$$

But the cyclic process is anticlockwise. Hence, the work done is negative.

22. (2) For path acb: $\Delta Q = \Delta U + \Delta W$

$\Rightarrow \quad 84 = \Delta U + 32 \Rightarrow \Delta U = 52$ kJ

Hence $\Delta U_{acb} = \Delta U_{ab} = \Delta U_{adb} = 52$ kJ

For path adb: $\Delta Q = \Delta U + \Delta W = 52 + 10.5 = 62.5$ kJ

So option (1) is correct.

For process ba, system will release the heat. So option (2) is wrong.

For path ad: $\Delta W_{adb} = \Delta W_{ad} + \Delta W_{db}$

$\Rightarrow \quad 10.5 = \Delta W_{ad} + 0$

$\Rightarrow \quad \Delta W_{ad} = 10.5$ kJ

$\Delta Q_{ad} = \Delta U_{ad} + \Delta W_{ad} = (42 - 0) + 10.5 = 52.5$ kJ

So option (3) is correct.

$\Delta Q_{adb} = 52 + 10.5 = 62.5$ kJ

$\Delta Q_{db} = \Delta Q_{adb} - \Delta Q_{ad} = 62.5 - 52.5 = 10$ kJ

So option (4) is correct.

Hence answer of this question is (2).

23. (3) Along path AB

$\Delta Q = \Delta U + \Delta W = U_B - U_A + \Delta W$

or $\quad -50 = U_B - U_A = U_B - 1500$

or $\quad U_B = 1450$ J

Along path BC

or $\quad \Delta Q = U_C - U_B + \Delta W$

or $\quad 0 = U_C - U_B + \Delta W = U_C - U_B + (-40)$

or $\quad 0 = U_C - 1450 - 40$ or $U_C = 1490$ J

24. (1) No work is done along path AB because the process is isochoric.

$\therefore \quad$ Work done $= P_B(V_D - V_A)$

$= 8 \times 10^4 (5 \times 10^{-3} - 2 \times 10^{-3}) = 240$ J

$(\Delta Q)_{AC} = (\Delta Q)_{AB} + (\Delta Q)_{BC} = 600 + 200 = 800$ J

$\therefore \quad (\Delta U)_{AC} = (\Delta Q) - (\Delta W) = 800 - 240 = 560$ J

25. (2) Work done, $W = \int_{V_1}^{V_2} \alpha V^2 dV$

where $\alpha = 3 \times 10^5$ Pa/m^6

$$\Rightarrow \quad W = \left[\frac{\alpha V^3}{3}\right]_{V_1=1m^3}^{V_2=2m^3} = \left[\frac{3\times10^5 V^3}{3}\right]_{V_1}^{V_2}$$

$$\Rightarrow \quad W = 10^5 (V_2^3 - V_1^3)$$

$$V_1 = 1 \text{ m}^3 \quad V_2 = 2 \text{ m}^3$$

$$W = 10^5 (8 - 1) = 7 \times 10^5 \text{ J}$$

26. (3) $\Delta Q = Q_1 + Q_2 + Q_3 + Q_4 = 5960 - 5585 - 2980 + 3645 = 1040$ J

$\Delta W = W_1 + W_2 + W_3 + W_4 = 220 - 825 - 1100 + W_4 = 275 + W_4$

For a cyclic process, $U_f = U_f$

$$\Delta U = Uf - U_i = 0$$

From the first law of thermodynamics, $\Delta Q = \Delta U + \Delta W$

$$1040 = 0 = 275 + W_4 \quad \text{or} \quad W_4 = 765 \text{ J}$$

27. (3) $P = 1$ atm $= 10^5$ N/m^2

$$T = 0°\text{C} = 273 \text{ K}$$

$$V = \frac{nRT}{P} = \frac{1\times8.3\times273}{10^5} = 0.0227 \text{ m}^3 = 22.7 \text{ L}$$

$$C_v = \frac{5}{2}R; \quad C_p = \frac{7}{2}R$$

Heat transferred

$$\Delta Q = nC_p\Delta T = n\frac{7R}{2}\Delta T = 13200 \text{ J}$$

Work done

$$nR\Delta T = \frac{13200\times2}{7} = P(V_f - V_i) = 3771$$

$$V_f - V_i = 3771 \times 10^{-5} = 0.0377 \text{ m}^3$$

$$V_f = V_i + 37.7 \text{ L} = 22.7 \text{ L} + 37.7 \text{ L} = 60.4 \text{ L} \approx 60 \text{ L}$$

28. (1) $dU = 0$

Therefore by the first law of thermodynamics $dQ_{\text{Cyclic}} = dW_{\text{Cyclic}}$

Since $B \rightarrow C$ is an isochoric process

$$\Rightarrow \quad dW_{B\rightarrow C} = 0$$

$$\Rightarrow \quad 15 = dW_{A\rightarrow B} + dW_{B\rightarrow C} + dW_{C\rightarrow A}$$

$$\Rightarrow \quad 15 = 10\,(2 - 1) + 0 + dW_{C\rightarrow A}$$

$$\Rightarrow \quad dW_{C\rightarrow A} = -5 \text{ J}$$

29. (3) For an adiabatic process, $PV^\gamma =$ constant

$TV^{\gamma-1} =$ constant and $T^\gamma P^{1-\gamma} =$ constant

Putting, $\gamma = 5/3$ (argon being a monatomic gas), the equation becomes:

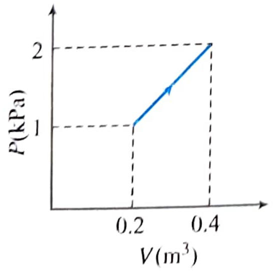
$$PV^{5/3} = \text{constant}$$

$$TV^{-2/3} = \text{constant}$$

$$T^{5/3} P^{-2/3} = \text{constant} \Rightarrow TP^{-2/5} = \text{constant}$$

30. (3) $\Delta W = \frac{1}{2}(1+2)\times10^3 \text{ N/m}^2 \times (0.4 - 0.2) \text{ m}^3 = 300$ J

$$\Delta U = nC_v\Delta T = \frac{(P_2V_2 - P_1V_1)}{\gamma - 1}$$



$$\Rightarrow \quad \Delta U = (3/2)\,(0.4 \times 2 - 0.2 \times 1) \times 103 \text{ J} = 900 \text{ J}$$

$$\Delta Q = \Delta W + \Delta U = 1200 \text{ J}$$

31. (2) Deal gas equation for m grams of gas is

$PV = mrT$ (where $r =$ specific gas constant)

$$\text{or} \quad P = \frac{m}{V}rT = \rho rT$$

$$\Rightarrow \quad r = \frac{P}{\rho T} = \frac{1.013\times10^5}{0.795\times273} = 466.7$$

Specific heat at constant volume

$$c_v = \frac{r}{\gamma - 1} = \frac{466.7}{\frac{4}{3} - 1} = 1400 \text{ J/kg-K}$$

($\gamma = 4/3$ for polyatomic gas)

32. (3) $C = C_v + W'$

where W' is the work done by the gas per unit mole per unit rise in temperature. So

$$W' = \alpha T - C_v = \alpha T - (5R/2)$$

$$\Delta W = \int W'dT = \int_{T_0}^{2T_0}\left(\alpha T - \frac{5R}{2}\right)dT = (3\alpha T_0 - 5R)\frac{T_0}{2}$$

33. (4) Since the gas is slowly heated, it remains in equilibrium (more or less) with the atmosphere, i.e., the process takes place at a constant pressure.

Now, from $PV = nRT$, $\quad PdV = nRdT$

$$\text{or,} \quad P\Delta V = nR\Delta T \qquad \text{...(i)}$$

But $P\Delta V$ is the work done by the gas.

So, $\quad \Delta W = nR\Delta T = (2 \text{ mol})\,(R)\,(4T_0 - T_0) = 6RT_0$

[From Eq. (i) $\Delta V \propto \Delta T$, i.e., if $\Delta V = 3V_0$, $\Delta T = 3T_0$]

34. (3) $C_p - C_v = m$, for hydrogen ($M_1 = 2$)

$C_p - C_v = n$, for nitrogen ($M_2 = 14$)

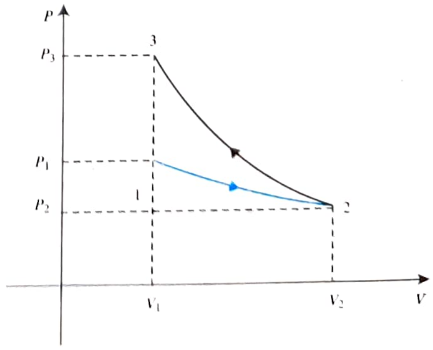
$$\text{For hydrogen,} \quad C_p - C_v = \frac{1}{M_1}\frac{dQ}{dT} = m$$

$$\text{For nitrogen,} \quad C_p - C_v = \frac{1}{M_2}\frac{dQ}{dT} = n$$

$$\therefore \quad \frac{m}{n} = \frac{\frac{1}{M}\frac{dQ}{dT}}{\frac{1}{M_2}\frac{dQ}{dT}} = \frac{M_2}{M_1} = \frac{14}{2} = 7$$

$$\therefore \quad m = 7n$$

35. (3) The two processes are shown in the following P–V diagram:



For isothermal process: $P_1V_1 = P_2V_2$

i.e.. $P_1 = \left(\frac{V_2}{V_1}\right)P_2$

For adiabatic process: $P_3V_1^{\gamma} = P_2V_2^{\gamma}$

i.e.. $P_3 = \left(\frac{V_2}{V_1}\right)^{\gamma} P_2$

As $\gamma > 1$, hence $P_3 > P_1$.

Further, as slope of adiabatic curve is greater than that of isothermal process curve, adiabatic curve will lie above the isothermal curve. That is, area under adiabatic curve > area under isothermal curve

i.e., Negative work > Positive work

i.e., $W < 0$

6. (2) $v_{\text{rms}} = \sqrt{\frac{3RT}{M}}$

or $v_{\text{rms}} \propto \sqrt{T}$

Since v_{rms} becomes half, so temperature is reduced to one-fourth of its previous value.

$$\frac{T'}{T} = \frac{1}{4}$$

During adiabatic process, $TV^{\gamma-1} = T'V'^{\gamma-1}$

or $\frac{V'}{V} = \left(\frac{T}{T'}\right)^{\frac{1}{\gamma-1}} = (4)^{\frac{1}{(1.5-1)}} = 4^2 = 16$

$\therefore$ $V' = 16\ V$

(1) $\Delta W = P\Delta V$; given: $PV = \mu RT + \alpha V$

$\therefore$ $P\Delta V = \mu R\Delta T + \alpha\Delta V$

or, $\Delta V = [(\mu R\Delta T)/P_0 - \alpha)]$

i.e., $\Delta W = \frac{P_0RT_0}{P_0 - \alpha}$ $(\mu = 1)$

38. (3) $Q = nC_v\,\Delta T = n\,(C_p - R\,)\,\Delta T$

$$= 5\left(7.03 - \frac{8.31}{4.2}\right) \times (20 - 10) = 250 \text{ cal}$$

(2) $R = \frac{2}{3}C_v$

We know, $C_p - C_v = R$

or $\gamma - 1 = \frac{R}{C_v}$

or $R = C_v(\gamma - 1)$

Comparing $\gamma = 5/3$

$P^{1-\gamma}T^{\gamma}$ = constant = K

or $P \propto T^{\gamma/(\gamma-1)}$ Given $\gamma = 5/3$

$\gamma/(\gamma - 1) = 5/2$ So, $P \propto T^{5/2}$

(4) $\Delta Q = \frac{K\gamma}{\gamma - 1} = \frac{KC_p/C_v}{(C_p/C_v) - 1} = \frac{KC_p}{C_p - C_v}$

$$= \frac{K(nC_p\Delta T)}{(nC_p\Delta T - nC_v\Delta T)} = \frac{K\Delta Q}{(\Delta Q - \Delta U)} = \frac{K\Delta Q}{\Delta W}$$

or $1 = \frac{K}{\Delta W}$ $\Rightarrow$ $K = \Delta W$

41. (4) C_v for hydrogen $= 5R/2$, C_v for helium $= 3R/2$, C_v for water vapour $= 6R/2$

$$\therefore \quad (C_v)_{\text{mix}} = \frac{4 \times \frac{5R}{2} + 2\frac{3R}{2} + 1 \times 3R}{4 + 2 + 1} = \frac{16R}{7}$$

$$\therefore \quad C_p = C_v + R = \frac{16R}{7} + R = \frac{23R}{7}$$

42. (4) For A: As piston is free to move, the process is isobaric.

$\Delta Q = \mu C_p\,(\Delta T)_1$

For B: As piston is held fixed, the process is isochoric.

$\therefore$ $\Delta Q = \mu C_v\,(\Delta T)_2$

Now $C_p(\Delta T)_1 = C_v(\Delta T)_2$

$$\frac{7R}{2} \times 30 = \frac{5R}{2}(\Delta T)_2$$

$\therefore$ $\Delta T_2 = 42$ K

43. (4) $P = 4.5 \times 10^5$ Pa; $dQ = 800$ kJ

$V_1 = 0.5\ \text{m}^3$; $V_2 = 2\ \text{m}^3$

$dW = P(V_2 - V_1) = 4.5 \times 10^5\,(2 \times 0.5) = 6.75 \times 10^5$ J

Change in internal energy

$dU = dQ - dW = 800 \times 10^3 - 6.75 \times 10^5 = 1.25 \times 10^5$ J

44. (2) $\frac{C_p}{C_v} = \gamma = \frac{7}{5}$

Work done $= \frac{\mu R}{\gamma - 1} \times \Delta T = \frac{8.3 \times 400 \times 5}{\frac{7}{5} - 1} = 41.5$ J

Work done = change in internal energy

($\because$ $\Delta Q = 0$ for adiabatic process)

Therefore, change in internal energy = 41.5 kJ.

45. (2) Let the initial pressure of the three samples be P_A, P_B and P_C, then

$P_A(V)^{3/2} = (2V)^{3/2}P$ $(\because\ P_B = P)$

or $P_A = P(2)^{3/2}$

$P_C(V) = P(2V)$

or $P_C = 2P$

$\therefore$ $P_A : P : P_C = (2)^{3/2} : 1 : 2 = 2\sqrt{2} : 1 : 2$

46. (2) $W = \int_{V_1}^{V_2} P\,dV = \int_{V_1}^{V_2} KV\,dV$ $\left(\because\ \frac{P}{V} = K = \text{constant}\right)$

$\therefore$ $W = \frac{1}{2}k(V_2^2 - V_1^2)$

$PV = RT$

But $p = KV$

$\therefore$ $KV^2 = RT$

or $K(V_2^2 - V_1^2) = R(T_2 - T_1)$

$\therefore$ $W = \frac{R}{2}(T_2 - T_1)$

47. (2) For 1 mol of gas, $\Delta Q = C_v\Delta T + P\Delta T$

At constant volume, $\Delta T = 0$

For 2 moles of gas, $\Delta = 2C_v\Delta T$

From $PV = nRT = 2R \times 300$ and $\frac{P}{2}V = 2RT_f$

$\therefore$ $T_f = 150$ K

$\therefore \quad \Delta Q = 2C_v(T_f - T_i) = 2C_v(150 - 300) = -300C_v$ J

In the next process, $\Delta Q = 2C_p\Delta T = 2C_p(300 - 150) = 300C_p$ J

$\therefore$ Net heat absorbed $= -300C_v + 300C_p$

$$= 300(C_p - C_v) = 300R \text{ J}$$

48. (1) For adiabatic process $PV_1^\gamma = P_A V_2^\gamma$

$$P_A = P\left(\frac{V_1}{V_2}\right)^\gamma \qquad \text{...(i)}$$

For isothermal process $PV_1 = P_B V_2$

$$P_B = P\frac{V_1}{V_2} \qquad \text{...(ii)}$$

From (i) and (ii), $P_A < P_B$ and by $PV = nRT$

$T_A < T_B$

49. (2) $\frac{C_V}{C_p} \times Q = nC_v dT$

$$dT = \frac{Q}{nC_p} = \frac{50}{2 \times \frac{5}{2} \times R} = 5 \text{ K}$$

50. (4) From the first law of thermodynamics $Q = W + \Delta U$

For an isobaric process

$$W = R\Delta T, \quad Q = C_P\Delta T = \left(\frac{f+2}{2}\right)R\Delta T$$

$$\frac{W}{Q} = \frac{2}{f+2}$$

where f is the number of degrees of freedom.

For an isochoric process, $W = 0, \ \frac{W}{Q} = 0$

The straight line 1 corresponds to an isochoric process. For monatomic gas $f = 3$ and for diatomic gas $f = 5$.

$$\frac{W}{Q} = \frac{2}{5} \qquad \text{(for monatomic gas)}$$

$$= \frac{2}{7} \qquad \text{(for diatomic gas)}$$

Straight line 2 corresponds to isobaric process for diatomic gas and straight line 3 corresponds to isobaric process for monatomic gas.

For isotherm $W = Q$, $\tan\theta_4 = 1$ straight line 4 corresponds to isothermal process because $W = Q$ only if $\Delta T = 0$ i.e., $\Delta U = 0$. For an adiabatic process $Q = 0$, the straight line 5 corresponds to it.

51. (4) $(\Delta Q)_{ab} = +7000 = \mu C_v(1000 - 300)$ ...(i)

For the process ca: $T_a = 300$ K, $T_c = T_b = 1000$ K

$$(\Delta Q)_{ca} = \mu C_p(300 - 1000) = -\mu C_p \times 700$$

$$= -\mu(C_v + R)700 \qquad \text{... (ii)}$$

For carbon monoxide: $T_a = 300$ K, $T_c = T_h = 1000$ K

$$(\Delta Q)_{ca} = \mu C_p(300 - 1000) = -\mu C_p \times 700$$

$$= -\mu(C_v + R)\,700$$

For carbon monoxide:

$$\gamma = \frac{7}{5}$$

$$C_v = \frac{R}{\gamma - 1} = \frac{R}{\frac{7}{5} - 1} = \frac{5R}{2} \qquad \text{...(iii)}$$

Hence, from Eq. (i)

$$7000 = \mu\frac{5R}{2} \times 700 \quad \text{or} \quad \mu R = \frac{20}{5} = 4$$

$$(\Delta Q)_{ca} = -(7000 + 4 \times 700) = -9800 \text{ J}$$

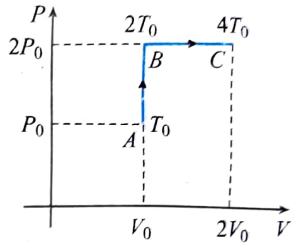
Negative sign shows that heat is ejected.

52. (3) As $\Delta U = 0$ in a cyclic process,

$\Delta Q = \Delta W =$ area of circle $= \pi r^2$

or $\Delta W = 10^2\pi$ J

53. (2) Let initial pressure, volume and temperature be P_0, V_0 and T_0, respectively, indicated by state A in P–V diagram. The gas is then isochorically taken to state B $(2P_0, V_0, 2T_0)$ and then taken from state B to state C $(2P_0, 2V_0, 4T_0)$ isobarically.



Total heat absorbed by 1 mol of gas

$$\Delta Q = C_v(2T_0 - T_0) + C_P(4T_0 - 2T_0)$$

$$= \frac{5}{2}RT_0 + \frac{7}{2}R \times 2T_0 = \frac{19}{2}RT_0$$

Total change in temperature from series A to C is $\Delta T = 3T_0$

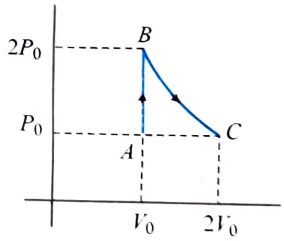
Therefore,

$$\text{Molar heat capacity} = \frac{\Delta Q}{\Delta T} = \frac{\frac{19}{2}RT_0}{3T_0} = \frac{19}{6}R$$

54. (3) $Q_{AB} = \Delta U_{AB} + W_{AB}$

$W_{AB} = 0$

$$\Delta U_{AB} = \frac{f}{2}nR\Delta T$$



$$\Delta U_{AB} = \frac{f}{2}(\Delta PV) \Rightarrow \Delta U_{AB} = \frac{5}{2}(\Delta PV)$$

$$Q_{AB} = 2.5\,P_0V_0$$

Process BC, $Q_{BC} + W_{BC} = 0 + 2\,P_0V_0 \log 2 = 1.4\,P_0V_0$

$$Q_{\text{net}} = Q_{AB} + Q_{BC} = 3.9\,P_0V_0$$

55. (2) For an adiabatic process, $0 = dU + PdV$

or $\quad d(a + bPV) + PdV = 0$

or $\quad bP\,dV + bV\,dP + p\,dV = 0$

or $\quad (b + 1)PdV + bV\,dP = 0$

or $\quad (b+1)\frac{dV}{V} + b\frac{dP}{P} = 0$

or $(b+1)\log V + b\log P = \text{constant}$

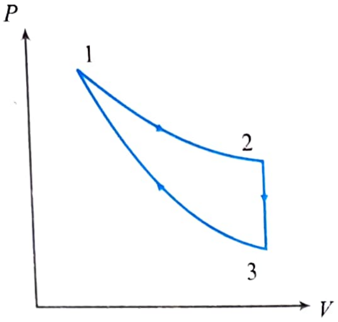
$V^{b+1}P^{b} = \text{constant}$

or $PV^{\frac{b+1}{b}} = \text{constant}$

$\therefore \quad \gamma = \dfrac{b+1}{b}$

(4) Work done on the system is negative.

For the process 1 → 2



Process	**ΔQ**	**ΔW**	**ΔU**
1–2			
2–3			
3–1		–20 J	–20 J
Cycle		10 J	

$$\Delta U_{1\to2} = 0$$

$$\therefore \quad \Delta Q_{1\to2} = \Delta W_{12}$$

For the process 2 → 3

$$\Delta W_{2\to3} = 0$$

$$\Delta Q_{2\to3} = -20\text{ J}$$

For the process 3 → 1

$$\Delta Q_{3\to1} = 20\text{ J}$$

$$\Delta U = \Delta U_{1\to2} + \Delta U_{2\to3} + \Delta U_{3\to1} = 0$$

$$\Delta W = \Delta W_{1\to2} + \Delta W_{2\to3} + \Delta W_{3\to1} = \Delta W_{1\to2} + 0 - 20\text{ J} = 10\text{ J}$$

Work done in the process 1→2 = 30 J

$$\Delta Q = \Delta Q_{1\to2} + \Delta Q_{2\to3} + \Delta Q_{3\to1} = 30\text{ J} - 20\text{ J} = 10\text{ J}$$

For the process 1 → 2, work is positive, i.e., work is done by the system.

57. (3) Work done by the balloon

$$W = \int_{15\text{m}^3}^{20\text{m}^3} P_0 dV + \int_{20\text{m}^3}^{25\text{m}^3} pdV$$

$$= \int_{15\text{m}^3}^{20\text{m}^3} P_0 dV + \int_{20\text{m}^3}^{25\text{m}^3} \left[P_0 + 2(V - V_0)^2\right] dV$$

$$= \left[100 \times 5 + 100 \times 5 + 2 \times \frac{(25-20)^3}{3}\right] \text{kJ} = 1083 \text{ kJ}$$

58. (4) Area of cycle = | W |

$$= \frac{\pi(4-3)(4-2)}{2} + \frac{\pi(2-1)(3-2.5)}{2}$$

$$= \frac{2.5\pi}{2} = \frac{5\pi}{4} \text{ atm L}$$

$$W = -\left(\frac{5\pi}{4}\right) \text{atm L}$$

59. (4) $\text{Process}_{1\to2}$ and $\text{Process}_{3\to4}$ are isochoric process.

$$W_{12} = 0$$

$$W_{34} = 0$$

$$W_{23} = nR(T_3 - T_2) = 3R(2400 - 800) = 4800 R$$

$$W_{41} = nR(T_1 - T_4) = 3R(400 - 1200) = -2400 R$$

$$W = (4800 - 2400)R = 2400 R = 20 \text{ KJ}$$

60. (2) Since PV^n = constant and also $PV = RT$, taking 1 mol of the gas for simplicity, $dU = C_V dT$

where $C_V \to$ molar specific heat at constant volume.

Now the molar specific heat in a polytropic process PV^n = constant is given by

$$C_V = \left(\frac{R}{\gamma - 1}\right) - \left(\frac{R}{n-1}\right) = \frac{(n-\gamma)R}{(n-1)(\gamma-1)} \quad \text{...(i)}$$

From this equation we see that C_v will be negative when $n < \gamma$ and $n > 1$, simultaneously, i.e., $1 < n < \gamma$. Since γ for all ideal gases is greater than 1, if $n > \gamma$ or $n < 1$, then C_V will be positive.

61. (1) Since the piston is moved slowly we assume isothermal condition for both the gases as thermal equilibrium is maintained throughout.

Let the total volume of chamber be V. Then volume of gas in X increases (expands) from $V/3$ to $2V/3$. The work done is positive and given (for 1 mol of monatomic gas) by

$$W_X = +RT \log_e \frac{2V/3}{V/3} = RT \log_e 2$$

The volume of gas in Y decreases (compressed) from $2V/3$ to $V/3$. The work done (isothermally) is negative and given (for 2 mol of diatomic gas) by

$$W_Y = 2RT \log_e \frac{V/3}{2V/3} = -2RT \log_e 2$$

Hence the total work done on the system is

$$W = W_X + W_Y = -RT \log_e 2$$

Substituting $R = 8.3$, $\log_e 2 = 0.6996$ and simplifying

$$W = -5.8\ T.$$

62. (2) The gas pressure $= \dfrac{\text{Weight of piston}}{\text{Area of cross-section}} + \text{atm. pressure}$

$$= \frac{8 \times 9.8}{60 \times 10^{-4}} + 1.00 \times 10^5 \text{ N/m}^2$$

$$= 1.13 \times 10^5 \text{ N/m}^2$$

During the heating pressure, the internal energy is changed by ΔU_1 and work ΔW_1 is done.

Therefore, $\Delta Q_1 = \Delta U_1 + \Delta W_1 = \Delta U_1 + PdV$

$$= \Delta U_1 + (1.13 \times 10^5)(0.20 \times 60 \times 10^{-4})$$

$$= \Delta U_1 + 136 \text{ J}$$

During the cooling process, no work is done as volume is constant, $\Delta W = 0$.

Hence, $\Delta Q_2 = \Delta U_2$. But ΔU_2 is negative as the temperature decreases, and since the gas returns to its original temperature, $\Delta U_2 = -\Delta U_1$

Hence $\left[\Delta Q_1 - |\Delta Q_2|\right] = (\Delta U_1 + 136 - \Delta U_1) = +136 \text{ J}$

63. (2) Temperature at state $P = T_0$, since P lies on the isothermal of temperature T_0. If T be the temperature at Q, then for the adiabatic process B, we have, $T_0 V_0^{\gamma-1} = T(2V_0)^{\gamma-1}$

$$T = \frac{T}{2^{\gamma-1}} = \frac{T_0}{2^{2/3}}$$

Change in the internal energy of the gas is

$$\Delta U = C_V(T - T_0) = \left(\frac{R}{\gamma-1}\right)\left(\frac{T_0}{2^{2/3}} - T_0\right)$$

$$= \frac{3RT_0(1 - 2^{2/3})}{2 \times 2} = -4.6\ T_0$$

64. (4) For adiabatic process:

Bulk modulus: $B = \gamma P$

for point p: $P = \frac{nRT}{V} = \frac{nR3T_0}{3V_0} = \frac{nRT_0}{V_0}$

$$\Rightarrow \quad B = \frac{\gamma nRT_0}{V_0} \qquad \text{...(i)}$$

Now $TV^{\gamma-1}$ = constant

$$\Rightarrow \quad (\gamma - 1)TdV + VdT = 0$$

$$\Rightarrow \quad \frac{dV}{dT} = \frac{-V}{(\gamma-1)T}$$

From graph, slope at point P, $\frac{-3V_0}{3T_0} = \frac{-(3V_0)}{(\gamma-1)(3T_0)}$

$$\Rightarrow \quad \gamma = 2$$

so from Eq. (i), $B = \frac{2nRT_0}{V_0}$

65. (2) At $x = \infty, C = \frac{3}{2}R$

from PV^x = constant and $P^{1/x}V$ = another constant

so at $x = \infty$, V = constant

hence $\quad C = C_v = \frac{3}{2}R$ and then $C_p = C_v + R = \frac{5}{2}R$

at $\quad x = 0$, P = constant and $C = C'$

hence $\quad C' = C_p = \frac{5}{2}R$ at $x = x'$, $C = 0$, so the process is adiabatic,

hence $\quad x' = \frac{C_P}{C_v} = \frac{5}{3}$

66. (1) In process AB, Pressure = constant; $V \propto T$

$V_B = 2V_0$

So, $\quad T_B = 2T_0 \qquad$ (if $T_A = T_0$)

$\Delta W = P_0(2V_0 - V_0) - P_0V_0 = RT_0$

$$\Delta Q = C_P\Delta T = \frac{5}{2}RT_0$$

$$\Delta U = \Delta Q - \Delta W = \frac{3}{2}RT_0$$

In process BC, Volume = constant $P \propto T$

$P_B = P_0$ but $T_B = 2T_0$

So, $\quad P_c = KP_0$, $T_c = KT_B = K \times 2T_0$

$\Delta W = 0$

$$\Delta Q = \Delta U = C_v\Delta T = \frac{3}{2}R \times 2T_0(K-1) = 3RT_0(K-1)$$

$$\Delta Q_{\text{Total}} = \frac{5}{2}RT_0 + 3RT_0(K-1) \qquad \text{...(i)}$$

$$\Delta U_{\text{Total}} = \frac{3}{2}RT_0 + 3RT_0(K-1) \qquad \text{...(ii)}$$

$$\Delta U/\Delta Q = 6/7 \qquad \text{...(iii)}$$

Compare Eqs. (i), (ii) and (iii), $K = 5/2$

So, $\quad P = \frac{5}{2}P_0$

67. (2) If parameters are temperature and pressure straight line will represent constant volume for which $dW = 0$. Hence, option (1) is wrong.

If parameters are volume and pressure, $\left(P \propto \frac{1}{V}\right)$ for constant temperature. $\Delta U = 0$ for constant temperature and $\Delta Q = \Delta W$. Hence, $dQ/dW \neq 5/2$. So, option (3) is wrong.

Obviously, option (4) is not possible

Thus, option (2) is correct. Analyse it.

68. (4) Straight line represents the constant pressure

$W_{12} = 0$

$dQ = nC_p.dT \Rightarrow dU = nC_vdT$

$dW = n(C_p - C_v)dT$

$$\frac{dQ}{dW} = \frac{5}{2} = \frac{nC_p.dT}{n(C_p - C_v)dT} = \frac{1}{1 - C_v/C_p} = \frac{\gamma}{\gamma - 1}$$

$$\gamma = \frac{5}{3} = 1.66$$

Multiple Correct Answers Type

1. (1), (4)

Vibrational kinetic energy of a monatomic gas = 0 at all temperatures. So, $C_v = 3R/2$ for a monatomic gas at high temperatures also. In case of a diatomic gas $C_v = 5R/2$ at low temperatures while, $C_v > 5R/2$ at high temperatures due to vibrational KE.

2. (1), (2), (3)

The molar heat capacity has the general definition

$$C = \frac{1}{n}\frac{\Delta Q}{\Delta T}$$

where n = number of moles

ΔQ = heat absorbed by the gas

ΔT = rise in temperature of gas

It is possible to obtain almost any set of values for ΔQ and ΔT by proper selection of a process.

3. (1), (2), (3), (4)

(1) $\Delta U = Q - W = nC_p\Delta T - P\Delta V$

$$= nC_p\Delta T - nR\Delta T = n(C_p - R)\Delta T = nC_v\Delta T = nC_v(T_2 - T_1)$$

(2) $\Delta Q = \Delta U + \Delta W$

But $\Delta Q = 0$ for adiabatic process; hence $\Delta U = -\Delta W$

or, $\quad |\Delta U| = |\Delta W|$

(3) $\Delta U = nC_v\Delta T = 0 \quad (\because \Delta T = 0)$

(4) $\Delta Q = 0$ (in adiabatic change)

4. (1), (3)

For a cyclic process, $\Delta U = 0$

i.e., $\quad \Delta U = \Delta U_1 + \Delta U_2 = 0$

From relation $\Delta Q = \Delta U + \Delta W$

As $\quad \Delta U = 0$

Hence, $\Delta Q = \Delta W$

or $\quad \Delta Q - \Delta W = 0$

5. (1), (2), (3)

Internal energy (U) depends only on the initial and final states. Hence, ΔU will be same in all the three paths. In all the three paths, work done by the gas is positive and the product PV or temperature T is increasing. Therefore, internal energy is also increasing. So, from the first law of thermodynamics, heat will be absorbed by the gas. Further, area under P–V graph is maximum in path 1 while ΔU is same for all the three paths. Therefore, heat absorbed by the gas is maximum in path 1. For temperature of the gas, we can see that product PV first increases in path 1 but whether it is decreasing or increasing later on we cannot say anything about it unless the exact values are known to us.

(1), (2), (4)

P–T graph is a straight line passing through origin. Therefore, V = constant.

$\therefore$ Work done on the gas = 0.

Further, $\rho = \frac{m}{V} \propto \frac{1}{V}$

Volume of the gas is constant. Therefore, density of gas is also constant.

$$PV = nRT$$

or $$P = \left(\frac{nR}{V}\right)T$$

i.e., slope of P–T line $\propto n$

(1), (3), (4)

T = constant

PV = constant (Boyle's law)

or $$P \propto \frac{1}{V}$$

Pressure of the gas is increasing. Therefore, volume should decrease. Work done by the gas is negative or work done on the gas is constant. Therefore, internal energy will remain constant.

8. (3), (4)

During expansion, an isotherm lies above an adiabat

Also $\begin{pmatrix}\text{Slope of}\\\text{an adiabat}\end{pmatrix} = \gamma\begin{pmatrix}\text{Slope}\\\text{of an}\\\text{isotherm}\end{pmatrix}$

$$\Rightarrow \quad m_2 = \frac{C_P}{C_V}(m_1)$$

$$m_2 C_v = m_1 C_p$$

Since $\gamma - 1$,

$$\Rightarrow \quad m_2 > m_1$$

9. (2), (4)

For an isothermal process, PV = constant

$$P = \frac{\text{constant}}{V} \text{ or } P \propto \frac{1}{V}$$

It means, for an isothermal process the graph between P and $1/V$ will be a straight line passing through origin. Hence, the straight line AB will pass through origin. Hence, option (1) is wrong.

During process AB, the pressure P remains constant but $1/V$ increases. It means, volume V decreases. Hence, AB is isobaric compression. Since volume of the gas decreases at constant pressure, therefore its temperature decreases. But temperature at C is equal to that of A. Hence during the process BC, the temperature of the gas increases. In fact, process BC is an isochoric heating. Therefore, option (2) is correct. Since during the process BC, volume remains constant, no work is done by the gas against external pressure. Therefore option (3) is wrong.

During process CA, $1/V$ decreases, it means volume V increases. Since the volume increases, work is done by the gas against external pressure.

Since process CA is an isothermal process, no change in internal energy of the gas takes place. Hence, according to the first law of thermodynamics, $Q = W + \Delta U$; heat supplied during this process is equal to work done by the gas against external pressure. Hence, option (4) is correct.

10. (1), (3)

For an adiabatic process, PV^γ = constant. Differentiating w.r.t. V, we get

$$\frac{dP}{dV}V^\gamma + P\gamma V^{\gamma-1} = 0$$

or $$\frac{dP}{dV} = -\frac{\gamma P}{V}$$

For isothermal process, PV = constant.

Hence, $$\frac{dP}{dV} = -\frac{P}{V}$$

Now, dP/dV is the slope of the (P–V) graph. Thus, the slope of the (P–V) graph for an adiabatic process is γ times that for an isothermal process. Hence, curves BC and DA both represent adiabatic process and curves AB and CD both represent isothermal process. Thus, the correct choices are (1) and (3).

11. (1), (2), (3)

The critical step is that work in path 1–3 is mean of that for 1–2–3 and 1–4–3 (considering areas).

For (1), $100 - 40 = x - 10$

or, $x = 70$ cal

For (2), It is also correct as half of (40 + 10) is 25 cal

For (3), dU is –60, work is –25

So $-60 = dQ - (-25)$

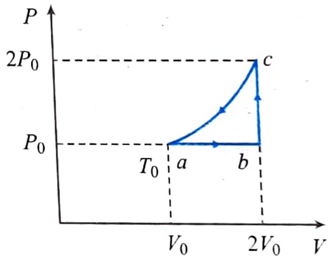
or $dQ = -85$ cal

For (4), $dU = 100 - 40 = 60$ cal

Hence, option (4) is not correct.

12. (1), (2), (3), (4)

One mole of an ideal monatomic gas (initial temperature T_0) is made to go through the cycle $abca$ shown in figure. U denotes the internal energy.



For the process ab, $\frac{P_0V_0}{T_0} = \frac{2P_0V_0}{T_0}$

$$T_b = 2T_0$$

$$T_b > T_a \Rightarrow U_b > U_a$$

$$U_b - U_a = C_v\Delta T = \frac{3R}{2}(2T_0 - T_0) = \frac{3RT_0}{2}$$

For the process bc,

$$\frac{P_0(2V_0)}{2T_0} = \frac{2P_0(2V_0)}{T_C} \Rightarrow T_C = 4T_0$$

$$T_c > T_0$$

$$U_c - U_b = \frac{3R}{2}(4T_0 - 2T_0) = 3RT_0$$

For the process ca,

$$U_c - U_a = \frac{3R}{2}(4T_0 - T_0) = \frac{9RT_0}{2}$$

13. (1), (2), (4)

During process A and B, pressure and volume both are decreasing. Therefore, temperature and hence internal energy of the gas will decrease ($T \propto PV$) or $\Delta V_{A\to B}$ = negative. Further, $\Delta W_{A\to B}$ is negative.

In process B to C, pressure of the gas is constant while volume is increasing. Hence, temperature should increase or $\Delta U_{B \to C}$ = positive. During C to A volume is constant while pressure is increasing.

Therefore, temperature and hence internal energy of the gas should increase or $\Delta U_{C \to A}$ = positive. During process CAB, volume of the gas is decreasing. Hence, work done by the gas is negative.

14. (2), (4)

Initial mass of air, $m = \dfrac{P_1V}{RT_1}$

Final mass of air $= \dfrac{m}{2} = \dfrac{P_2V}{RT_2}$

$$\frac{P_1V}{2RT_1} = \frac{P_2V}{RT_2}$$

$$\left(\frac{T_2}{T_1}\right) = 2\left(\frac{P_2}{P_1}\right)$$

As the tank is insulated, the process is adiabatic with $\gamma = \dfrac{5}{3}$

$$T_2 = T_1\left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}} \Rightarrow \left(\frac{T_2}{T_1}\right)^{\frac{5}{2}} = \left(\frac{P_2}{P_1}\right)$$

$$\frac{T_2}{T_1} = 2\left(\frac{T_2}{T_1}\right)^{\frac{5}{2}} \Rightarrow T_2 = \frac{T_1}{2^{\frac{2}{3}}}$$

$$= \frac{320}{2^{\frac{2}{3}}} = 200\text{ K} = -73°\text{C} \quad [\text{Given } \sqrt[3]{4} = 1.6]$$

15. (1), (2), (4)

(i) Work done = area under the curve,

$$W_1 = \left(P + \frac{P}{2}\right)(2V - V) = \frac{3}{2}PV$$

Work done under isothermal process,

$$W_2 = RT \times 2.3026 \log\left(\frac{2V}{V}\right) = 0.693\,RT = 0.693PV$$

$\therefore$ $W_1 > W_2$, i.e., option (1) is correct.

(ii) Let P_0 and V_0 be the intercepts on the P and V axes. Now the equation of straight line would be

$$P = -\frac{P_0}{V_0} \times V + P_0$$

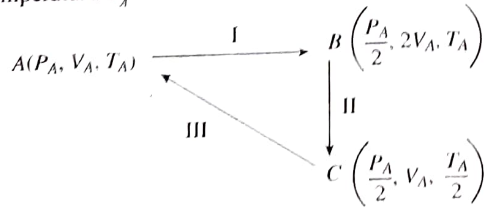
$(\because \; y = mx + c)$

Further $PV = RT$ or $P = \dfrac{RT}{V}$

(This represents parabola, so option (2) is also correct.)

16. (1), (3)

Let the process start from initial pressure P_A, volume V_A and temperature T_A.



I: Isothermal expansion (PV = constant) at temperature T_A to twice the initial volume V_A.

II: Compression at constant pressure $P_A/2$ to original volume V_A (i.e., $V \propto T$)

III: Isochoric process (at volume V_A) to initial conditions (i.e., $P \propto T$)

17. (2), (3)

For adiabatic process 'bc'

$$T_1V_b^{\gamma-1} = T_2V_c^{\gamma-1} \quad \text{...(i)}$$

For adiabatic process 'da'

$$T_2V_d^{\gamma-1} = T_1V_a^{\gamma-1} \quad \text{...(ii)}$$

Multiplying Eqs. (i) and (ii), we get

$$\Rightarrow \quad T_1T_2(V_bV_d)^{\gamma-1} = T_1T_2(V_aV_c)^{\gamma-1}$$

$$\Rightarrow \quad V_bV_d = V_aV_c$$

Since adiabatic expansion leads to cooling,

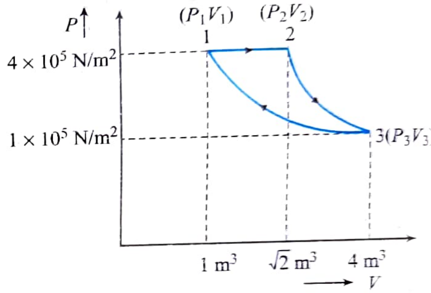
so $\quad T_1 > T_2$

18. (2), (3)

For the process 3 → 1

$$P_1V_1 = P_3V_3 \Rightarrow V_3 = \frac{P_1V_1}{P_3} = \frac{4\times10^5\times1}{1\times10^5} = 4\text{ m}^3$$

For the process 2 → 3



$$P_2V_2^\gamma = P_3V_3^\gamma \Rightarrow V_2 = \left(\frac{P_3}{P_2}\right)^{\frac{1}{\gamma}} V_3 = 4\left(\frac{1}{4}\right)^{\frac{3}{4}} = 4^{\frac{1}{4}}\text{ m}^3 = \sqrt{2}\text{ m}^3$$

For the process 1 → 2, $W_{12} = P_1(V_2 - V_1) = \left(\sqrt{2} - 1\right)4\times10^5\text{ J}$

For the process 2 → 3,

$$W_{23} = \frac{(P_2V_2 - P_3V_3)}{(\gamma - 1)} = \frac{(4\sqrt{2} - 1\times4)\times10^5}{\left(\frac{4}{3} - 1\right)} = 12(\sqrt{2} - 1)\times10^5\text{ J}$$

For the process 3 → 1,

$$W_{31} = -P_1V_1\ \ln\frac{V_3}{V_1} = -4\times10^5\times1\times\ln\left(\frac{4}{1}\right)$$

$$\Delta U = 0$$

$$\underset{1\to2\to3\to1}{\Delta Q} = \Delta U + \Delta W = 4\times10^5\,(\sqrt{2} - 1) + 12\times10^5\,(\sqrt{2} - 1) - 4\times10^5\times1.386$$

$$\simeq 1.08\times10^5\text{ J}$$

$$\Delta Q = \Delta W \simeq 1.08\times10^5\text{ J}$$

19. (1), (2), (3), (4)

For insulated chambers

$n_1 + n_2 = n_1' + n_2'$ (final pressures become equal)

$$\frac{PV}{RT} + \frac{2P\cdot2V}{RT} = \frac{P}{RT}[3V] \Rightarrow P' = \frac{5P}{3}$$

For left chamber $PV = P'V' = \dfrac{5P}{3}V' \Rightarrow V' = \dfrac{3V}{5}$

For right chamber $4PV = P'V' = \dfrac{5P}{3}V' \Rightarrow V' = \dfrac{12V}{5}$

(3), (4)

According to the process equation for adiabatic process $PV^{\gamma} =$ constant for helium and neon which are monoatomic gases ratio of specific heats is same so final pressure will be same but for oxygen which is diatomic γ is different so its pressure will also be different.

(1), (2), (3), (4)

At equilibrium net force on piston is zero or the force on the two sides must be equal so we use $PS = kx_0$ and the energy stored by the spring is equal to work done by the gas as chamber is insulated and no heat supply is there. As chamber is insulated process will be nearly adiabatic so change in internal energy will be equal to work done by the gas and in adiabatic expansion temperature decreases.

(1), (2), (3), (4)

We use $P_A V_A = RT_0$ as $P_B = 2P_A$

$$\Rightarrow \quad V_B = V_C = 2V_A \text{ and } T_B = \frac{P_B V_B}{R} = 4T_0$$

From graph work done is $\Delta W = \left(\frac{V_C - V_A}{2}\right)(P_B - P_C) = 0.5\,RT_0$

$$\text{and} \quad \Delta W_{AB} = \frac{1}{2}(P_A + P_B)(V_B - V_A) = \frac{3}{2}P_A V_A = 1.5\,RT_0$$

$$\text{and} \quad \Delta U_{AB} = \frac{3}{2}R(T_B - T_A) = 4.5\,RT_0$$

$$\text{and} \quad \Delta Q_{AB} = \Delta W_{AB} + \Delta U_{AB} = 6RT_0$$

$$\Rightarrow \quad C = \left(\frac{\Delta Q_{AB}}{T_B - T_A}\right) = 2R$$

Thus cycle efficiency is given as $\eta = \frac{\Delta W_{net}}{\Delta Q_{AB}} = \frac{1}{12} = 8.33\%$

nked Comprehension Type

r Problems 1–6

(3), 2. (1), 3. (2), 4. (3), 5. (4), 6. (2)

Gas is monatomic; hence

$$C_v = \frac{3}{2}R \quad \text{and} \quad C_p = \frac{5}{2}R$$

Number of moles, $n = 2$.

$T_A = 27°\text{C} = 300$ K

Process $A \to B$ is a straight line passing through origin; hence it is an isobaric process.

$$\frac{V_A}{T_A} = \frac{V_B}{T_B}$$

$$T_B = \left(\frac{V_B}{V_A}\right)T_A = 2 \times 300 \text{ K} = 600 \text{ K}$$

Process $A \to B$ is isobaric.

$$Q_p = nC_p(T_B - T_A)$$

$$Q_p = 2\left(\frac{5}{2}R\right)(600 - 300) = 1500\,R$$

Heat is added to the system.

Process $B \to C$ is isothermal.

From the first law of thermodynamics,

$$Q_{BC} = W_{BC} \qquad (\text{as } \Delta U = 0)$$

$$W = nRT_B \ln\left(\frac{V_C}{V_B}\right) = (2)(R)(600)\ln\left(\frac{4V_A}{2V_A}\right)$$

$$= (1200\,R)\ln(2)$$

Heat is added to the system.

Process $C \to D$ is isochoric.

From the first law of thermodynamics,

$$Q_{CD} = nC_v(T_D - T_C) = n\left(\frac{3}{2}R\right)(T_A - T_B)$$

$$= 2\left(\frac{3}{2}R\right)(300 - 600) = -900\,R$$

Negative sign implies that heat is rejected by the system.

Process $D \to A$ is isothermal.

$$Q_{DA} = W_{DA} = nRT_D \ln\left(\frac{V_A}{V_D}\right)$$

$$= (2)(R)300\ln\left(\frac{V_A}{4V_A}\right) = 600\,R\ln\left(\frac{1}{4}\right)$$

Negative sign implies that heat is rejected by the system.

In a cyclic process, $\Delta U = 0$

$$Q_{\text{net}} = W_{\text{net}}$$

$$W_{\text{net}} = Q_{AB} + Q_{BC} + Q_{CD} + Q_{DA}$$

$$= 1500\,R + 831.6\,R - 900\,R - 831.6\,R = 600\,R$$

For Problems 7–9

7. (4) Since AB is rectangular hyperbola, therefore

$\therefore \quad U\rho = \text{Constant}$

$$\Rightarrow \quad nC_vT\,\frac{PM}{RT} = C$$

$\Rightarrow \quad P = \text{Constant}$

$\therefore \quad AB \to$ isobaric

$BC \to$ isothermal as U is constant

$CA \to$ isochoric as ρ is constant

8. (2) Given $U_C = 3000\,R = nC_vT_C$

$$\Rightarrow \quad 3000\,R = 2 \times \frac{3}{2}R \times T_C$$

$\Rightarrow \quad T_C = 1000$ K

Also $U_B = U_C$ and $U_A = 2 \times \frac{3}{2}R \times 300 = 900R$

$W_{AB} = P\Delta V = nR\Delta T = 2 \times R\,(1000 - 300) = 1400\,R$

$U_{AB} = U_B - U_A = 3000R - 900R = 2100R$

$\therefore \quad \Delta Q = \Delta W + \Delta U = 1400R + 2100R = 3500R$

9. (4) CA is isochoric. Therefore W_{CA} is zero.

$\therefore \quad Q_{CA} = U_{CA} = U_A - U_C = 900\,R - 3000\,R = -2100\,R$

For Problems 10–12

10. (2) $\Delta U = 2\Delta(PV) = 2R\Delta T$

$\Rightarrow \quad C_V = 2R$

$\Rightarrow \quad C_P = 3R$

11. (4) $W = RT\log_e\frac{2V_0}{V_0} = PV\log_e 2 = \frac{U_1 - U_0}{2}\log_e 2$

12. (3) $C_V = 2R$, which is between the C_V values of mono and diatomic gases.

For Problems 13–15

13. (3) Heat given for gas A, $\Delta Q = n_1 C_{v_1}\Delta T$

For gas $B - \Delta Q = n_2 C_{v_2}\Delta T$

($\therefore$ For same heat given, temperature rises by same value for both the gases) ...(i)

$$n_1 C_{v_1} = n_2 C_{v_2}$$

Also, $(\Delta P_B)V = n_2 R\Delta T$ and $(\Delta P_A)V = n_1 R\Delta T$

$$\frac{n_1}{n_2} = \frac{\Delta P_A}{\Delta P_B} = \frac{2.5}{1.5} = \frac{5}{3}$$

$$n_1 = \frac{5}{3} n_2$$

Substituting in Eq. (i),

$$\frac{5}{3} n_2 C_{v_1} = n_2 C_{v_2} \quad \Rightarrow \quad \frac{C_{v_2}}{C_{v_1}} = \frac{5}{3} = \frac{\left(\frac{5}{2}R\right)}{\left(\frac{3}{2}R\right)}$$

Hence, gas B is diatomic and gas A is monatomic.

14. (4) Since $n_1 = 5/3\, n_2$, $\frac{125}{M_A} = \frac{5}{3}\left(\frac{60}{M_B}\right)$

(From experiment 1: $W_A = 25$ g and $W_B = 60$ g)

$$5M_B = 4M_A$$

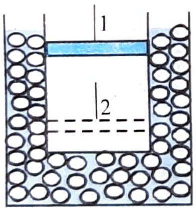
The above relation holds if gas A is Ar and gas B is O_2

15. (3) It could also be seen directly that temperature finally will be 300 K, since no heat exchange takes place between those gases as their initial temperatures are same.

Since volume remains same but number of moles increases, pressure increases.

For Problems 16–18

16. (4) Since the piston is pushed very quickly from position 1 to position 2, the process AB is an adiabatic compression. Process BC is isochoric and process CA is an isothermal expansion. These are shown on the P–V diagram correctly in option (4).



17. (1) If ΔW is the work done in the whole cycle, we have

$$\Delta Q = \Delta W + \Delta U$$

But $\Delta U = 0$ for the entire cycle.

Hence $\Delta W = \Delta Q = mL = 100 \times 80 = 8000$ cal $= 8$ kcal

18. (4) Work done during the entire cycle = 8 kcal = W

But $W = W_{ad} + W_v + W_{iso} = \frac{R(T_2 - T_1)}{\gamma - 1} + 0 + RT \ln \frac{V_2}{V_1}$

This is not equal to any of options given.

For Problems 19–21

19. (3) An AB is isobaric, in $B \rightarrow C$ magnitude of $\frac{dT}{dv}$ increases and OA is isochoric process.

Hence option (3) is correct.

20. (3) In cyclic process $\Delta U = 0$

$$W_{AB} + W_{BC} + W_{CA} = -1000 \text{ J}$$
$$R\,200 + W_{BC} + 0 = -1000 \text{ J}$$
$$W_{BC} = -7640 \text{ J}$$

21. (1) The volume of gas at state C is

$$V_C = V_A = \frac{nRT_A}{P_A} = 0.0332 \text{ m}^3$$

For Problems 22–24

22. (1) It is given that there is decrease in the internal energy while the gas expands absorbing some heat, which is numerically equal to the decrease in internal energy.

Hence, $dQ = -dU$

Thus, $dQ = dW + dU = -dU$

$dw = -2dU$ (remembering that both dW and dU are negative)

$PdV = 2CdT \quad (dU = -CdT)$

where C is the molar specific heat of the gas. ...(i)

Since the gas is ideal, we have

$$P = \frac{RT}{V} \qquad \text{...(ii)}$$

Hence Eq. (i) gives

$$\frac{RT}{V} dV = 2CdT$$

$$\frac{dV}{V} = \frac{2C}{R}\frac{dT}{T}$$

Solving we get $(VT^{-2C/R}) =$ constant ...(iii)

Also since $dQ = -dU$, $C = \left(\frac{dQ}{dT}\right)_{\substack{\text{constant}\\ \text{volume}}} = \left(\frac{dU}{dT}\right) = -C_v$

Hence, $C = -C_v = -\frac{R}{\gamma - 1} = -\frac{R}{\frac{7}{5} - 1} = -\frac{5R}{2}$

Since the gas is diatomic.

23. (2) From Eq. (iii)

$$TV^{\frac{-R}{2C}} = \text{constant}$$

Putting $C = -5R/2$, $TV^{\frac{1}{5}} = \text{constant}$ ...(iv)

24. (2) Under the given process, the final volume V is 32 times the initial volume V_0.

Thus, $W = \int_{V_0}^{32V_0} \frac{PdV}{2} = \int_{V_0}^{32V_0} \frac{RT}{2V} dV = \int_{V_0}^{32V_0} \frac{R}{2V} TdV$

Now, $TV^{\frac{1}{5}} = T_0 V_0^{\frac{1}{5}}$ gives $T = \frac{T_0 V_0^{\frac{1}{5}}}{V^{\frac{1}{5}}}$

Hence, $W = \int_{V_0}^{32V_0} \frac{R}{2V} \frac{T_0 V_0^{\frac{1}{5}}}{V^{1/5}} dV = RT_0 V_0^{1/5} \int_{V_0}^{32V_0} \frac{V^{-6/5}}{2} dV$

$$= RT_0 V_0^{1/5} \left[\frac{-5V^{-1/5}}{2}\right]_{V_0}^{32V_0}$$

$$W = -\frac{5RT_0}{2}$$

Work done by the gas $= \frac{5RT_0}{2}$

For Problems 25–27

25. (1), 26. (3), 27. (1)

For adiabatic process BC, ...(i)

$$P_B V_B^\gamma = P_C V_C^\gamma$$

For isothermal process CA, ...(ii)

$$P_A V_A = P_C V_C$$

From Eqs. (i) and (ii)

$$V_C = \left[\frac{V_B^\gamma}{V_A}\right]^{\frac{1}{\gamma - 1}} = 64 \text{ m}^3$$

$$P_C = \frac{P_A V_A}{V_C} = \frac{10^5}{64}\ \text{N/m}^2$$

Work done, $W = W_{AB} + W_{BC} + W_{CA}$

$$= P(V_B - V_A) + \frac{1}{\gamma - 1}\left[PV_B - P_C V_C + PV_A \ln(V_A/V_C)\right]$$

Putting the values, $W = 4.9 \times 10^5$ J

or Problems 28–31

(2), 29. (2), 30. (1), 31. (2)

Process CB is done at minimum temperature, while process DA is done at maximum temperature.

Process

$A \to B$ (isobaric)

$$\frac{V_A}{V_B} = \frac{T_A}{T_B} = \frac{800}{400} = 2$$

hence $V_B = 2V_0$

$B \to C$ (isothermal)

$C \to D$ (isobaric)

$$\frac{V_D}{V_C} = \frac{T_D}{T_C} \Rightarrow V_D = 2V_0$$

$D \to A$ (isothermal)

$$W_{AB} = P(V_B - V_A) = nR\,DT = nR(-400) = -400nR$$

$$W_{BC} = nRT\ln\frac{V_C}{V_B} = nR\,400\ln\frac{V_0}{2V_0} = -400\,nR\ln 2$$

$$W_{CD} = P(V_D - V_C) = nR(400) = 400nR$$

$$W_{DA} = nRT\ln\frac{V_A}{V_D} = nR \times 800 \times \ln\frac{2V_0}{V_0} = 800\,nR\ln 2$$

$$\Delta W = 400\,nR\ln 2$$

ΔQ heat is extracted

$$Q_{CD} = nC_p\Delta T = n\frac{5}{2}R\,400 = 1000nR$$

$$\Delta Q = 1000nR + 800nR\ln 2$$

$$\text{Efficiency} = \frac{\Delta W}{\Delta Q}$$

$$\eta = \frac{2\ln 2}{5 + 4\ln 2} \times 100\%$$

For Problems 32–33

32. (3), 33. (4)

Process 1→2 (Isochoric process)

$$W_{12} = 0$$

$$Q_{12} = \Delta U_{12} = nC_v(T_2 - T_1)$$

$$= n\frac{3}{2}R(T_2 - T_1) = \frac{3}{2}[P_0V_0 - (P_0/2)V_0]$$

$$= \frac{3P_0V_0}{4}\ \text{(heat absorbed)}$$

Process 3→1,

$$W_{3\to1} = \text{Area under process } 3 \to 1 = -\left(\frac{P_0}{2} + P_0\right)\cdot\frac{V_0}{2} = -\frac{3P_0V_0}{4}$$

Work done during 3→1 will be negative as volume is decreasing.

$$\Delta U_{3\to1} = n\frac{3R}{2}(T_1 - T_3) = \frac{3}{2}\left(\frac{P_0V_0}{2} - P_0 2V_0\right) = -\frac{9P_0V_0}{4}$$

$$Q_{3\to1} = \Delta U_{3\to1} + W_{3\to1} = -3P_0V_0\ \text{(heat rejected)}$$

Process 2→3,

The temperature will be maximum at $x = 2P_0$, $T_x = \dfrac{3P_0V_0}{2R}$

$$W_{2\to x} = W_{x\to3} = \frac{1}{4}\pi\frac{P_0V_0}{2} + \frac{P_0V_0}{2} = \frac{P_0V_0}{2}\left(\frac{\pi}{4} + 1\right)$$

$$W_{2\to3} = \frac{P_0V_0}{4}(\pi + 4)$$

$$\Delta U_{2\to x} = n\frac{3}{2}R(T_x - T_2) = \frac{3}{2}(3P_0V_0 - P_0V_0) = 3P_0V_0$$

$$\Delta U_{x\to3} = n\frac{3}{2}R(T_3 - T_x) = \frac{3}{2}(2P_0V_0 - 3P_0V_0) = -\frac{3}{2}P_0V_0$$

$$\therefore\quad \Delta Q_{2\to x} = \Delta U_{2\to x} + W_{2\to x} = 3P_0V_0 + \frac{P_0V_0}{2}\left(\frac{\pi}{4} + 1\right)$$

$$= P_0V_0\left(3 + \frac{1}{2} + \frac{\pi}{8}\right) = \frac{P_0V_0}{2}\left(7 + \frac{\pi}{4}\right)$$

$$\Delta Q_{x\to3} = \Delta U_{x\to3} + W_{x\to3}$$

$$= -\frac{3}{2}P_0V_0 + \frac{P_0V_0}{2}\left(\frac{\pi}{4} + 1\right) = \frac{-P_0V_0}{2}\left(2 - \frac{\pi}{4}\right)$$

Total heat rejected in one cycle

$$= 3P_0V_0 + \frac{P_0V_0}{2}\left(2 - \frac{\pi}{4}\right) = 4P_0V_0 - \frac{\pi P_0V_0}{8}$$

$$= P_0V_0\left(4 - \frac{\pi}{8}\right) = \frac{P_0V_0(32 - \pi)}{8}$$

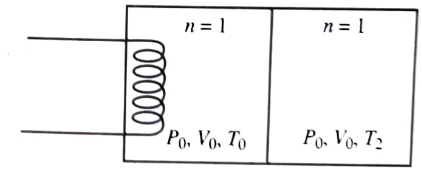
For Problems 34–35

34. (4), 35. (2)

For the right part, $P_0V_0^{\gamma} = PV_2^{\gamma}$

$$V_2 = \left(\frac{P_0}{P}\right)^{1/\gamma} V_0 = \left(\frac{32}{243}\right)^{3/5} V_0$$

$$V_2 = \frac{8}{27}V_0$$



$$V_1 = 2V_0 - \frac{8}{27}V_0 = \frac{46}{27}V_0$$

For left part, $\dfrac{P_0V_0}{T_0} = \dfrac{PV_1}{T_1}$

$$T_1 = \frac{PV_1}{P_0V_0}T_0 = \frac{\frac{243P_0}{32} \times \frac{46}{27}}{P_0V_0}T_0$$

$$T_1 = \frac{207}{16}T_0$$

Now applying ideal gas equation in right part,

$$T_2 = \frac{PV_2}{P_0V_0}T_0 = \frac{9}{4}T_0$$

For right part, $\Delta Q = 0 = dU + dW$

$$dW = -dU = -1\,C_v\,RT$$

$$= -\frac{3}{2}R\left(\frac{9}{4}T_0 - T_0\right) = -\frac{15}{8}RT_0$$

For Problems 36–38

36. (3) First adiabatic compression takes place

$$\gamma = 1.5$$

$$V_0 = Ah, \quad V_1 = \frac{Ah}{2}$$

$$T_0V_0^{\gamma-1} = T_1V_1^{\gamma-1} \Rightarrow \sqrt{2}T_0 > T_0$$

37. (2) Heat given in adiabatic compression is used to work against elastic force and against atmospheric force.

38. (3) Final pressure, $P_f = \left(P_0 + \frac{Kh/16}{A}\right)$

$$\frac{P_fV_f}{T_f} = \frac{P_0V_0}{T_0}$$

$$\frac{\left(P_0 + \frac{kh}{16A}\right)A \times \left(\frac{h}{2} + \frac{h}{16}\right)}{T_f} = \frac{P_0Ah}{T_0}$$

$$\frac{\left(10^5 + \frac{3700h}{16 \times 27 \times 10^{-4}}\right)\frac{9}{16}}{\frac{4}{3} \times 273} = \frac{10^5}{273}$$

$$(1 + 0.856\,h) = \frac{16}{9} \times \frac{4}{3}$$

Solving, $h = 1.6$ m

For Problems 39–41

39. (1), 40. (2), 41. (1)

For 'C', $P_0V_0^\gamma = \left(\frac{125}{27}\right)P_0V_C^\gamma$

$$V_C = V_0\left(\frac{9}{25}\right)$$

$$V_B = 2V_0 - \frac{9V_0}{25} = \frac{(50-9)}{25}V_0$$

$$V_B = \frac{41}{25}V_0$$

$$T_{0A} = T_{0B} = \frac{P_0V_0}{R}$$

$$T_C = \frac{\frac{125}{27}P_0 \times \frac{9}{25}V_0}{R} = \frac{5P_0V_0}{3R}$$

$$T_B = \frac{\frac{125}{27}P_0 \times \frac{41}{25}V_0}{R} = \frac{205}{27}\frac{P_0V_0}{R} = T_A$$

$$Q = \Delta U$$

$$Q = n_AC_v(T_A - T_0) + n_BC_v\Delta T_B + n_CC_v\Delta T_C$$

$$Q = 2.2R\left(\frac{205}{27} - 1\right)\frac{P_0V_0}{R} + 1.2R\left(\frac{205}{27} - 1\right)\frac{P_0V_0}{R} + 1.2R\left[\frac{5}{3}\frac{P_0V_0}{R} - \frac{P_0V_0}{R}\right]$$

$$= 2P_0V_0\left[\frac{178 \times 2}{27} + \frac{178}{27} + \frac{2}{3}\right] = \frac{368}{9}P_0V_0$$

Matrix Match Type

1. i. → d.; ii. → b.; iii. → c.

(d) AE is isothermal process

$$\frac{V_0}{T_0} = \frac{2V_0}{2T_0} \rightarrow AC \text{ – isobaric process}$$

For adiabatic process, $T_0V_0^{\gamma-1} = T(2V_0)\gamma^{-1}$, $\gamma = \frac{5}{3}$

$$T = T_0\left(\frac{1}{2}\right)^{\gamma-1} = \frac{T_0}{\substack{2\\2^3}} = 0.63T_0 \rightarrow AF \text{ – adiabatic process}$$

2. i. → a., d.; ii. → d.; iii. → a., d.; iv. → b., c.

(a) $PV = nRT$

$$P = (nRT)\frac{1}{V} = (\text{constant})\ \frac{1}{V}$$

$T =$ constant i.e. isothermal process

$\because$ V increases, ΔW is positive

and $\Delta Q = \Delta U + \Delta U = \Delta W > 0$

(b) $\Delta Q = 0$

$pdV = \Delta W =$ positive

(c) $PV = nRT$

As volume increases, T also increases

i.e., $\Delta U > 0$

$PdV = \Delta W > 0$ So $\Delta Q > 0$

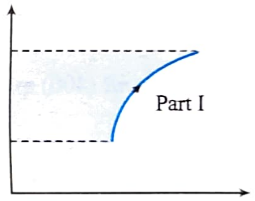
(d) For cyclic process $\Delta U = 0$

$\Delta W < 0$ (anticlockwise)

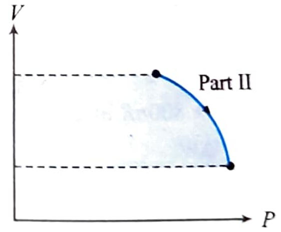
$\Delta Q < 0.$

3. i. → d.; ii. → b.; iii. → c.; iv. → b.

In (i), V is on the vertical axis.



As V is increasing, W is positive



V is decreasing, W is negative

As negative work in Part II is greater than positive work in Part I, net work during the process is negative. Using $PV = nRT$ and as V remains same for initial and final points of the process, it is obvious that final temperature is greater than initial temperature as pressure has increased. Therefore dU is positive. Hence option (4) is connected with (i).

Similar arguments can be applied to other graphs.

4. i. → a.; ii. → d.; iii. → c.; iv. → b.

$$W_{iaf} + \Delta U_{if} = H_{iaf}$$

$$20 + \Delta U_{if} = 50$$

$$\Delta U_{if} = 30 \text{ and } W_{ibf} + \Delta U_{if} = H_{ibf}$$

$$W_{ibf} + 30 = 36$$

$$W_{ibf} = 6$$

(i) $W_{ibf} = 6$

(ii) $H_{fi} = W_{fi} + \Delta U_{fi} = 13 + 30 = 43$

(iii) $\Delta U_{if} = 30$ $U_i = 10$ $U_f = 40$

(iv) $H_{ib} = W_{ib} + \Delta U_{ib} = 6 + (22 - 10) = 18$

i. → b.; ii. → c.; iii. → a.

(i) $dQ = dU + dW$

Here $dQ = +\frac{500 \times 4184}{1000}\text{ J} = 2092\text{ J}$

$dW = +400$ J. Since the work is done by the system,

$$dU = dQ - dW = 2092 - 400 = 1700\text{ J}$$

(ii) $\Delta U = \Delta Q - \Delta W = (300)(4.184) - (-420) = 1680\text{ J}$ (approx.)

(iii) $\Delta U = (\Delta Q - \Delta W) = (-1200)(4.184) - 0 = -5000\text{ J}$

(Note that ΔQ is positive when heat is added to the system and ΔW is positive when the system does the work.)

i. → b.; ii. → c.; iii. → d.; iv. → a.

$$C_p - C_v = 29.1\text{J/K} = nR$$

For molar heat capacities $C_p - C_v = R$

Number of moles in the gas, $n = \frac{29.1}{8.31} = 3.5\text{ mol}$

For monatomic gas, $\frac{C_p}{C_v} = \frac{5}{3}$

$$\frac{5}{3}C_v - C_v = 29.1\text{ J/K}$$

$$C_v = \left(\frac{3}{2} \times 29.1\right)\text{J/K} = 43.7\text{ J/K}$$

$$C_p = 72.7\text{ J/K}$$

For rigid diatomic gas the number of degree of freedom is 5.

$$\frac{C_p}{C_v} = \frac{7}{5}$$

$$C_p - \frac{5}{7}C_p = 29.1$$

$$C_p = 102\text{ J/K}$$

For vibrating diatomic ideal gas, the total number of degrees of freedom is 7.

Internal energy, $U = \frac{7}{2}nRT$

$$C_v = \frac{7}{2}nR \quad \text{and} \quad C_p = \frac{9}{2}nR$$

$$C_p - C_v = nR \quad \Rightarrow C_p = \frac{9}{2} \times 29.1 = 131\text{J/K}$$

7. i. → a., c., d.; ii. → b.; iii. → a., c., d.; iv. → b., c.

(i) If $P = 2V^2$, from ideal gas equation, we get $2V^3 = nRT$

(1) Hence as volume increases temperature will also increase.

(2) dW = positive

Hence $dQ = dU + dW$ = positive

(ii) If PV^2 = constant, from ideal gas equation, we get $VT = K$ (constant)

Hence with increase in volume, temperature decreases.

Now $dQ = dU + PdV = nC_v dT - \frac{PK}{T^2}dT \quad \left[PdV = -\frac{K}{T^2}dT\right]$

$$= nC_v dT - \frac{PVT}{T^2}dT = n(C_v - R)dT \quad [\because K = VT]$$

As temperature increases, dT = positive.

And since $C_v > R$ for monatomic gas, dQ = positive as temperature is increased.

(iii) $dQ = nCdT = nC_v + PdV$

$$n(C_v + 2R)dT = nC_v dT + pdV$$

$$2nRdT = PdV$$

$$\therefore \quad \frac{dV}{dT} = +\text{ve}$$

Hence with increase in temperature, volume increases and vice versa.

$$\therefore \quad dQ = \Delta U + dW + PdV$$

$$n(C_v - 2R)dT = nC_r dT + pdV$$

$$-2nRdT = PdV$$

$$\therefore \quad \frac{dV}{dT} = -\text{ve}$$

Therefore, with increase in volume, temperature decreases.

Also $dQ = n(C_v - 2R)\,dT$

With increase in temperature dT=+ve but $C_v < 2R$ for monatomic gas. Therefore dQ = –ve with increase in temperature.

8. i. → c.; ii. → d.; iii. → a.; iv. → b.

(i) Heat, ΔH added to the gas along the straight line path AC.

(i) Work ΔW_{AC} done by the gas along the straight line path AC = area $\left(\int_A^C pdv\right)$ under the path.

$$\Delta W_{AC} = P_A V_A + \frac{1}{2}P_A V_A = \frac{3}{2}P_A V_A$$

(ii) Increase of internal energy ΔU_{AC} is

$$\Delta U_{AC} = \frac{3}{2}nR(T_C - T_A) = \frac{3}{2}(P_C V_C - P_A V_A) = \frac{9}{2}P_A V_A$$

Hence heat added,

$$\Delta H_{AC} = \Delta W_{AC} + \Delta U_{AC} = \frac{3}{2}P_A V_A = \frac{9}{2}P_A V_A = 6P_A V_A$$

(ii) From above, $U_{AC} = \frac{9}{2}P_A V_A$

(iii) Work done

$$\Delta W_{ABC} = \Delta W_{AB} + \Delta W_{BC} = 0 + P_C(V_C - V_A) = 2P_A V_A]$$

(iv) Heat ΔH added to the gas along ABC,

$$\Delta H_{ABC} = \Delta H_{AB} + \Delta H_{BC} = \frac{9}{2}P_A V_A + 2P_A V_A = \frac{13}{2}P_A V_A$$

Numerical Value Type

1. (6) For process I: $dQ = dU + dW$

$\Rightarrow \quad 4 = dU + 5 \Rightarrow dU = -1$

For process II : $5 = dU + dW$

$\Rightarrow \quad 5 = -1 + dW \Rightarrow dW = +6\text{ J}$

2. (9) Here $\Delta Q = 15000$ J (given)

In an isobaric process $\Delta Q = nC_p\Delta T, \Delta U = nC_v\Delta T$ (always)

$$\frac{\Delta U}{\Delta Q} = \frac{nC_v\Delta T}{nC_p\Delta T} = \frac{1}{\gamma} \Rightarrow \Delta U = \frac{\Delta Q}{\gamma}$$

or $\quad \Delta U = \frac{3}{5} \times 15 = 9\text{ kJ}$

3. (4) As here volume of gas remains constant,

$(\Delta Q)_v = \mu C_v \Delta T$, Here $C_v = 5$ cal/mol K

And $\Delta T = (400 - 300) = 100$ K

And so for ideal gas $PV = \mu RT$,

$$\mu = \frac{(10)^5 \times (0.2)}{8.31 \times 300} = 8\text{ mol}$$

$(\Delta Q)_v = 8 \times 5 \times 100 = 4$ kcal

4. (3) $P \propto V$ since graph is a straight line

$$PV^{-1} = \text{constant}$$

For the process $PV^x = $ Constant, molar heat capacity is given by

$$C = \frac{R}{(\gamma - 1)} + \frac{R}{(1 - x)}$$

But here $\gamma = 1.4$ and $x = -1$

$$\text{So, } C = \frac{R}{(1.4 - 1)} + \frac{R}{(1 + 1)} = 3R = nR \text{ (Given)}$$

So, $n = 3$

5. (3) $PV^{\gamma} = \text{constant}$

$PV^{4/3} = \text{constant}$

$\ln(V^4) + \ln(P^3) = \text{constant}$

$$4\left(\frac{dV}{V}\right) = -3\left(\frac{dP}{P}\right)$$

$4P(dV) + 3V(dP) = 0$

$3(PdV + VdP) + PdV = 0$ ...(i)

For adiabatic process, $dQ = 0 = dU + dW = dU + PdV$ ...(ii)

Compare Eqs. (i) and (ii), $dU = 3(PdV + VdP)$

$$U = K + 3PV$$

So, $n = 3$

6. (6) The specific heat of the process, $C = \dfrac{13R}{6}$

The specific heat at constant volume, $C_v = \dfrac{3}{2}R$

$$\text{Now, } C = \frac{Q}{\Delta T} = \frac{(\Delta U + W)}{\Delta T} = \frac{\Delta U}{\Delta T} + \frac{W}{\Delta T}$$

$$\Rightarrow \quad C = C_v + \frac{W}{\Delta T}$$

$$\text{So, } \frac{W}{\Delta T} = C - C_v = \frac{13R}{6} - \frac{3R}{2} = \frac{2R}{3}$$

$$\Delta T = T_2 - T_1 = \frac{(nP_0)(5V_0)}{R} - \frac{(3P_0)V_0}{R}$$

$$\Rightarrow \quad \Delta T = \frac{P_0V_0}{R}(5n - 3)$$

Work done in process $A \rightarrow B$,

$$W = \frac{(5V_0 - V_0)(nP_0 + 3P_0)}{2} = 2\,P_0V_0(n + 3)$$

$$\Rightarrow \quad \frac{W}{\Delta T} = \frac{2(n+3)R}{(5n - 3)} = \frac{2R}{3}$$

On solving we get, $n = 6$

7. (3) Let x moles of diatomic gas be associated. x moles of diatomic gas become $2x$ moles of a monatomic gas after dissociation.

Internal energy of n moles of an ideal gas

$$= \frac{3}{2}nRT \text{ for monatomic gas}$$

Internal energy of n moles of an ideal gas

$$= \frac{5}{2}nRT \text{ (for diatomic gas)}$$

So, [Internal energy of $2x$ moles of a monatomic gas + internal energy of $(4 - x)$ moles of a diatomic gas] – internal energy of 4 moles of a diatomic gas

$$= \frac{3}{2}RT \text{ (given)}$$

$$\Rightarrow \quad (2x)\left(\frac{3RT}{2}\right) + (4 - x)\left(\frac{5}{2}RT\right) - (4)\left(\frac{5}{2}RT\right) = \frac{3}{2}RT$$

On solving, $x = 3$ moles.

8. (2) For two gases, we can write

$$\frac{n_1}{\gamma_1 - 1} + \frac{n_2}{\gamma_2 - 1} = \frac{n_1 + n_2}{\gamma_{\text{mean}} - 1}$$

Here, $n_1 = 1$ and $n_2 = ?$

$\gamma_1 = 5/3$ and $\gamma_2 = 7/5$

Substituting these values in the above equation, we get

$$\frac{1}{\frac{5}{3} - 1} + \frac{n_2}{\frac{7}{5} - 1} = \frac{1 + n_2}{\frac{19}{13} - 1}$$

$$\text{or} \quad \frac{3}{2} + \frac{5n_2}{2} = \frac{13(1 + n_2)}{6}$$

$$\text{or} \quad 3(3 + 5n_2) = 13(1 + n_2)$$

$$9 + 15n_2 = 13 + 13n_2$$

$$n_2 = 2 \text{ g mol}$$

9. (3.14)

The cycle is clockwise; thus net work is done by the gas. As in a cyclic process no change in internal energy takes place, heat supplied is equal to the work done by the gas in one complete cycle. So in this case heat supplied to the gas is given as

$Q = $ work done by the gas, $W = $ area of ellipse $= -\pi ab$

where a and b are semi-major and semi-minor axes of the ellipse, respectively, which are given from the figure as

$a = 1.0 \times 10^5$ N/m and $b = 100 \times 10^{-6}$ m^3

Thus area of ellipse is $Q = W = \pi \times 1.0 \times 10^5 \times 100 \times 10^{-6}$

$= 3.14 \times 10 = 3.14$ J

10. (60)

Given that $\Delta Q_1 = 100\,\text{J}, \Delta W_1 = -20\,\text{J}, \Delta Q_2 = -20\,\text{J}$

For a cyclic process we use $\Delta Q_1 - \Delta W_1 = -(\Delta Q_2 - \Delta W_2)$

$$\Rightarrow \quad 100 - 20 = -(-20 - \Delta W_2)$$

$$\Rightarrow \quad \Delta W_2 = 80 - 20 = 60\,\text{J}$$

11. (207.5)

Given that $\Delta Q = nC_V(10) = 25R$

Work done in adiabatic process is

$W = -\Delta U = -nC_V\Delta T = nC_V(10) = 25R$

12. (1590)

Given that $\Delta Q_{A\rightarrow B} = 50\,\text{J}$, $\Delta Q_{B\rightarrow C} = 0$, $\Delta Q_{C\rightarrow D} = -70\,\text{J}$

$\Delta W_{B\rightarrow C} = 40\,\text{J}$, $U_A = 1500\,\text{J}$

$U_B = U_A + \Delta U_{A\rightarrow B} = 1500 + 50 = 1550\,\text{J}$

$U_C = B_B + W_{B\rightarrow C} = 1500 + 40 = 1590\,\text{J}$

13. (16.6)

Rate of collisions with vessel wall is $N_C = \dfrac{1}{6}n_0V_{\text{rms}} = \text{constant}$

$$\Rightarrow \quad \frac{1}{6}\frac{N}{V}\cdot\sqrt{\frac{3RT}{m}} = \text{constant}$$

$$\Rightarrow \quad \frac{\sqrt{T}}{V} = \text{constant}$$

$$\Rightarrow \quad T = kV^2$$

$$\Rightarrow \quad \frac{dT}{dV} = 2kV$$

Molar heat capacity is $C = C_V + \dfrac{PdV}{ndT}$

$$C = \frac{3R}{2} + \frac{RT}{V}\cdot\frac{dV}{dT}$$

$$C = \frac{3}{2}R + \frac{R(kV)^2}{V}\cdot\frac{1}{2kV} = 2R$$

(24.9)
Rate of collisions with vessel wall is

$$N_C = \frac{1}{6} n_0 V_{rms} = \frac{1}{6}\left(\frac{N}{V}\right)\sqrt{\frac{3RT}{M}}$$

$$\Rightarrow \quad N_C \alpha \frac{\sqrt{T}}{V} = \text{constant}$$

$$\Rightarrow \quad T = kV^2$$

$$\Rightarrow \quad \frac{dT}{dV} = 2kV$$

$\Rightarrow$ molar heat capacity is given as $C = C_V + \frac{RT}{V}\frac{dV}{dT}$

$$\Rightarrow \quad C = \frac{5R}{2} + \frac{R(kV^2)}{V} \cdot \frac{1}{2kV} = 3R$$

(1050)

As here piston is open to atmosphere, the gas is under a constant pressure given as

$$P_{\text{gas}} = P_{\text{atm}} + \frac{Mg}{S} = 10^5 + \frac{50 \times 10}{100 \times 10^4}\,\text{Pa}$$

$$= 10^5 + 5 \times 10^4 = 1.5 \times 10^5\,\text{Pa}$$

It is given that the piston moves out by 20 cm, thus increment in its volume is given as

$$\Delta V = S\Delta x = 100 \times 10^{-4} \times 20 \times 10^{-2}\,\text{m}^3 = 2 \times 10^{-3}\,\text{m}^3$$

Thus in the process work done by gas is

$$W = P_{\text{gas}}\Delta V \quad [\text{As } P_{\text{gas}} = \text{constant}]$$

$$= 1.5 \times 10^5 \times 2 \times 10^{-3} = 300\,\text{J}$$

For a process in which gas pressure is constant, work done can also be given as

$$W = nR\Delta T = P_{\text{gas}}\Delta V$$

or $\quad nR\Delta T = 300\,\text{J}$

As in the process gas pressure is constant thus heat supplied in raising the temperature by ΔT is given as

$$Q = nC_P\Delta T = n\left(\frac{7}{2}R\right)\Delta T$$

[As for O_2 as being diatomic gas $C_P = \frac{7R}{2}$]

$$= - \times 300 = 1050\,\text{J}$$

Archives

E Main

ingle Correct Answer Type

1. (3) $W_{ab} = \Delta Q - \Delta U = nC_p\,dT - nC_v\,dT$ (at constant pressure)

$$= n(C_p - C_v)dT = nRdT = 2 \times R \times (500 - 300) = 400R$$

2. (1) At constant temperature (isothermal process),

$$W_{DA} = nRT \log\left(\frac{P_1}{P_2}\right) = 2.303 \times 2R \times 300 \log\left(\frac{10^5}{2 \times 10^5}\right)$$

$$= 2.303 \times 600\,R \log\left(\frac{1}{2}\right) = 0.693 \times 600R = -414R$$

3. (2) Net work done in a cycle is $W_{AB} + W_{BC} + W_{CB} + W_{BA}$

$$= 400R + 2 \times 2.303 \times 500R \ln 2 - 400R - 414R$$

$$= 1000R \times \ln 2 - 600R \times \ln 2 = 400R \times \ln 2 = 276R$$

4. (2) The efficiency of cycle is $\eta = 1 - \frac{T_2}{T_1}$ for adiabatic process

$TV^{\gamma-1} = \text{constant}$

For a diatomic gas, $\gamma = 7/5$.

$$T_1V_1^{\gamma-1} = T_2V_2^{\gamma-1}$$

$$\Rightarrow \quad T_1 = T_2\left(\frac{V_2}{V_1}\right)^{\gamma-1}$$

$$\Rightarrow \quad T_1 = T_2(32)^{\frac{7}{5}-1} = T_2(2^5)^{2/5} = T_2 \times 4$$

$$\Rightarrow \quad T_1 = 4T_2$$

$$\therefore \quad \eta = \left(1 - \frac{1}{4}\right) = \frac{3}{4} = 0.75$$

5. (4) $\frac{1}{2}Mv^2 = 1 \times C_V\Delta T$

$$\Rightarrow \quad \frac{1}{2}Mv^2 = \frac{R}{\gamma - 1}\cdot\Delta T$$

$$\Rightarrow \quad \Delta T = \frac{Mv^2(\gamma-1)}{2R} = \frac{(\gamma-1)Mv^2}{2R}\,\text{K}$$

6. (1) $\eta = 1 - \frac{T_2}{T_1} = \frac{1}{6} \Rightarrow \frac{T_2}{T_1} = 1 - \frac{1}{6} = \frac{5}{6}$

$$\frac{1}{3} = 1 - \frac{(T_2 - 62)}{T_1} \Rightarrow \frac{T_2 - 62}{T_1} = \frac{2}{3}$$

$$\Rightarrow \quad \frac{5(T_2 - 62)}{6T_2} = \frac{2}{3} \Rightarrow T_2 = 310\,\text{K}$$

and $\quad T_1 = \frac{6 \times 310}{5} = 372\,\text{K}$

7. (3) For 1st case, Efficiency $= \eta = \left(1 - \frac{T_1}{T_2}\right) \times 100$

$$\left(1 - \frac{T_1}{500}\right) \times 100 = 40$$

$$T_1 = 300\,\text{K}$$

For 2nd case, $\eta = \left(1 - \frac{300}{T_2}\right) \times 100 = 60 \qquad T_2 = 750\,\text{K}$

8. (1) Heat is extracted from the source in path DA and AB is

$$\Delta Q = \frac{3}{2}R\left(\frac{P_0V_0}{R}\right) + \frac{5}{2}R\left(\frac{2P_0V_0}{R}\right) = \frac{13}{2}P_0V_0$$

9. (2) $\Delta U_{AB} = nC_V(T_B - T_A) = 1 \times \frac{5R}{2}(800 - 400) = 1000\,R$

$$\Delta U_{BC} = nC_V(T_C - T_B) = 1 \times \frac{5R}{2}(600 - 800) = -500\,R$$

$$\Delta U_{\text{total}} = 0$$

$$\Delta U_{CA} = nC_V(T_A - T_C) = 1 \times \frac{5R}{2}(400 - 600) = -500\,R$$

10. (2) We define the entropy change of the body $dS = \frac{dQ}{T} = ms\frac{dT}{T}$

Hence change in entropy for changing temperature from T_1 to T_2

$$\Delta S = \int_{T_1}^{T_2} ms\left(\frac{dT}{T}\right)$$

In case-I: $\Delta S_1 = \int_{100}^{200} 1\left(\frac{dT}{T}\right) = \ln 2$

In case-II: $\Delta S_2 = \int_{100}^{200} 1\left(\frac{dT}{T}\right) = \ln 2$

11. (3) $\tau = \dfrac{\lambda}{v_{rms}} = \dfrac{1}{\sqrt{2}\pi d^2\left(\dfrac{N}{V}\right)\sqrt{\dfrac{3RT}{M}}}$ ...(i)

$$\tau \propto \frac{V}{\sqrt{T}} \quad ...(ii)$$

$$TV^{\gamma-1} = k \quad ....(iii)$$

$$\Rightarrow \quad \tau \propto V^{\frac{\gamma+1}{2}}$$

12. (2) We define molar heat capacity C,

$$C = C_V + \frac{PdV}{ndT} = C_V + \frac{R}{1-n} = \frac{R}{\gamma-1} + \frac{R}{1-n}$$

$$C = \frac{R}{\dfrac{C_P}{C_V} - 1} + \frac{R}{1-n} \text{ on solving we get, } n = \frac{C-C_P}{C-C_V}$$

13. (1) $C_p - C_v = R$

where C_p and C_v are molar specific heat capacities. As per the question

$$a = \frac{R}{2} b = \frac{R}{28} = 14b$$

JEE Advanced

Single Correct Answer Type

1. (1) Number of moles of He = 5.6/22.4 = 1/4

Now $T(5.6)^{\gamma-1} = T_2(0.7)^{\gamma-1}$

$$T_1 = T_2\left(\frac{1}{8}\right)^{2/3} \Rightarrow 4T_1 = T_2$$

$$\text{Work done} = -\frac{nR[T_2 - T_1]}{\gamma - 1} = \frac{\frac{1}{4}R[3T_1]}{\frac{2}{3}} = -\frac{9}{8}RT_1$$

2. (4) $\Delta Q = nC_P\Delta T = 2\left(\dfrac{f}{2}R + R\right)\Delta T$

$$= 2\left[\frac{3}{2}R + R\right]\times 5 = 2\times\frac{5}{2}\times 8.31\times 5 = 208 \text{ J}$$

3. (3)

$PV^{5/3} = C \Rightarrow \gamma = 5/3$

$$\Delta Q_1 = nC_p\Delta T = n\frac{R}{\gamma-1}\gamma\Delta T = \frac{\gamma P\Delta V}{\gamma-1} = \frac{\left(\frac{5}{3}\right)}{\left(\frac{5}{3}-1\right)}10^5(7\times10^{-3})$$

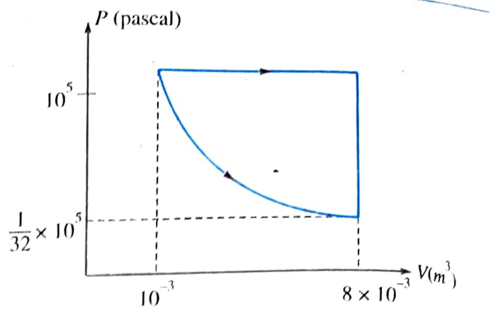
$$= \frac{5}{2}\times 7\times 10^2 = \frac{35}{2}\times 10^2 \text{ J}$$

$$\Delta Q_2 = nC_V\Delta T = \frac{nR}{\gamma-1}\Delta T = \frac{(\Delta P)V}{\gamma-1} = \frac{\left(\frac{1}{32}-1\right)10^5\times 8\times 10^{-3}}{\frac{5}{3}-1}$$

$$= \frac{-31}{32}\times\frac{8\times10^2}{2}\times 3 = \frac{-93}{8}\times 10^2 \text{ J}$$

$$\Delta Q_2 = \Delta Q_1 + \Delta Q_2 = \left(\frac{35}{2} - \frac{93}{8}\right)\times 10^2$$

$$\Delta Q_2 = \frac{140-93}{8}\times 100 = \frac{4700}{8} = 587.5 \text{ Joule}$$



Multiple Correct Answers Type

1. (2), (4)

For monatomic gas, $C_v = \dfrac{3}{2}R$, $C_p = \dfrac{5}{2}R$

For diatomic gas, $C_v = \dfrac{5}{2}R$, $C_p = \dfrac{7}{5}R$

2. (2), (4)

$$\Delta Q = \Delta U + W$$

For process $B \rightarrow C \rightarrow D$

ΔU is negative and W is also negative, so ΔQ is also negative, hence heat flows out during this process.

A to B, work is +ve. B to C work is −ve. But +ve work is more, so net work is not zero in process $A \rightarrow B \rightarrow C$.

3. (1), (2)

Process AB is isothermal. Temperature at A and B is the same, so internal energy at A and B is the same.

$$\Delta W_{AB} = nRT_0 \ln(V_B/V_A)$$

For point A: $P_0V_0 = nRT_0$

So $\quad \Delta W_{AB} = P_0V_0\ln(4V_0/V_0) = P_0V_0\ln(4)$

Nothing can be said about pressure and temperature at C.

4. (1), (2), (3)

We will assume that the gas on the side of container having spring is connected to atmosphere. Hence, its pressure remains constant at P_1.

In final position of Piston, force balance gives: $Kx + P_1A = P_2A$

Where A = cross sectional area of piston.

So, $\quad Kx = (P_2 - P_1)A$ ...(i)

Also, $\dfrac{P_1V_1}{T_1} = \dfrac{P_2V_2}{T_2}$

If $\quad V_2 = 2V_1, T_2 = 3T_1$, then $P_2 = \dfrac{3}{2}P_1$

Also, $V_2 - V_1 = Ax$ ...(ii)

(i) and (ii) give: $Kx = (P_2 - P_1)\dfrac{(V_2 - V_1)}{x}$

$$\therefore \quad \frac{1}{2}Kx^2 = \frac{1}{2}(P_2 - P_1)(V_2 - V_1) \quad ...(iii)$$

$$= \frac{1}{2}\left(\frac{P_1}{2}\right)(V_1) = \frac{P_1V_1}{4}$$

Hence, (1) is correct.

$$\Delta U = \frac{f}{2}(P_2V_2 - P_1V_1) = \frac{3}{2}\left[\frac{3}{2}\times 2P_1V_1 - P_1V_1\right] = 3P_1V_1$$

Hence, (2) is correct as well.

Net work done on piston by all forces is zero. So,

$$W_{\text{gas}} = \frac{1}{2}Kx^2 - P_1(V_2 - V_1) = 0$$

$$\therefore \quad W_{gas} = \frac{P_1V_1}{3} + 2P_1V_1$$

$$W_{gas} = \frac{7}{3}P_1V_1$$

So, (3) also turns out to be correct.

$$Q = \Delta U + W$$

$$\Delta U = \frac{3}{2}(P_2V_2 - P_1V_1) = \frac{3}{2}\left(\frac{4}{3}P_1 \cdot 3V_1 - P_1V_1\right)$$

$$\Rightarrow \quad \Delta U = \frac{9}{2}P_1V_1$$

$$\therefore \quad Q = \frac{9}{2}P_1V_1 + \frac{7}{3}P_1V_1 \quad \Rightarrow Q = \frac{41}{6}P_1V_1$$

Hence (4) is not correct.

(2), (3), (4)

Process-I is not isochoric, V is decreasing, Hence option (1) is wrong.

Process-II is isothermal expansion

Hence $\Delta U = 0$ also $W > 0$

According to first law of thermodynamics $Q = \Delta U + W$

It means $Q > 0$ or gas absorbs heat. Hence option (2) is correct.

Process-IV is isothermal compression and temperature is decreasing, hence heat is released. It means option (3) is correct.

Process-I and III are NOT isobaric because in isobaric process $T \mu V$ hence isobaric T–V graph will be linear.

Hence option (4) is correct.

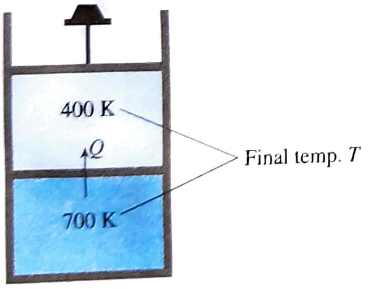
ked Comprehension Type

1. (4) Let final temperature of both compartments is T. Heat given by lower compartment

$$Q = nC_v\Delta T = 2 \times \frac{3}{2}R \times (700 - T) \qquad \text{...(i)}$$

Heat obtained by upper compartment

$$Q = nC_p\Delta T = 2 \times \frac{7}{2}R \times (T - 400) \qquad \text{...(ii)}$$



equating (i) and (ii)

$$3(700 - T) = 7(T - 400)$$

$$2100 - 3T = 7T - 2800$$

$$4900 = 10T \Rightarrow T = 490 \text{ K}$$

2. (4) Let final temperature is T. Heat given by lower compartment

$$Q = nC_p\Delta T = 2 \times \frac{5}{2}R \times (700 - T) \qquad \text{...(i)}$$

Heat obtained by upper compartment

$$Q = nC_p\Delta T = 2 \times \frac{7}{2}R \times (T - 400) \qquad \text{...(ii)}$$

By equating (i) and (ii)

$$5(700 - T) = 7(T - 400)$$

$$3000 - 5T = 7T - 2800$$

$$6300 = 12T$$

$$T = 525 \text{ K}$$

$\therefore$ Work done by lower gas $W_L = P\Delta V = nR\Delta T = -350\,R$

Work done by upper gas $W_U = P\Delta V = nR\Delta T = +250\,R$

Net work done $W_L + W_U = -350 + 250 = -100\,R$

Matrix Match Type

1. i. → a., c., e.; ii. → a., c.; iii. → b., d.; iv. → c., e.

(i) $A \rightarrow B$: $V\downarrow$, P constant $\rightarrow T\downarrow$, $U\downarrow$ and ΔW is –ve.

(ii) $B \rightarrow C$: V is same, $P\downarrow T\downarrow$

No work is done.

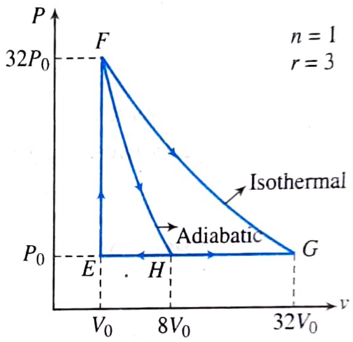
(iii) $C \rightarrow D$: $V\uparrow \Rightarrow T\uparrow$, $\Delta U \Rightarrow$ +ve

$\Delta W =$ +ve

(iv) $D \rightarrow A$: V decreases so $\Delta W \Rightarrow$ –ve

Final temperature is same so $\Delta U = 0$ and then $\Delta Q \Rightarrow$ –ve.

2. (1) i. → d.; ii. → c.; iii. → b.; iv. → a.



Apply $PV^{1+2/3}$ = constant for F to H.

$$(32P_0)V_0^{5/3} = P_0V_H^{5/3} \Rightarrow V_H = 8V_0$$

For path FG, $\quad PV$ = constant

$$(32P_0)V_0 = P_0V_G \Rightarrow V_G = 32V_0$$

Work done in $GE = 31\,P_0V_0$

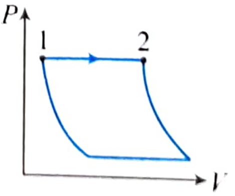
Work done in $GH = 24\,P_0V_0$

$$\text{Work done in} \quad FH = \frac{P_HV_H - P_FV_F}{(-2/f)} = 36P_0V_0$$

$$\text{Work done in} \quad FG = RT\ln\left(\frac{V_G}{V_F}\right) = 160\,P_0V_0\ \ln 2$$

3. (3) (Note that $W_{1\rightarrow 2}$ is work done on the gas).

Process 1 to 2 represents isobaric process



$$W_{1\rightarrow 2} = P(V_1 - V_2) = PV_1 - PV_2$$

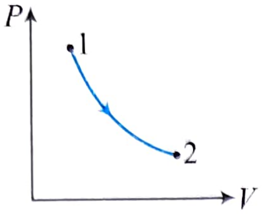
$\Delta U = \Delta Q - P\Delta V$ is correct.

No other combination is possible.

4. (1) Laplace correction is done in the determination of the speed of sound in an ideal gas, which states that the process assumed was not Isothermal but it is Adiabatic.

$$W_{\text{adiabatic}\,1\to 2} = \frac{1}{\gamma - 1}(P_2V_2 - P_1V_1)$$

Corresponding graph is



5. **(2)**

(1) Incorrect, since in Isochoric process $W = 0$

(2) Correct, as $W_{\text{isochoric}} = 0$
Graph is also correct.

(3) Not correct combination, since $W_{1\to 2} = -PV_2 + PV_1$ is wrong for Adiabatic process.

(4) Incorrect combination of graph and process.

6. **(3)** Process-I is an adiabatic process

$$\Delta Q = \Delta U + W \quad \Delta Q = 0 \; W = -\Delta U$$

Volume of gas is decreasing

$\Rightarrow$ $W < 0$

$\Delta U > 0$

$\Rightarrow$ Temperature of gas increases.

$\Rightarrow$ No heat is exchanged between the gas and surrounding.

Process-II is an isobaric process (Pressure remains constant)

$$W = P\,\Delta V = 3P_0[3V_0 - V_0] = 6P_0V_0$$

Process-III is an isochoric process (Volume remains constant)

$$\Delta Q = \Delta U + W$$
$$W = 0$$
$$\Delta Q = \Delta U$$

Process-IV is an isothermal process (Temperature remains constant)

$$\Delta Q = \Delta U + W$$
$$\Delta U = 0$$

Numerical Value Type

1. **(2)** $\Delta Q_{iaf} = \Delta U_{iaf} + W_{iaf} \Rightarrow 500 = \Delta U_{iaf} + 200\text{ J}$

$\Delta U_{iaf} = 300\text{ J} \therefore \Delta U_{ibf} = 300\text{ J}$

$W_{ibf} = W_{ib} + W_{bf} = 50 + 100 = 150\text{ J}$

$Q_{ibf} = W_{ibf} + \Delta U_{ibf} = 150 + 300 = 450$

$\Delta Q_{ibf} = 450\text{ J} = Q_{ib} + Q_{bf}$

$U_i = 100\text{ J}\; U_b = 200\text{ J}\; \Delta U_{ib} = 100\text{ J}\; W_{ib} = 50\text{ J}$

$Q_{ib} = \Delta U_{ib} + W_{ib} \therefore Q_{ib} = 150\text{ J}$

$\therefore$ $450 = Q_{ib} + Q_{bf} \Rightarrow 450 = 150\text{ J} + Q_{bf}$

$$Q_{bf} = 300\text{ J} \Rightarrow \frac{Q_{bf}}{Q_{ib}} = \frac{300}{150} = 2$$

Chapter 5

Concept Application Exercises

ercise 5.1

1. (i) Let $y = \sin\omega t - \cos\omega t = \sqrt{2}\left(\frac{1}{\sqrt{2}}\sin\omega t - \frac{1}{\sqrt{2}}\cos\omega t\right)$

$$\Rightarrow \quad y = \sqrt{2}\left(\cos\frac{\pi}{4}\sin\omega t - \sin\frac{\pi}{4}\cos\omega t\right)$$

$$\Rightarrow \quad y = \sqrt{2}\left(\sin\omega t\cos\frac{\pi}{4} - \cos\omega t\sin\frac{\pi}{4}\right)$$

$$\Rightarrow \quad y = \sqrt{2}\sin\left(\omega t - \frac{\pi}{4}\right)$$

[as $\sin A\cos B - \cos A\sin B = \sin(A - B)$]

Obviously, ($\sin\omega t - \cos\omega t$) represents SHM.

The period of SHM is $\frac{2\pi}{\omega}$.

(ii) Let $y = \sin^3\omega t = [\sin\omega t]^3 = [\sin(\omega t + 2\pi)]^3 = [\sin\omega(t + 2\pi/\omega)]^3$

Thus, $\sin^3\omega t$ is a periodic motion with period $2\pi/\omega$.

Further, as d^2y/dt^2 is not proportional to $(-y)$, the function $y = \sin^3\omega t$ does not represent SHM.

(iii) $y = 3\cos\left(\frac{\pi}{4} - 2\omega t\right) = 3\cos\left(2\omega t - \frac{\pi}{4}\right)$

[as $\cos(-\theta) = \cos\theta$]

$$y = 3[\cos 2\omega t - \pi/4 + 2\pi]$$

$$\Rightarrow \quad y = 3[\cos 2\omega\,(t + 2\pi/2\omega) - \pi/4]$$

Thus, the function represents SHM with time period

$$T = \frac{2\pi}{2\omega} = \frac{\pi}{\omega}$$

(iv) $y = \cos\omega t + \cos 3\omega t + \cos 5\omega t$

$\cos\omega t = \cos(\omega t + 2\pi) = [\cos\omega\,(t + 2\pi/\omega)] = \cos\omega\,(t + T)$

$\Rightarrow \quad \cos 3\omega t = \cos(3\omega t + 2\pi) = \cos[3\omega(t + 2\pi/3\omega)]$

$= \cos 3\omega(t + T/3)$

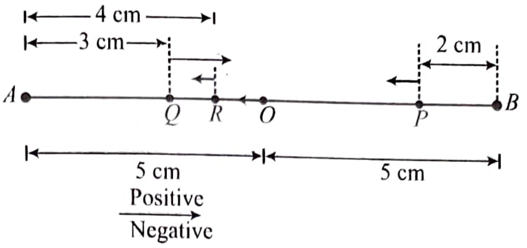
$\Rightarrow \quad \cos 5\omega t = \cos(5\omega t + 2\pi) = \cos[5\omega(t + 2\pi/5\omega)]$

$= \cos 5\omega(t + T/5)$

Clearly, each term in the given function represents a periodic function with periods $T = \frac{2\pi}{\omega}, \frac{T}{3} = \frac{2\pi}{3\omega}$ and $\frac{T}{5} = \frac{2\pi}{5\omega}$, respectively. Thus, the given function which is the sum of three terms, is periodic with a speed $\frac{2\pi}{\omega}$. During this period, $\cos\omega t$ repeats once, $\cos 3\omega t$ repeats three times and $\cos 5\omega t$ repeats five times. But this function is not SHM as d^2y/dt^2 is not proportional to $(-y)$,

(v) $y = (1 + \omega t + \omega^2t^2)$ represents a non-periodic function because its value increases when t increases, i.e., the function $\to \infty$ as $t \to \infty$.

2. Figure shows different positions of the particle in linear SHM between two points A and B, 10 cm apart. The velocity of the particle is zero at the extreme positions A and B. For a particle moving in the chosen positive direction, velocity is positive. Further, acceleration and hence force are always directed towards the mean position O and are zero at O. Keeping this in mind:



(a) At point A: Velocity is zero, acceleration is +ve, force is +ve. (0, +, +).

(b) At point B: Velocity is zero, acceleration is negative, force is negative. (0, –, –).

(c) At point O (midpoint of AB): While going towards A, velocity is negative, acceleration is zero, force is zero. (–, 0, 0).

(d) At point P (2 cm from B): While going towards A, velocity is negative, acceleration is negative, force is negative. (–, –, –).

(e) At point Q (3 cm from A): While going towards B, velocity is positive, acceleration is positive, force is positive. (+, +, +).

(f) At point R (4 cm from B): While going towards A, velocity is negative, acceleration is negative, force is negative. (–, –, –).

3. (i) For $x = A\cos(\omega t + \phi)$:

when $t = 0$, $x = 1$ cm; from $x = A\cos(\omega t + \phi)$,

$\Rightarrow \quad A\cos\phi = 1 \qquad 1 = A\cos(\omega \times 0 + \phi)$

Further, $v = \frac{dx}{dt} = -\omega A\sin(\omega t + \phi)$

At $t = 0$, $v = \pi$ cm/s Thus, $\pi = -\pi A\sin(\omega \times 0 + \phi)$

or $\quad A\sin\phi = -1$ (as $\omega = \pi$ per sec)

Clearly, $A^2\cos^2\phi + A^2\sin^2\phi = 1 + 1$

or $\quad A^2 = 2$ or $A = \sqrt{2}$ cm

As $\quad \frac{A\sin\phi}{A\cos\phi} = -1$, $\tan\phi = -1$ or $\phi = \frac{7\pi}{4}$

(ii) For $x = B\sin(\omega t + \alpha)$: when $t = 0$, $x = 1$ cm;

from $x = B\sin(\omega t + \alpha)$, $1 = B\sin(\omega \times 0 + \alpha)$

$\Rightarrow \quad B\sin\alpha = 1$

Further, $v = \frac{dx}{dt} = \omega B\cos(\omega t + \alpha)$

At $t = 0$, $v = \pi$ cm/s.

Thus, $\pi = \pi B\cos(\omega \times 0 + \alpha)$ or $B\cos\alpha = 1$

Clearly, $B^2\sin^2\alpha + B^2\cos^2\alpha = 1 + 1$

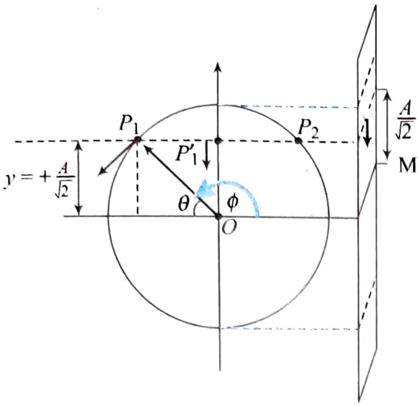
or $\quad B^2 = 2$ or $B = \sqrt{2}$ cm

As $\quad \frac{B\sin\alpha}{B\cos\alpha} = -1$, $\tan\alpha = -1$ or $\alpha = \frac{\pi}{4}$

4. At $t = 0$, particle was at $\frac{A}{\sqrt{2}}$ and was going towards mean position.

The situation is described by making reference point on a circle of radius A as shown in figure.

The reference point can be located at any of the two positions P_1 and P_2 as shown in figure for having displacement $\frac{A}{\sqrt{2}}$, but to move towards mean position, it should be P_1. Position P_1 corresponds to angle $\frac{3\pi}{4}$.



Thus, $\phi=\frac{3\pi}{4}$ and $x=A\sin\left(\omega t+\frac{3\pi}{4}\right)$.

Alternatively: Putting $t = 0$ in equation $x = A\sin(\omega t + \phi)$, we get

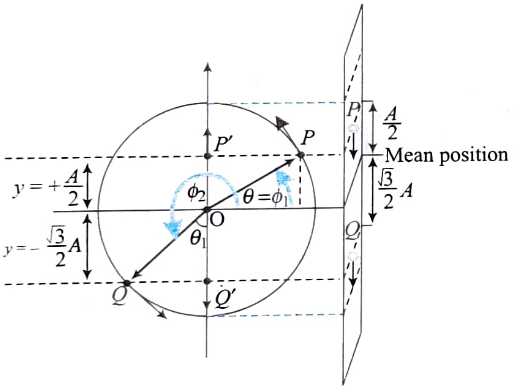
$$\frac{A}{\sqrt{2}}=A\sin\phi$$

Thus, $\phi=\frac{\pi}{4},\frac{3\pi}{4}$

But at $t = 0$; $v = 0$ so $\frac{\pi}{4}$ is rejected.

$$\therefore \qquad x=A\sin\left(\omega t+\frac{3\pi}{4}\right)$$

5. Let us make the situation of the particle on phasor circle.



For initial phase of particle P, $\sin\theta=\frac{1}{2}$ or $\theta=\phi_1=\frac{\pi}{6}$

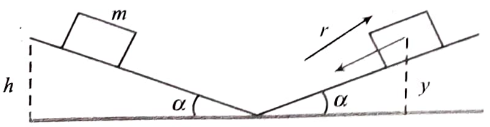
For initial phase of particle Q, $\cos\theta_1=\frac{\sqrt{3}}{2}$ or $\theta_1=\frac{\pi}{6}$

Hence, initial phase of particle Q, $\phi_2=\frac{3}{2}\pi-\frac{\pi}{6}=\frac{4}{3}\pi$

Hence, required phase difference, $\Delta\phi=\phi_2-\phi_1=\frac{4}{3}\pi-\frac{\pi}{6}=\frac{7}{6}\pi$

6. Restoring force $F = mg\sin\alpha$

$$F=mg\frac{y}{r}$$



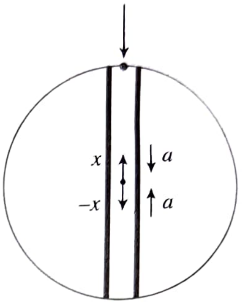
As far as motion a is constant motion is not SHM. As motion is restricted between two points and separated itself hence oscillatory. Time taken for the block to slide down a height R

$$\frac{h}{\sin\alpha}=\frac{1}{2}g\sin\alpha t^2$$

$$t^2=\frac{2h}{g}\frac{1}{\sin^2\alpha},\quad t=\frac{1}{\sin\alpha}\sqrt{\frac{2h}{g}}$$

Hence period, $4t=\frac{4}{\sin\alpha}\sqrt{\frac{2h}{g}}$

7. The acceleration of the particle always pointing towards centre of earth and displacement of the particle is not proportional to displacement from earth surface hence the motion is not SHM as seen from earth surface. It is periodic.



8. (a) $x=5\sin\left(20t+\frac{\pi}{3}\right)$

$$\frac{dx}{dt}=100\cos\left(20t+\frac{\pi}{3}\right)$$

For particle to come into rest $\frac{dx}{dt}=0$

$$\cos\left(20t+\frac{\pi}{3}\right)=\cos\frac{\pi}{2}$$

$$20t=\frac{\pi}{2}-\frac{\pi}{3}=\frac{\pi}{6}$$

$$t=\frac{\pi}{120}$$

(b) and (c) The particle will have zero acceleration and maximum speed. When it will be at equilibrium position.

As $\omega = 20$, $2\pi/T = 20$

$$\Rightarrow \quad T=\frac{\pi}{10}\text{ s}$$

Hence particle will come to equilibrium position.

$$t=\frac{\pi}{120}+\frac{\pi}{40}=\frac{\pi}{40}\left(\frac{1}{3}+1\right)=\frac{\pi}{30}\text{ s}$$

9. Given $|v_{\max}|=a\omega=20$ cm/s ...(i)

and $|f_{\max}|=a\omega^2=100\pi$ cm/s^2 ...(ii)

Dividing Eq. (ii) by Eq. (i), we get $\omega = 5\pi$ rad/s

Therefore, time period $T=\frac{2\pi}{\omega}=\frac{2\pi}{5\pi}=0.4$ s

10. Let equation of SHM be $x = A\sin\omega t$

when $x = 0$, $t = 0$

when $x = A/2$

$A/2 = A\sin\omega t$

or $\sin\omega t = ½$

$\omega t = \pi/6$

$$\frac{2\pi}{T}t=\frac{\pi}{6}$$

$t = T/12$

Hence, time taken is $T/12$, where T is time period of SHM.

1. $F = 8 - 2x$

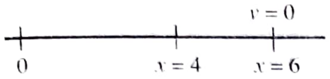
or $F = -2(x - 4)$ for equilibrium position $F = 0$

⇒ $x = 4$ m is equilibrium position

Hence the motion of the particle is SHM with force constant 2 and equilibrium position $x = 4$ m.

(a) Yes, motion is SHM.

(b) Equilibrium position is $x = 4$ m



(c) At $x = 6$ m, particle is at rest, i.e., it is one of the extreme position.

Hence amplitude is $A = 2$ m and initially particle is at the extreme position.

∴ Equation of SHM can be written as

$x - 4 = 2\cos\omega t$, where $\omega = \sqrt{\dfrac{k}{m}} = \sqrt{\dfrac{2}{2}} = 1$ i.e., $x = 4 + 2\cos t$

(d) Time period, $T = \dfrac{2\pi}{\omega} = 2\pi$ s

2. In the case of SHM, $x = a\sin(\omega t + \phi)$

Here, $a = 1$ m, $\omega = \dfrac{2\pi}{T} = \dfrac{2\pi}{2} = \pi$ rad/s

At $t = 0$, net force on the particle is zero.

Hence, particle is at mean position at $t = 0$.

$$\therefore \quad x = 0 \text{ at } t = 0$$

$$\therefore \quad 0 = a\sin(\omega \times 0 + \phi)$$

$$\Rightarrow \quad \phi = 0$$

$$x = a\sin\omega t$$

$$x = 1\cdot\sin\omega t$$

$$x = \sin\omega t$$

$$v_{max} = a\omega = 1 \times \pi = \pi \text{ m/s}$$

$$a_{max} = -\omega^2 a = -\pi^2 \text{ m/s}^2$$

$$\therefore \quad |a_{max}| = \pi^2 \text{ m/s}^2$$

4. Remember that whenever motion starts from origin or the mean position, then $x = a\sin(\omega t)$ and whenever it starts from extreme position then $x = a\cos(\omega t)$.

Because at $t = 0$, $x = 0$ for $x = a\sin(\omega t)$

and at $t = 0$, $x = a$ for $x = a\cos(\omega t)$

$$\frac{a}{2} = a\sin(\omega t_1) \quad \Bigg| \quad \frac{a}{2} = a\cos(\omega t_2)$$

$$\sin(\omega t_1) = \frac{1}{2} \quad \Bigg| \quad \cos(\omega t_2) = \frac{1}{2} \quad \Rightarrow \quad \frac{t_1}{t_2} = \frac{1}{2}$$

$$\omega t_1 = \frac{\pi}{6} \quad \Bigg| \quad \omega t_2 = \frac{\pi}{3}$$

15. We have $V = \omega\sqrt{a^2 - y^2}$

$$\therefore \quad \frac{V_2}{V_1} = \frac{\sqrt{a^2 - y_2^2}}{\sqrt{a^2 - y_1^2}} \quad \text{or} \quad \frac{49}{100} = \frac{a^2 - 16}{a^2 - 9}$$

$$\therefore \quad a = 4.76 \text{ cm}$$

Length of the path $= 2a = 9.5$ cm

Exercise 5.2

1. $x = 30\sin(\pi t + \pi/6)$

$$\frac{1}{2}Kx^2 = 2\left(\frac{1}{2}m\omega^2(A^2 - x^2)\right)$$

$$\Rightarrow \quad \frac{x^2}{2} = (0.09 - x^2) \Rightarrow x^2 = 0.06 \Rightarrow x = \frac{\sqrt{6}}{10} \text{ m.}$$

2. $x = A\sin\omega t$

$$v = \frac{dx}{dt} = A\omega\cos\omega t$$

$$\therefore \quad U_k = \frac{1}{2}mv^2 = \frac{1}{2}mA^2\omega^2\cos^2\omega t$$

$$\text{and} \quad U_P = \frac{1}{2}mA^2\omega^2\sin^2\omega t$$

$$\therefore \quad \frac{U_k}{U_P} = \frac{\cos^2\omega t}{\sin^2\omega t} = \cot^2\omega t$$

$$\Rightarrow \quad \frac{U_k}{U_P} = \cot^2\frac{2\pi}{T}\times\frac{T}{12} \Rightarrow \frac{U_k}{U_P} = \cot^2\frac{\pi}{6} \Rightarrow \frac{U_k}{U_P} = 3:1$$

3. $U_k = \dfrac{1}{2}m\omega^2(A^2 - x^2)$

$$\text{or} \quad \frac{1}{4}\times\frac{1}{2}mA^2\omega^2 = \frac{1}{2}m\omega^2(A^2 - x^2)$$

$$\text{or} \quad \frac{A^2}{4} = (A^2 - x^2)$$

$$\text{or} \quad x^2 = A^2 - \frac{A^2}{4} = \frac{3A^2}{4}$$

$$\therefore \quad x = \frac{\sqrt{3}}{2}A$$

Hence distance between Q and S is $\sqrt{3}A$.

4. Maximum kinetic energy; $\dfrac{1}{2}mv_{max}^2 = 9 - 5 = 4$ J

$$\text{or} \quad mv_{max}^2 = 8$$

$$\text{or} \quad 2v_{max}^2 = 8$$

$$\text{or} \quad v_{max} = 2 \text{ m/s} = A\omega$$

$$\therefore \quad \omega = \frac{v_{max}}{A} = \frac{2}{0.01} = 200 \text{ rad/s}$$

$$\text{or} \quad \frac{2\pi}{T} = 200$$

$$\therefore \quad T = \frac{2\pi}{200} = \frac{\pi}{100} \text{ s}$$

5. We define potential energy of the object executing SHM as

$$\frac{1}{2}Kx^2 = 0.4 \text{ J} \Rightarrow x^2 = \frac{0.8}{K}$$

Substituting $x = 0.04$ m, we get $K = 500$ N/m

Angular frequency, $\omega = 2\pi \times \dfrac{25}{\pi} = 50$ rad/s

Kinetic energy of the particle, $K = \dfrac{1}{2}m\omega^2(A^2 - x^2) = 0.5$ J

$$\Rightarrow \quad A^2 = x^2 + \frac{1}{500}$$

Hence, $A = 0.06$ m

6. $K = U$ (given)

$$K_0\cos^2\omega t = U_0\sin^2\omega t$$

$$\tan\omega t = 1 \qquad (K_0 = U_0)$$

$$\text{or} \quad \omega t = \frac{\pi}{4} \quad \text{or} \quad t = \frac{T}{8}$$

7. Differentiating $U = \dfrac{a}{x^{12}} - \dfrac{b}{x^{6}}$ with respect to x, we have

$$\frac{dU}{dx} = \frac{-12a}{x^{13}} + \frac{6b}{x^{7}}$$

Since, $F = \dfrac{-dU}{dx} = 0$, for the particle to stay in equilibrium, the equilibrium point $x = x_0$ can be given as: $\dfrac{-12a}{x_0^{13}} + \dfrac{6b}{x_0^{7}} = 0$

This gives $x_0 = \sqrt[6]{\dfrac{2a}{b}}$

Differentiating $\dfrac{dU}{dx}$ with respect to x again, we have,

$$\frac{d^2U}{dx^2} = (-12) \times (-13)\frac{a}{x^{14}} + \frac{(6b)(-7)}{x^{8}},$$

Evaluating $k_{\text{eff}} = \dfrac{d^2U}{dx^2}$ at $x = x_0 = \sqrt[6]{\dfrac{2a}{b}}$,

We have $k_{\text{eff}} = \dfrac{156a}{\left(\sqrt[6]{\dfrac{2a}{b}}\right)^{14}} - \dfrac{42b}{\left(\sqrt[6]{\dfrac{2a}{b}}\right)^{8}}$

This gives $k_{\text{eff}} = \dfrac{18b^2}{a}\left(\dfrac{b}{2a}\right)^{1/3}$

8. Potential energy (U) of a body is given by $U = (x^2 - 4x + 3)$ J

Force acting on the body, $F = -\dfrac{dU}{dx} = -2x + 4$

$\Rightarrow \quad F = -2(x - 2)$ N

(a) For equilibrium, $F = 0 \Rightarrow x = 2$ m

(b) $\dfrac{dU}{dx} = 2x - 4$ and $\dfrac{d^2U}{dx^2} = 2 > 0$

Hence, $x = 2$ m is stable equilibrium position, hence the body will perform SHM about $x = 2$ m.

Effective force constant, $k_{\text{eff}} = \left(\dfrac{d^2U}{dx^2}\right)_{x=2\text{ m}} = 2$ N/m

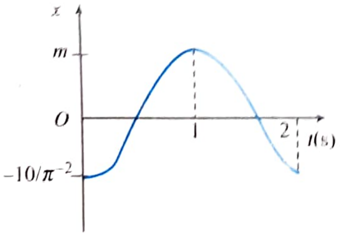
$$k_{\text{eff}} = m\omega^2 \Rightarrow \omega = \sqrt{\frac{k_{\text{eff}}}{m}}$$

Hence, $\omega = \sqrt{\dfrac{2}{1}} = \sqrt{2}$ rad/s $\Rightarrow T = \dfrac{2\pi}{\sqrt{2}} = \sqrt{2}\pi$ s

(c) In SHM, maximum potential energy is equal to maximum kinetic energy. The maximum potential energy occurs at amplitude position and maximum kinetic energy occurs at mean position.

$$\frac{1}{2}KA^2 = \frac{1}{2}mv^2 \Rightarrow A = 2\sqrt{3}\text{ m}$$

9. (i)



(ii) 0.5 s^{-1} (iii) $10/\pi^2$ m (iv) zero

(v) 0.5 s, 1.5 s (vi) 0; 1 s, and 2 s

10. At mean position, potential energy is minimum and kinetic energy is maximum. Hence, $U_{\text{min}} = 50$ J (at mean position)

And $K_{\text{max}} = E - U_{\text{min}} = 200 - 50$

$K_{\text{max}} = 150$ J (at mean position)

At extreme positions, kinetic energy is zero and potential energy is maximum.

$\therefore \quad U_{\text{max}} = E = 200$ J (at extreme position)

11. (a) $F = -\dfrac{dU}{dx} = -2(x - 2)$

By assuming $x - 2 = X$, we have $F = -2X$

Since, $F \propto -X$

The motion of the particle is simple harmonic.

(b) The mean position of the particle is $X = 0$ or $x - 2 = 0$, which gives $x = 2$ m

(c) Maximum kinetic energy of the particle is

$$K_{\text{max}} = E - U_{\text{min}} = 36 - 20 = 16\text{ J.}$$

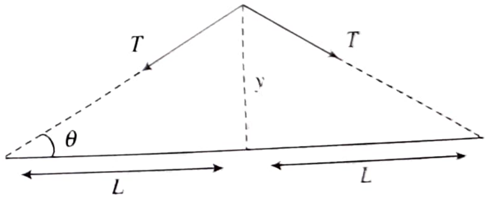
Exercise 5.3

1. When the elevator is stationary, the spring is stretched to support the block. If the extension is x, the tension is kx which should balance the weight of the block.

Thus, $x = mg/k$. As the cable breaks, the elevator starts falling with acceleration 'g'. We shall work in the frame of reference of the elevator. Then we have to use a psuedo force mg upward on the block. This force will 'balance' the weight. Thus, the block is subjected to a net force kx by the spring when it is at a distance x from the position of unstretched spring. Hence, its motion in the elevator is simple harmonic with its mean position corresponding to the unstretched spring. Initially, the spring is stretched by $x = mg/k$, where the velocity of the block (with respect to the elevator) is zero. Thus, the amplitude of the resulting simple harmonic motion is mg/k.

2. (a) If we displace the block upward the net tension towards mean position will provide the restoration force on the block.

$$\sum \vec{F} = -2T\sin\theta, \quad \text{where} \quad \theta = \tan^{-1}(y/L)$$



Since for a small displacement, $\sin\theta \approx \tan\theta = \dfrac{y}{L}$

and the resultant force is $\sum \vec{F} = \left(-\dfrac{2Ty}{L}\right)$

(b) Since there is a restoring force that is proportional to the position, it causes the system to move with simple harmonic motion like a block–spring system. Thus, we can think of

$$\sum F = -\left(\frac{2T}{L}\right)y$$

as an example of $\Sigma F = -kx$. By comparison, we identify the effective spring constant as $k = 2T/L$. Then the angular frequency is

$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{2T}{mL}}$$

Not everything with a stable equilibrium position will execute simple harmonic motion when disturbed and released, but a lot of things do when the disturbance is small.

3. The period of oscillation of a mass M attached to a spring of force constant K is

$$T = 2\pi\sqrt{\left(\frac{M}{K}\right)}$$

the 1st case, $T = 2$ s

$$2 = 2\pi\sqrt{\left(\frac{M}{K}\right)} \qquad \text{...(i)}$$

the 2nd case, $T = 3$ s

$$3 = 2\pi\sqrt{\left(\frac{M+2}{K}\right)} \qquad \text{... (ii)}$$

ividing Eq. (ii) by Eq. (i), we get $\frac{3}{2} = \sqrt{\left(\frac{M+2}{K}\right)}$

quaring, $\frac{3}{2} = \sqrt{\left(\frac{M+2}{K}\right)}$

$$\frac{2}{M} = \frac{9}{4} - 1 = \frac{5}{4}$$

$$M = \frac{4\times 2}{5} = \frac{8}{5} = 1.6 \text{ kg}$$

$$T_0 = 2\pi\sqrt{\frac{M}{k}} \qquad \text{... (i)}$$

Let v_0 be the velocity of mass M as it passes the equilibrium position. If a_0 is the initial amplitude, then

$$\frac{1}{2}Mv^2 = \frac{1}{2}ka_0^2$$

$$\Rightarrow \quad v = \left(\sqrt{\frac{k}{M}}\right)a_0$$

If V is the combined velocity of $(M + m)$ system in equilibrium position, then by law of conservation of linear momentum, we have

$$Mv = (M + m)\,V$$

$$\Rightarrow \quad V = \left(\frac{M}{M+m}\right)v$$

Let a be the new amplitude, then $\frac{1}{2}ka^2 = \frac{1}{2}(M+m)V^2$

$$= \frac{1}{2}(M+m)\left(\frac{M}{M+m}\right)^2 v^2 = \frac{1}{2}(M+m)\frac{M^2}{(M+m)^2}\left(\frac{k}{M}\right)a_0^2$$

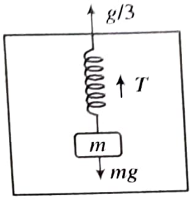
$$\Rightarrow \quad a = a_0\sqrt{\frac{M}{M+m}}$$

New time period is $T = 2\pi\sqrt{\frac{M+m}{k}}$

$$= \sqrt{\frac{M+m}{M}}T_0 \qquad \left\{\because\ T_0 = 2\pi\sqrt{\frac{M}{k}}\right\}$$

5. The angular frequency under all circumstances is

$$\omega = \sqrt{(k/m)} = \sqrt{(200/1)} = 14 \text{ rad/s}$$



When elevator is moving up, the equation of motion

$$T - mg = \frac{mg}{3} \quad \Rightarrow T = \frac{4mg}{3}$$

This tension elongates the spring by x.

$$T = kx$$

$$\Rightarrow \quad x = \frac{4mg}{3k} = 0.07 \text{ m}$$

6. We know, $T = 2\pi\sqrt{(m/k)} = 2\pi\sqrt{\left(\frac{m+6}{400}\right)}$

$$\Rightarrow \quad 0.75 = 2\pi\sqrt{\left(\frac{m+6}{400}\right)}$$

$$m = 4.0 \text{ kg}$$

Maximum acceleration of SHM is $a_{\max} = \omega^2 A$ (A = amplitude).

To avoid slipping, maximum force of mass m is $m\omega^2 A$ which is being provided by the force of friction between blocks A and B

$$\mu_s mg \geq m\omega^2 A$$

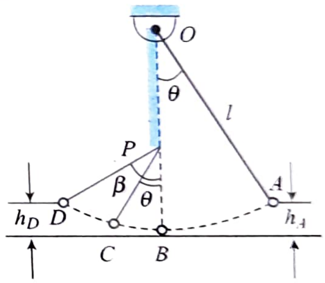
$$\Rightarrow \quad \mu_s \geq (\omega^2 A)/g$$

$$\geq (2\pi/T)^2(A/g)$$

$$\geq \left(\frac{2\pi}{1.0}\right)^2\left(\frac{0.05}{10}\right) \qquad (A = 50 \text{ mm})$$

$$= 0.20$$

7. (a) Let us assume that the bob swings a maximum angle β in the left hand side of the mean position in the absence of the portion PC of the wall.



Conserving energy at A and D, we have $U_A + K_A = U_D + K_D$

where $U_A = mgh_A, U_B = mgh_B$ and $K_A = K_D = 0$

because the bob comes to rest at the extreme positions.

Then, we have $h_A = h_D$

Substituting $h_A = OA(1 - \cos\theta)$

and $\quad h_D = PD(1 - \cos\beta)$, we have

$$OA(1 - \cos\theta) = PD(1 - \cos\beta)$$

Substituting $OA = l$, $PD = (l - d)$, we have $\beta = \left(\sqrt{\frac{l}{l-h}}\right)\theta$

(b) Since the bob swings from A to B, $t_{AB} = \frac{T}{4}$

where $T = 2\pi\sqrt{\frac{l}{g}}$

Then, $t_{AB} = \frac{\pi}{2}\sqrt{\frac{l}{g}}$

The time of motion from B to C can be found by substituting

$$t = t_{BC},\ \theta = \theta$$

and $\quad \theta_0 = \left(\sqrt{\frac{l}{l-h}}\right)\theta$ in the equation $\theta = \theta_0 \sin\omega t$, where $\omega = \sqrt{g/(l-h)}$

for the swinging position of the string from B to C. This gives

$$\theta = \left(\sqrt{\frac{l}{l-h}}\right)\theta \sin\sqrt{\frac{g}{l-h}}t_{BC}$$

Then, we have $t_{BC} = \sqrt{\frac{l-h}{g}}\sin^{-1}\sqrt{\frac{l-h}{l}}$

Finally, substituting t_{AB} and t_{BC} in the expression of total time, $T = 2(t_{AB} + t_{BC})$, we have

$$T = \pi\sqrt{\frac{l}{g}} + \sqrt{\frac{l-h}{l}}\sin^{-1}\sqrt{\frac{l-h}{l}}$$

8. $T \propto 1/\sqrt{K}$

Now, $\frac{1}{k_{\text{eff}}} = \frac{1}{k_1} + \frac{1}{k_2} + \frac{1}{k_3} + \frac{1}{k_4}$

or, $\frac{1}{k} = \frac{4}{k'}$

[Here $k_{\text{eff}} = k$ and $k_1 = k_2 = k_3 = k_4 = k$]

$k' = 4k$

Therefore, time period will be half the original value.

9. As we know, spring constant is inversely proportional to its natural length. Therefore,

$$k = \frac{C}{l};\ k_1 = \frac{C}{l_1} \quad\text{and}\quad k_2 = \frac{C}{l_2}$$

$\therefore\ \frac{k_1}{k_2} = \frac{l_2}{l_1} = 3$

$\therefore\ k_1 = 3\,k_2$

But $\frac{1}{k} = \frac{1}{k_1} + \frac{1}{k_2}$ or $\frac{1}{k} = \frac{1}{3k_2} + \frac{1}{k_2} = \frac{1+3}{3k_2}$

$\therefore\ k = \frac{3k_2}{4}$

$\therefore\ k_2 = \frac{4k}{3}$

$\therefore\ k_1 = 3k_2 = 4k$

But $T = 2\pi\sqrt{\frac{m}{k}}$; $T_1 = 2\pi\sqrt{\frac{m}{k_1}}$ and $T_2 = 2\pi\sqrt{\frac{m}{k_2}}$

$\therefore\ \frac{T_1}{T_2} = \sqrt{\frac{k_2}{k_1}} = \sqrt{\frac{4k}{3\times 4k}}$

$\frac{T_1}{T_2} = \sqrt{\frac{1}{3}} = 1:\sqrt{3}$

10. Common velocity after collision $= \frac{u}{2}$

Hence, $\text{KE} = \frac{1}{2}(2M)\left(\frac{u}{2}\right)^2 = \frac{1}{4}Mu^2$

It is also the total energy of vibration because the string is unstretched at this moment; hence, if A is the amplitude, then

$$\frac{1}{2}KA^2 = \frac{1}{4}Mu^2 \quad\text{or}\quad A = \sqrt{\frac{M}{2K}}\,u$$

11. When block P is released from rest from a distance $3l/2$ toward right from mean position, this will be the amplitude of oscillation. So velocity of block when passing from its mean position is given as

$$v = A\omega = \frac{3l}{2}\sqrt{\frac{k}{m}} \quad \left[\text{as } \omega = \sqrt{\frac{k}{m}}\right]$$

If mass m_1 is added to it and just after if velocity of combined block becomes v_1, from momentum conservation, we have

$mv = (m + m_1)v_1$

or $v_1 = \frac{m}{(m+m_1)}\left(\frac{3l}{2}\sqrt{\frac{k}{m}}\right)$

If this is the velocity of combined block at mean position, it must be given as

$$v_1 = A\omega_1 \quad \left[\text{now } \omega_1 = \sqrt{\frac{k}{m+m_1}}\right]$$

where A_1 and ω_1 are the new amplitude and angular frequency, respectively, of SHM of the block. It is given that combined block just reaches the left wall; thus the new amplitude of oscillation must be l. So we have

$$\frac{m}{(m+m_1)}\cdot\frac{3l}{2}\sqrt{\frac{k}{m}} = l_1\sqrt{\frac{k}{m+m_1}}$$

or $\frac{3\sqrt{m}}{2\sqrt{m+m_1}} = 1$

or $9m = 4m + 4m_1$ or $m_1 = \frac{5}{4}m$

12. When second mass sticks to the lower mass, due to such an inelastic collision, the velocity of combined block is reduced by half, that is, $u/2$, to conserve momentum. Now at mean position the velocity of block can be written as

$$\frac{u}{2} = A\omega \quad (\text{if } A \text{ is the amplitude of oscillation})$$

or $\frac{u}{2} = A\sqrt{\frac{k}{2m}}$

[here for combined block new angular frequency of SHM is $\omega\sqrt{k/2m}$]

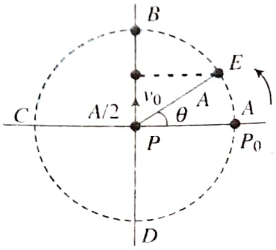
or $A = \frac{u}{2}\sqrt{\frac{2m}{k}} = u\sqrt{\frac{m}{2k}}$

As oscillation starts from mean position, in reaching its extreme position, particle has to cover a phase of $\pi/2$ radians; thus time taken by particle to reach its extreme position is $t = \frac{\pi/2}{\omega} = \frac{\pi}{2}\sqrt{\frac{k}{2m}}$.

13. Initially in equilibrium if block is at a depth h below the natural length of spring, then we have $mg = kh$.

If at mean position block is imparted a velocity v_0, this would be the maximum velocity of block during its oscillation. If its amplitude of oscillation is A, then it is given as

$$v_0 = A\omega \quad \left[\text{where } \omega = \sqrt{\frac{k}{m}}\right] \quad\text{or}\quad A = \frac{v_0}{\omega} = v_0\sqrt{\frac{m}{k}}$$



Now to find the time taken by block to reach its half of amplitude point, we consider the corresponding circular motion of the SHM as shown in the figure.

Here block P will reach to half of its amplitude when particle P_0 will reach point E shown in the figure at an angular displacement θ from mean position relative to point A, thus time taken by it is

$$t=\frac{\theta}{\omega}=\frac{\sin^{-1}(1/2)}{\sqrt{k/m}}=\frac{\pi}{6}\sqrt{\frac{m}{k}}$$

ercise 5.4

1. (a) For minimum time period, we have $r=k$ and $k=\sqrt{I_C/m}$

Substituting $I_C=ml^2/12$ for the rod, we have $r=\dfrac{l}{2\sqrt{3}}$

Then, $x=l/2-r$, where $r=l/2\sqrt{3}$

This gives $x=\dfrac{l}{2}\left(1-\dfrac{1}{\sqrt{3}}\right)$

(b) $T_{\min}=2\pi\sqrt{\dfrac{2k}{g}}$ where $k=\dfrac{l}{2\sqrt{3}}$

This gives $T_{\min}=2\pi\sqrt{\dfrac{l}{\sqrt{3}g}}$

2. $T=2\pi\sqrt{\dfrac{I}{mgl}}=\sqrt{\dfrac{3mr^2}{2mgr}}$

or $\quad T=2\pi\sqrt{\dfrac{3r}{2m}}$ or $2\pi\sqrt{\dfrac{l}{g}}=2\pi\sqrt{\dfrac{3r}{2g}}$

$\therefore \quad l=\dfrac{3}{2}r=\dfrac{3}{2}\times 1=1.5\text{ m}$

3. $I=\dfrac{ml^2}{3}+\dfrac{ml^2}{12}+ml^2=\dfrac{17}{12}ml^2$

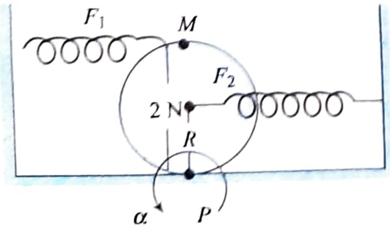
$d=\dfrac{3l}{4}$

$$T=2\pi\sqrt{\frac{17ml^2}{12\times 2mg\times\frac{3}{4}l}}\Rightarrow T=2\pi\sqrt{\frac{17l}{18g}}$$

4. Since $T_0=2\pi\sqrt{\dfrac{I_0}{C}}$ and $T=2\pi\sqrt{\dfrac{I}{C}}$, where $I=I_0+I_m$

We have $I_m=\dfrac{C}{4\pi^2}(T^2-T_0^2)=\dfrac{12.5}{4\times 10}[(2.5)^2-2^2]=0.90\text{ kg m}^2$

5. Let us displace the CM by a small distance x towards right. As a result the body rolls on the horizontal surface. Hence spring 1 is elongated by $x_1=2R\theta$ and spring 2 is compressed by $x_2=R\theta$, where θ is the angle of rotation of the rolling body.



The spring forces F_1 and F_2 produce anticlockwise torques $F_1(2R)$ and $F_2(R)$, respectively about P. Summing up the spring torques about P, we have

$$\tau_{\text{net}}=F_1(2R')+F_2(R)$$

where $F_1=k_1x_1=k_1(2R\theta)$ and $F_2=k_2x_2=k_2R\theta$

This gives $\quad \tau_{\text{net}}=-(4R^2k_1+R^2k_2)\theta=-(4k_1+k_2)R^2\theta$

This torque produces anticlockwise acceleration α for clockwise displacement θ, which can be given as

$$\alpha=\frac{\tau_{\text{net}}}{I_P}=\frac{-(4k_1+k_2)R^2\theta}{I_P}$$

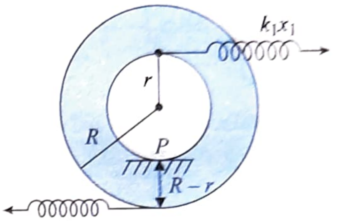
Substituting $I_P=mR^2+I_C=mR^2+\dfrac{mR^2}{2}=\dfrac{3mR^2}{2}$

We have $\alpha=\dfrac{-2(4k_1+k_2)}{3m}\theta$

Comparing the above equation with $\alpha=-\omega^2\theta$, we have

$$\omega=\sqrt{\frac{2(4k_1+k_2)}{3m}}$$

6. The torque about P is $\tau_P=k_1x_12r+k_2x_2(R-r)=I_P\alpha$



But putting $x_1=2r\theta,\ x_2=(R-r)\theta$

and $\quad I_P=mk^2+mr^2$, we have $\alpha=\dfrac{4k_1r^2+k_2(R-r)^2}{m(k^2+r^2)}\theta$

$$\omega=\sqrt{\frac{4k_1r^2+k_2(R-r)^2}{m(k^2+r^2)}}$$

7. The net torque produced by the spring when the disc is rotated through an angle θ is

$$\tau_C=(kx)R=(I_C+mr^2)\alpha$$

where $x=R\theta$ and $I_C=MR^2$

or $\quad \alpha=\dfrac{kR^2}{\left(\dfrac{MR^2}{2}+mr^2\right)}\theta \quad$ or $\quad \omega=\sqrt{\dfrac{2kR^2}{MR^2+2mr^2}}$

8. Taking torque of spring force about P for a small displacement of O, we have

$$Kx(R+r)=I_p\alpha,\text{ where } x=(R+r)\theta$$

or $\quad k(R-r)2\theta=(mr^2+I_0)\omega^2\theta$

$$\omega=\sqrt{\frac{k(R+r)^2}{mr^2+I_0}}$$

$$f=\frac{\omega}{2\pi}=\frac{1}{2\pi}\sqrt{\frac{k(R+r)^2}{I_0+mr^2}}$$

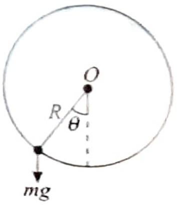
9. In equilibrium the particle is at the lowest position. Consider the system at an angular position θ.

$$\tau=mgR\sin\theta\simeq mgR\theta\ (\text{for small }\theta)$$

$$I\alpha=\tau$$

$$\left[\frac{1}{2}(2m)R^2+mR^2\right]\alpha=-mgR\theta$$

$$\therefore\ a=-\left(\frac{g}{2R}\theta\right)\quad \therefore\ f=\frac{1}{2\pi}\sqrt{\frac{g}{2R}}$$



10. (a) The moment of inertia about the axis of oscillation is

$$I_A = \frac{2}{3}Ma^2$$

Distance of COM from point of suspension, $l = \frac{a}{\sqrt{2}}$

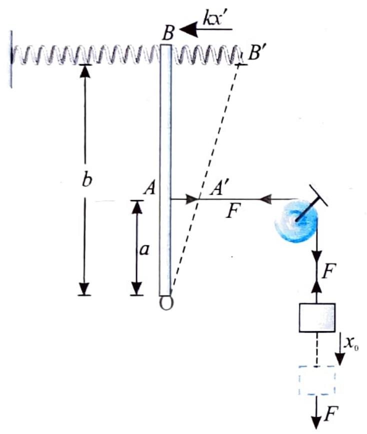
$$\therefore \quad T = 2\pi\sqrt{\frac{I_A}{mgl}}$$

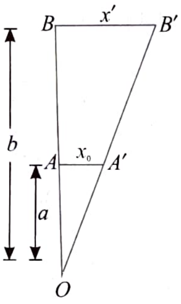
$$\therefore \quad T = 2\pi\sqrt{\frac{\frac{2}{3}Ma^2}{Mg\frac{a}{\sqrt{2}}}} = 2\pi\sqrt{\frac{2\sqrt{2}}{3}\frac{a}{g}}$$

(b) In this case $I_A = \frac{7}{12}Ma^2$

$$\therefore \quad T = 2\pi\sqrt{\frac{\frac{7}{12}Ma^2}{Mg\frac{a}{\sqrt{2}}}} = 2\pi\sqrt{\frac{7}{6\sqrt{2}}\frac{a}{g}}$$

11. Let the block m is displaced downward by distance x_0 beyond its equilibrium position by applying a force F and the system achieve new equilibrium position.





Due to displacement of the block (x_0) the spring further stretch a distance x'.

Using geometry we can write $\frac{x'}{b} = \frac{x_0}{a} \Rightarrow x' = \frac{x_0 b}{a}$ ...(i)

Taking torque of the forces applied after initial equilibrium about O.

$$kx' \cdot b = F \cdot a$$

or $\quad kx_0\frac{b}{a}\cdot b = F\cdot a$

$\Rightarrow \quad \frac{F}{x_0} = k\frac{b^2}{a^2} = k_{\text{eff}}$

Hence $m\omega^2 = k\frac{b^2}{a^2}$

or $\quad \omega = \sqrt{\frac{k\, b^2}{m\, a^2}} \Rightarrow T = 2\pi\frac{a}{b}\sqrt{\frac{m}{k}}$

Exercise 5.5

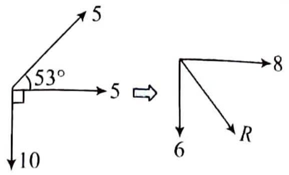
1. $x_1 = 5\sin\omega t$

$x_2 = 5\sin(\omega t + 53°)$

$x_3 = -10\cos\omega t$

We can write $x_3 = 10\sin(\omega t + 270°)$

Finding the resultant amplitude by vector notation.



Resultant amplitude, $|R| = \sqrt{8^2 + 6^2} = 10$

2. First write all SHMs in terms of sine functions with positive amplitude. Keep ωt with positive sign. Therefore,

$x_1 = 3\sin\omega t$

$x_2 = 4\sin(\omega t + \pi/2)$

$$A = \sqrt{3^2 + 4^2 + 2\times3\times4\cos\frac{\pi}{2}} = \sqrt{9+16} = \sqrt{25} = 5$$

$$\tan\phi = \frac{4\sin\frac{\pi}{2}}{3 + 4\cos\frac{\pi}{2}} = \frac{4}{3} \Rightarrow \phi = 53°$$

$$x = 5\sin(\omega t + 53°)$$

3. (a) At $\quad t = 0, x_1 = A_1\sin\omega t = 0$

and $\quad x_2 = A_2\sin(\omega t + \pi/3) = A_2\sin\left(\frac{\pi}{3}\right) = \frac{A_2\sqrt{3}}{2}$

Thus, the resultant displacement at $t = 0$ is

$$x = x_1 + x_2 = A_2\frac{\sqrt{3}}{2}$$

(b) The resultant of the two motions is a simple harmonic motion of the same angular frequency ω. The amplitude of the resultant motion is

$$A = \sqrt{A_1^2 + A_2^2 + 2A_1A_2\cos(\pi/3)} = \sqrt{A_1^2 + A_2^2 + A_1A_2}$$

The maximum speed is $u_{\max} = A\omega = \omega\sqrt{A_1^2 + A_2^2 + A_1A_2}$

(c) The maximum acceleration is

$$a_{\max} = A\omega^2 = \omega^2\sqrt{A_1^2 + A_2^2 + A_1A_2}$$

4. The particle is moving along the y-axis.

$y(t) = a\cos\omega t + b\sin\omega t$

$y = \sqrt{a^2+b^2}\sin(\omega t + \phi_0)$

where $\tan\phi_0 = a/b \Rightarrow y = \sqrt{a^2+b^2}\sin\left(\omega t + \tan^{-1}a/b\right)$

Comparing with $y = a\sin(\omega t + \phi)$, $T = \frac{2\pi}{\omega} = \frac{2\pi}{2} = \pi$ s

$A = \sqrt{a^2+b^2} = \sqrt{3^2+4^2} = 5$ m

$v_{\max} = \omega A = 2\times5 = 10$ m/s

$\Rightarrow \quad a_{\max} = \omega^2 A = 4\times5 = 20$ m/s^2

5. For the equation $y_1 = 10\sin(4\pi t + \pi/2)$, the amplitude $a_1 = 10$ units

For equation $y_2 = 5\left[\sin 2\pi t + \sqrt{8}\cos 2\pi t\right]$,

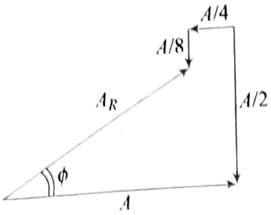
Multiplying and dividing by $\sqrt{1+8} = 3$

$$y_2 = \left[15\sin(2\pi t)\times\frac{1}{3} + \cos(2\pi t)\frac{\sqrt{8}}{3}\right] = 15\sin(2\pi t + \phi)$$

where $\tan\phi = \sqrt{8}$ whose amplitude a_2 is 15 units.

Therefore, the required ratio $= \frac{a_1}{a_2} = \frac{10}{15} = \frac{2}{3}$

om the vector diagram, we can see that the resultant amplitude is

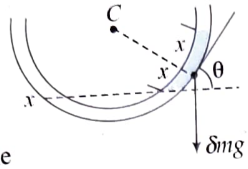


$$A_R=\sqrt{\left(A-\frac{A}{4}\right)^2+\left(\frac{A}{2}-\frac{A}{8}\right)^2}$$

$$A_R=A\sqrt{\left(\frac{3}{4}\right)^2+\left(\frac{3}{8}\right)^2}=\frac{3\sqrt{5}}{8}A$$

nd $\tan\phi=\dfrac{(A/4)-(A/8)}{A-(A/4)}=\dfrac{1}{2}$

ethod 1: Displace the liquid slightly through a distance x from e equilibrium position, in left hand limbs. In the right hand limb, quid of same length will rise.



ence, the restoring force is

$$F=-\delta mg\sin\theta-A\rho(2x)g\sin\theta$$

his accelerates the total mass M of the quid with an acceleration,

$$a=\frac{-F}{M}$$

here $F=2A\rho gx\sin\theta$ and $M=2A\rho R\theta$

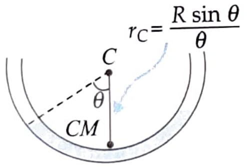
his gives $a=-\dfrac{g\sin\theta}{R\theta}x$

Comparing the above equation with $a=-\omega^2x$, we have

$$\omega=\sqrt{\frac{g\sin\theta}{R\theta}}$$

Method 2: Since, the liquid moves in the rigid tube, it behaves as a physical pendulum even though the liquid is not rigid. The time period of physical pendulum is

$$T=2\pi\sqrt{\frac{I_P}{mgr_C}}$$



here $I_p=mR^2$ and $r_C=\dfrac{R\sin\theta}{\theta}$

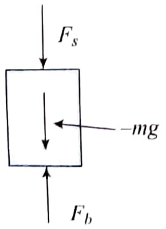
This gives $T=2\pi\sqrt{\dfrac{R\theta}{g\sin\theta}}$

Then $\omega=\sqrt{\dfrac{g\sin\theta}{R\theta}}$

$F_s+F_b=mg$

(i) $kx_0+A\rho gh=mg$

$$\Rightarrow\quad h_0=\frac{mg-kx_0}{A\rho g}$$



(ii) If the cylinder is pulled down by an additional distance x, the net force is

$$\Delta F=kx+A\rho gx=(k+A\rho g)x$$

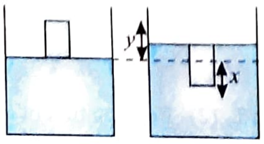
Then, $K_{eq}=k+A\rho g$

or $\quad f=\dfrac{1}{2\pi}\sqrt{\dfrac{k}{m}}=\dfrac{1}{2\pi}\sqrt{\dfrac{k+A\rho g}{m}}$

. (a) When the cylinder moves down by x, the liquid level rises by y.

$$3Ay=Ax\Rightarrow y=\frac{x}{3}$$

Since volume of liquid is constant



∴ Buoyancy force on cylinder

$$=\left(\frac{4x}{3}A\right)(2\rho)g$$

In equilibrium $=\dfrac{8x_0}{3}A\rho g=AL\rho g\Rightarrow x_0=\dfrac{3L}{8}$

Since, equilibrium position is x_0 below the original position (where, cylinder was at rest), hence amplitude is $A=x_0=\dfrac{3L}{8}$

(b) When the cylinder is displaced x from its equilibrium position, restoring force on it will be the extra buoyancy force

$$F_{\text{restoring}}=B_{\text{extra}}=\left(A\frac{4x}{3}\right)(2\rho)g$$

$$\therefore\quad ma=-\left(\frac{8}{3}A\rho g\right)x\Rightarrow AL\rho g\frac{d^2x}{dt^2}=-\frac{8}{3}A\rho g\cdot x$$

$$\therefore\quad a=-\left(\frac{8}{3}\frac{g}{L}\right)x$$

$$\therefore\quad \omega=\sqrt{\frac{8}{3}\frac{g}{L}}\Rightarrow T=2\pi\sqrt{\frac{3L}{8g}}$$

Exercises

Single Correct Answer Type

1. (1) From graph of acceleration and time graph

$$f=-f_{\max}\cos\omega t=-(\omega^2A)\cos\omega t$$

and we have relationship between acceleration and position as $f=-\omega^2x\Rightarrow x=A\cos\omega t$

which gives $v=\dfrac{dx}{dt}=-A\omega\sin\omega t=-v_{\max}\sin\omega t$

Hence option (1) is correct.

2. (4) Since acceleration of the particle at initial moment is maximum possible and is negative; therefore, the particle is at right extreme position at this moment.

When the particle is released, it starts to move to the left. It means, velocity starts to increase from zero initial value to negative value and its magnitude becomes maximum possible at mean position (at $t=T/4$). It means at $t=T/4$, kinetic energy is equal to maximum possible.

At $t=T/2$, the particle comes to instantaneous rest at left extreme position. It means at $t=T/2$, v is equal to zero. Hence kinetic energy is equal to zero. At $t=3T/4$, particle comes back to mean position and now moves to the right. Therefore, velocity is positive and has maximum possible magnitude. Therefore, kinetic energy is maximum possible.

At $t=T$, particle comes back to initial position (extreme right position). Velocity and kinetic energy become equal to zero.

3. (3) Let T be the time period; then $\dfrac{T}{2}=\dfrac{5\pi}{64}-\dfrac{\pi}{64}=\dfrac{4\pi}{64}$

$$T=\frac{\pi}{8}\text{ s}$$

$$\therefore\quad \omega=\frac{2\pi}{T}=\frac{2\pi}{(\pi/8)}=16\text{ rad/s}$$

Also, $A=10$ cm (from the graph)

The equation of the sinusoidal wave can be written as $y=10\sin(16t+\phi)$ cm, where ϕ is the initial phase.

From the graph, corresponding to the crest = 10 cm; when $t=3\pi/64$.

$$10\text{ cm}=10\sin\left[16\left(\frac{3\pi}{64}\right)+\phi\right]\text{cm}$$

$$\sin\left[\frac{3\pi}{4}+\phi\right]=1 \quad \text{or} \quad \frac{3\pi}{4}+\phi=\frac{\pi}{2} \Rightarrow \phi=-\frac{\pi}{4}$$

$$\therefore \quad y=10\sin\left(16t-\frac{\pi}{4}\right)$$

4. (1) For A, time period $T_A = 16$ s (distance between two adjacent crests)

For B, time period $T_B = 2\,(20-8) = 24$ s (length between the crest and trough shown = 20 s − 8 s = 12 s)

Also, amplitudes $a_A = 10$ cm, $a_B = 5$ cm

$$\text{Now,} \quad \frac{(V_{\max})_A}{(V_{\min})_B}=\frac{\omega_A a_A}{\omega_B a_B}$$

$$\frac{\left(\frac{2\pi}{T_A}\right)a_A}{\left(\frac{2\pi}{T_B}\right)a_B}=\frac{T_B a_A}{T_A a_B}=\frac{24\times 10}{16\times 5}=\frac{3}{1}$$

5. (2) The slope of the length is $\frac{F}{x}=-\frac{0.5}{5}=-0.1$ N/cm $=-10$ N/m

But $F=-m\omega^2 x$ or $F/x=-m\omega^2$

so, $-m\omega^2=-10$ or $m\omega^2=10$

or, $\omega^2 = 10\,/m$

$$\therefore \quad \omega^2=\frac{10}{4\times 10^{-1}} \Rightarrow \omega=\frac{10}{2}=5$$

$$\therefore \quad f=\frac{\omega}{2\pi}=\frac{5}{2\pi}/\text{s}$$

6. (4) $x = A \sin \omega t$

$$v=\frac{dx}{dt}=A\omega\cos\omega t=\omega\sqrt{A^2-x^2}$$

$$a=\frac{dv}{dt}=-A\omega^2\sin\omega t=\frac{dv}{dt}=-\omega^2 x$$

$$\text{But} \quad x=-\frac{a}{\omega^2}$$

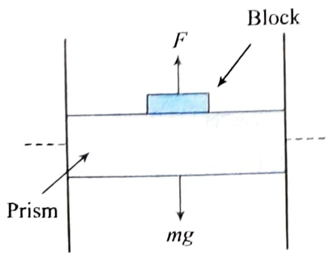
$$\therefore \quad v=\omega\sqrt{A^2-\frac{a^2}{\omega^4}}$$

$$\text{or} \quad v^2=\omega^2\left(A^2-\frac{a^2}{\omega^4}\right)$$

7. (2) The situation when the block is just below the mean position is shown in the figure, the restoring forces acting on the piston cause a normal reaction F to act on the block. For the block to separate $F \geq mg$

i.e., $m\omega^2 A \geq mg$

(where ω = angular frequency and A = amplitude)



$$\text{or} \quad A\geq\frac{g}{\omega^2}=\frac{g}{\left(\frac{2\pi}{T}\right)^2} \quad \text{or} \quad A\geq\frac{gT^2}{4\pi^2}$$

Now, the maximum velocity $v_{\max}$ at that instant $=\omega A$

$$v_{\max}=\left(\frac{2\pi}{T}\right)\left(\frac{gT^2}{4\pi^2}\right)=\frac{10}{\pi}\text{ m/s}$$

8. (3) $E=\frac{1}{2}mA^2\omega^2=\frac{1}{2}mA^2\left(\frac{k}{m}\right)=\frac{1}{2}A^2k$ (Independent of mass)

9. (3) Let the acceleration be f; $f=-\omega^2 x$

Therefore, distance of the particle from the centre at any time t is given by

$x = r\cos(\omega t)$, where r is the amplitude

when $t = 1$ s, $x = r - a$

$\therefore \quad (r-a) = r\cos\omega$

$$\cos\omega=\frac{r-a}{r} \qquad \text{...(i)}$$

When $t = 2$ s, $x = r - a - b$, therefore $r - a - b = \cos 2\omega$

$\therefore \quad r-a-b=r\,(2\cos^2\omega-1)$ ....(ii)

Substituting the value of $\cos\omega$ from Eq. (i) in Eq. (ii), we get

$$r-a-b=r\left[2\frac{(r-a)^2}{r^2}-1\right]=\frac{2(r-a)^2}{r}-r$$

$$\therefore \quad r(3a-b)=2a^2 \Rightarrow r=\frac{2a^2}{3a-b}$$

10. (2) $x=a\sin\omega t$ with $\omega=\frac{2\pi}{T}=\frac{2\pi}{8}=\frac{\pi}{4}$ rad/s

$$x=a\sin\frac{\pi t}{4}$$

In 2 s (which is equal to $T/4$), one amplitude will be covered. In 1st second

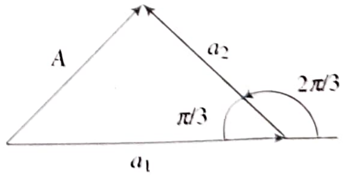
$$x=a\sin\frac{\pi}{4}=\frac{a}{\sqrt{2}}$$

$$\text{Required ratio}=\frac{\frac{a}{\sqrt{2}}}{a-\frac{a}{\sqrt{2}}}=\frac{1}{\sqrt{2}-1}$$

11. (4) $x=a_1\sin\omega t+a_2\cos\left(\omega t+\frac{\pi}{6}\right)$

$$=a_1\sin\omega t+a_2\sin\left(\omega t+\frac{\pi}{6}+\frac{\pi}{2}\right)$$

$$=a_1\sin\omega t+a_2\sin\left(\omega t+\frac{2\pi}{3}\right)$$



$$\therefore \quad A^2=a_1^2+a_2^2+2a_1a_2\cos\frac{2\pi}{3}$$

$$\Rightarrow \quad A^2=a_1^2+a_2^2-2a_1a_2\cos\frac{\pi}{3}$$

$$\therefore \quad A=\sqrt{a_1^2+a_2^2-2a_1a_2\cos\frac{\pi}{3}}$$

12. (2) Let equation of simple harmonic motion is

$x = A\sin(\omega t+\delta)$

It is given, $A = 0.5$ m and $\omega = \frac{2\pi}{T} = \frac{2\pi}{4}\text{s}^{-1} = \frac{\pi}{2}\text{s}^{-1}$

At $\quad t = 0.5$ s, $x = 0.3$m,

so $\quad 0.3 = 0.5 \sin(\omega t + \delta)$

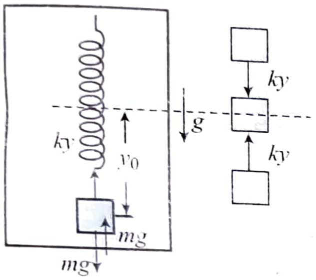
$\Rightarrow \quad \sin\left(\frac{\pi}{2} \times \frac{1}{2} + \delta\right) = \frac{3}{5} \Rightarrow \frac{\pi}{4} + \delta = 37°$

$\Rightarrow \quad \delta = 37° - 45° = -8°$

So, equation of motion is $x = (0.5 \text{ m}) \sin\left[\frac{\pi t}{2} - 8°\right]$

(4) When the elevator is at rest, the elongation in spring is given by $ky_0 = mg$

or $\quad y_0 = \frac{mg}{k}$



At the instant the elevator starts falling down with acceleration g, the block is at rest w.r.t. elevator and the net force acting on it is ky_0 in the upward direction w.r.t. lift frame of reference. Due to this force, the block moves up and as a result elongation in the spring decreases, and the force experienced by the block becomes zero when spring and stops momentarily till compression in the spring becomes mg/k.

Hence the block will always have net force towards relaxed position of spring and the block will perform simple harmonic notion with time period $T = 2\pi\sqrt{\frac{m}{k}}$ and with amplitude mg/k.

(3) At mean position, the speed will be maximum

$$\frac{kx_0^2}{2} = \frac{mv^2}{2}$$

$\Rightarrow \quad v_{\max} = \sqrt{\frac{k}{m}}\, x_0$

and this is attained at $t = T/4$.

Time period of motion is $T = 2\pi\sqrt{\frac{m}{k}}$

So required time is $t = \frac{\pi}{2}\sqrt{\frac{m}{k}}$

(1) $v = \frac{dx}{dt} = 2\pi A \cos\left(2\pi t + \frac{\pi}{3}\right)$

or $\quad \pm 2\pi A = 2\pi A \cos\left(2\pi t + \frac{\pi}{3}\right)$

or $\quad \cos\left(2\pi t + \frac{\pi}{3}\right) = -1 = \cos \pi$

$$2\pi t + \frac{\pi}{3} = \pi \Rightarrow t = \frac{1}{3}\text{ s}$$

16. (2) Let spring constant of each spring be k, then the equivalent spring constant of the two spring system in parallel is $2k$.

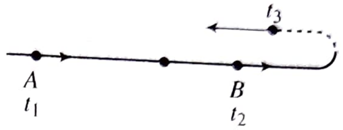
Without mass, $T = 3\text{ s} = 2\pi\sqrt{\frac{12}{2k}}$

With mass, $T_1 = 6\text{s} = 2\pi\sqrt{\frac{12+m}{2k}}$

$$\frac{T}{T_1} = \frac{1}{2} = \sqrt{\frac{12}{12+m}}$$

$$m = 36 \text{ kg}$$

17. (4) The situation is illustrated in the following figure.



$$t_2 - t_1 = 2\text{ s},\ t_3 - t_2 = 2\text{ s}$$

i.e., $\quad t_3 - t_1 = 4$ s which is half of the time period.

So, $\quad T = 8$ s

18. (3) Let spring constant of two springs are k_1 and k_2, respectively, then

$$T_1 = 2\pi\sqrt{\frac{m}{k_1}} \quad \text{and} \quad T_2 = 2\pi\sqrt{\frac{m}{k_2}}$$

$$k_1 = \frac{4\pi^2 m}{T_1^2} \quad \text{and} \quad k_2 = \frac{4\pi^2 m}{T_2^2}$$

When the two springs are connected in series, then $T = 2\pi\sqrt{\frac{m}{k_{eq}}}$

where $k_{eq} = \frac{k_1 k_2}{k_1 + k_2}$

$$T = \sqrt{T_1^2 + T_2^2}$$

19. (1) Let at $t = 0$, the particle is at extreme position, then the equation of SHM can be written as

$$x = A\cos(\omega t) = A\cos\left(\frac{2\pi}{T}t\right)$$

At $\quad t = T/8$,

$$x = A\cos\frac{\pi}{4} = \frac{A}{\sqrt{2}}$$

Acceleration $= -\omega^2 x = -\left(\frac{2\pi}{T}\right)^2 \times \frac{A}{\sqrt{2}}$

Magnitude of acceleration $= \frac{4\pi^2 A}{\sqrt{2T^2}}$

20. (2) Velocity at any displacement x is given by $v = \omega\sqrt{A^2 - x^2}$

So, the required velocity $= \frac{2\pi}{T}\sqrt{A^2 - \frac{A^2}{2}} = \frac{\sqrt{2}\pi A}{T}$

21. (3) Due to impulse, the total energy of the particle becomes

$$\frac{1}{2}m\omega^2 A^2 + \frac{1}{2}m\omega^2 A^2 = m\omega^2 A^2$$

Let A' be the new amplitude,

$$\frac{1}{2}m\omega^2 (A')^2 = m\omega^2 A^2 \Rightarrow A' = \sqrt{2}\, A$$

22. (1) Conserving momentum, $2V = 3V'$

$$V' = \frac{2}{3}V$$

$$E_i = \frac{1}{2}m_1v_1^2 = \frac{1}{2}\times 2V^2 = V^2 = \frac{1}{2}KA^2$$

$$E_f = \frac{1}{2}\times m_2V_2^2 = \frac{1}{2}\times 3\times\frac{2}{3}\times\frac{2}{3}V^2 = \frac{2}{3}V^2$$

$$\frac{1}{2}KA'^2 = \frac{2}{3}V^2 = \frac{2}{3}E_i \qquad (\because E_i = V^2 \text{ from above})$$

$$\frac{1}{2}KA'^2 = \frac{2}{3}\left(\frac{1}{2}KA^2\right) \Rightarrow A' = \sqrt{\frac{2}{3}}A$$

23. (4) The equation $a = -100x + 50$ ($a = -kx$ form) itself shows that the particle performs SHM. Hence (4) is correct answer.

24. (1) $v\frac{dv}{dx} = -100x + 50$

$$\Rightarrow \int v\,dv = \int (50 - 100x)\,dx$$

$$\Rightarrow \frac{v^2}{2} = 50x - \frac{100x^2}{2} + c$$

At $x = 2$; $v = 0$

$0 = 100 - 200 + c \Rightarrow c = 100.$

$$v^2 = 2\left[50x - \frac{100x^2}{2} + 100\right]$$

At $x = 0$,

$$v = \sqrt{2[100]} = 10\sqrt{2}\text{ m/s}$$

25. (4) $\frac{1}{2}mv_{\max}^2 = 9 - 5 = 4$

or $mv_{\max}^2 = 8$

or $2v_{\max}^2 = 8$

or $v_{\max} = 2\text{ m/s} = A\omega$

$\therefore\ \omega = \frac{v_{\max}}{A} = \frac{2}{0.01} = 200$

or $\frac{2\pi}{T} = 200$

$\therefore\ T = \frac{2\pi}{200} = \frac{\pi}{100}$

26. (1) For a displacement x, the kinetic and potential energies are

$$\text{KE} = \frac{1}{2}m(A^2 - x^2)\omega^2 \text{ and PE} = \frac{1}{2}mx^2\omega^2$$

Each of these $= 10/2 = 5$ J when $x = 1$ cm

Hence $\frac{1}{2}mx^2\omega^2 = \frac{1}{2}\times 0.1\times(1\times10^{-2})^2\omega^2 = 5$

This gives $\omega^2 = \frac{10}{0.1\times10^{-4}} = 10^6$

Giving $\omega = 10^3 = 1000$ rad/s

Hence $T = \frac{2\pi}{\omega} = \frac{\pi}{500}$ s

And frequency $n = \frac{500}{\pi}$ s

Also, $\text{KE} = 5 = \frac{1}{2}\times 0.1[A^2 - (1\times10^{-2})^2](10^6)$

$$A^2 - 10^{-4} = 10^{-4}$$

$$A^2 = 2\times10^{-4} \Rightarrow A = \sqrt{2}\times10^{-2}\text{ m} = \sqrt{2}\text{ cm}$$

27. (2), 28. (2), 29. (3), 30. (3)

From the graph $T = (5 - 1) = 4$ s (distance between the two adjacent crests shown in the figure)

and $v_{\max} = 5$ m/s; $\omega A = 5$ m/s

$$\left(\frac{2\pi}{T}\right)A = 5 \Rightarrow A = \frac{5T}{2\pi} = \frac{5\times4}{2\pi} = \frac{10}{\pi}\text{ m}$$

Also, $\omega = \frac{2\pi}{4} = \frac{\pi}{2}$ rad/s

The equation of velocity can be written as $v = 5\sin\left(\frac{\pi}{2}t\right)$ m/s

At extreme position, $v = 0$; $\sin\left(\frac{\pi}{2}t\right) = 0$ or $t = 2$ s

Phase of the particle velocity at that instant corresponding to the above equation $= \pi$.

Therefore, when a phase change of $\pi/6$ takes place, the resulting phase $= \pi + \pi/6$.

$$v = 5\sin\left(\pi + \frac{\pi}{6}\right) = -5\sin\frac{\pi}{6} = -5\left(\frac{1}{2}\right)$$

$$= 2.5\text{ m/s (numerically)}$$

$$\frac{dy}{dt} = v \Rightarrow dy = vdt$$

$$dy = \int 5\sin\left(\frac{\pi t}{2}\right)dt = \frac{10}{\pi}\left[-\cos\frac{\pi t}{2}\right] + C$$

Since at $t = 0$, the particle is at the extreme position, therefore at $t = 0$; $y = -\frac{10}{\pi}$

$$-\frac{10}{\pi} = -\frac{10}{\pi}\cos\theta + C \Rightarrow C = 0$$

$$y = -\frac{10}{\pi}\cos\frac{\pi t}{2}$$

Clearly, a phase change of $\pi/6$ corresponds to a time difference of

$$\frac{T}{2\pi}\left(\frac{\pi}{6}\right) = \frac{T}{12} = \frac{4}{12} = \frac{1}{3}\text{ s}$$

$$y = -\frac{10}{\pi}\cos\frac{\pi}{6} = -\frac{10}{\pi}\left(\frac{\sqrt{3}}{2}\right) = \frac{5\sqrt{3}}{\pi}\text{ m (numerically)}$$

Acceleration, $a = \frac{dv}{dt} = \frac{d}{dt}\left(5\sin\frac{\pi t}{2}\right) = \frac{5\pi}{2}\cos\frac{\pi t}{2}$

$$a \text{ at } \left(t = \frac{1}{3}\text{ s}\right) = \frac{5\pi}{2}\cos\frac{\pi}{6} = \frac{5\pi\sqrt{3}}{4}\text{ m/s}^2$$

Maximum displacement, $x_{\max} = A = \frac{10}{\pi}$ m

and maximum acceleration,

$$a_{\max} = \omega^2 A = \left(\frac{\pi}{2}\right)^2\times\frac{10}{\pi} = \frac{5\pi}{2}\text{ m/s}^2$$

31. (2) The angular frequency is $\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{200}{1}} \approx 14$ rad/s

3) $T = \frac{25}{50} = \frac{1}{2}\text{ s} \quad \Rightarrow \omega = \frac{2\pi}{T} = 4\pi\text{ rad/s}$

Spring constant $k = m\omega^2 = 5 \times (4\pi)^2 = 80\pi^2$ N/m

Force required to stretch the spring by 5 cm is

$$F = kx = 80\pi^2 \cdot 0.05\text{ N} = 4\,\pi^2$$

1) Since $F = -\frac{dU}{dx} = -2kx\exp(-x^2)$

$F = 0$ (at equilibrium as $x = 0$)

U is minimum at $x = 0$ and $U_{\text{min}} = 0$

U is maximum at $x \to \pm\infty$ and $U_{\text{max}} = k$

The particle would oscillate about $x = 0$ for small displacement from the origin and it is in stable equilibrium at the origin.

(1) Let $x_1 = a\sin\omega t$ and $x_2 = a\sin(\omega t + \delta)$ be two SHMs.

$$\frac{a}{3} = a\sin\omega t \quad \text{and} \quad -\frac{a}{3} = a\sin(\omega t + \delta)$$

$$\sin\omega t = 1/3 \quad \text{and} \quad \sin(\omega t + \delta) = -1/3$$

Eliminating t, $\frac{1}{3}\cos\delta + \sqrt{1 - \frac{1}{9}}\sin\delta = -\frac{1}{3}$

$$9\cos^2\delta + 2\cos\delta - 7 = 0$$

$$\cos\delta = -1 \quad \text{or} \quad \frac{7}{9}$$

i.e., $\delta = 180°$ or $\cos^{-1}\left(\frac{7}{9}\right)$

If we put 180°, we find that v_1 and v_2 are of opposite signs. Hence $\delta = 180°$ is not applicable.

$$\therefore \quad \delta = \cos^{-1}\left(\frac{7}{9}\right)$$

(3) $U = U_0 - U_0\cos ax$

$$F = -\frac{dU}{dx} = -U_0\sin ax = m\frac{d^2x}{dt^2}$$

This is the equation of SHM $\Rightarrow T = \frac{2\pi}{a}\sqrt{\frac{m}{U_0}}$

6. (1) Equation of SHM $= y\sin(\omega t + \phi)$

When $y = \frac{a}{2}$, $t = \frac{T}{4} = \frac{2\pi}{4\omega} = \frac{\pi}{2\omega}$

$v = a\omega\cos(\omega t + \phi)$, velocity is negative

$$\frac{\pi}{2} < (\omega t + \phi) < \frac{3\pi}{2}$$

$$\frac{a}{2} = a\sin(\omega t + \phi) \quad \Rightarrow \sin(\omega t + \phi) = \frac{1}{2}$$

$$\left(\frac{\pi}{2} + \phi\right) = \frac{5\pi}{6}$$

Substituting in the above equation, we get $\phi = \pi/3$.

37. (2) Amplitude = 0.15 m

$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{400}{4}} = 10\text{ rad/s}$$

$$\Rightarrow \quad f = \frac{\omega}{2\pi} = \frac{5}{\pi}\text{ Hz}$$

Energy of the particle executing SHM = KE + PE

$$= \frac{1}{2}mv^2 + \frac{1}{2}kx^2 = \frac{1}{2}kA^2$$

Therefore, $\frac{1}{2}mv^2 = \frac{1}{2}kA^2 - \frac{1}{2}kx^2 = \frac{1}{2}k(A^2 - x^2)$

$$= \frac{1}{2} \times 400(0.15^2 - 0.1^2) = 2.5\text{ J}$$

38. (2) Period $= 2\frac{\pi}{\omega} = 2$ s (or) $\omega = \pi$ rad/s, amplitude = 10 cm

If the system is released, the equation of motion is $y = A\cos\omega t$

$5 = 10\cos\pi t_1$ when $y = 5$ cm

$-5 = 10\cos\pi t_2$ when $y = -5$ cm

$$\pi t_1 = \cos^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{3} \quad \text{or} \quad t_1 = \frac{1}{3}\text{ s}$$

$$\pi t_2 = \cos^{-1}\left(-\frac{1}{2}\right) = \frac{2\pi}{3} \quad \text{or} \quad t_2 = \frac{2}{3}\text{ s}$$

$$\therefore \text{ Time interval} = t_2 - t_1 = \frac{2}{3} - \frac{1}{3} = \frac{1}{3}\text{s}$$

39. (1) Let x_1 and x_2 be the distances of the two positions from centre. Then with usual notations

$$u^2 = \omega^2(A^2 - x_1^2) \quad \text{...(i)}$$

$$v^2 = \omega^2(A^2 - x_2^2) \quad \text{...(ii)}$$

$$a = \omega^2 x_1 \quad \text{...(iii)}$$

$$b = \omega^2 x_2 \quad \text{...(iv)}$$

Subtracting Eq. (ii) from Eq. (i),

$$u^2 - v^2 = \omega^2\left(x_2^2 - x_1^2\right) \quad \text{...(v)}$$

Adding Eqs. (iii) and (iv), $a + b = \omega^2(x_1 + x_2)$ ...(vi)

Dividing Eq. (v) by Eq. (vi) $\frac{u^2 - v^2}{a + b} = x_2 - x_1$

40. (4) Equations are $x_1 = a\cos\left(\omega t + \frac{\pi}{6}\right)$ and $x_2 = a\cos\left(\omega t + \frac{\pi}{3}\right)$

The first will pass through the mean position when $x_1 = 0$ i.e., for instants t for which $\left(\omega t + \frac{\pi}{6}\right) = \frac{n\pi}{2}$, where n is an integer.

The smallest value of t is $n = 1$, $\omega t_1 = (\pi/2) - (\pi/6) = \pi/3$. The second will pass through the mean position when $x_2 = 0$, i.e., for instants t for which $\left(\omega t + \frac{\pi}{3}\right) = \frac{m\pi}{2}$

where m is an integer.

The smallest value of t is $m = 1$, $= (\pi/2) - (\pi/3) = \pi/6$.

The smallest interval between the instants $x_1 = 0$ and $x_2 = 0$ is therefore,

$$\omega(t_1 : t_2) = \left(\frac{\pi}{3} - \frac{\pi}{6}\right) = \frac{\pi}{6} \Rightarrow t_1 : t_2 = \frac{\pi}{6\omega}$$

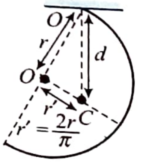
41. (4) $T = 2\pi\sqrt{\frac{I_0}{mgd}}$; $I_0' = I_C + mr'^2 \Rightarrow mr^2 = I_C + mr'^2$

$$\Rightarrow \quad I_C = m(r^2 - r'^2)$$

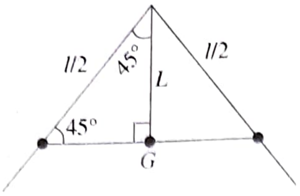
$$I_0 = I_C + md^2 = m(r^2 - r'^2) + m(r^2 + r'^2) = 2mr^2$$

$$d = \sqrt{r^2 + \left(\frac{2r}{\pi}\right)^2} = r\sqrt{1 + \frac{4}{\pi^2}}$$

$$\Rightarrow \quad T = 2\pi\sqrt{\frac{2r}{g\left(1 + \frac{4}{\pi}\right)^{\frac{1}{2}}}}$$



42. (2) $T = 2\pi\sqrt{\dfrac{I}{mgL}}$



Here $I = \dfrac{ml^2}{3} + \dfrac{ml^2}{3} = \dfrac{2ml^2}{3}$

From figure, $\sin 45^\circ = \dfrac{L}{l/2}$

$$\therefore \quad L = \frac{l}{2\sqrt{2}} \quad T = 2\pi\sqrt{\frac{2ml^2}{3 \times \dfrac{l}{2\sqrt{2}} 2mg}} = 2\pi\sqrt{\frac{2\sqrt{2}\,l}{3g}}$$

43. (1) Hence $\sin\omega t = \dfrac{x}{2}$

$$\therefore \quad \cos\omega t = \frac{\sqrt{4-x^2}}{2}$$

$$\therefore \quad y = 2\sin\omega t\cos\frac{\pi}{4} + 2\cos\omega t\sin\frac{\pi}{4}$$

$$= \frac{2}{\sqrt{2}}\sin\omega t + \frac{2}{\sqrt{2}}\cos\omega t = \sqrt{2} \times \frac{x}{2} + \sqrt{2}\,\frac{\sqrt{4-x^2}}{2}$$

$$= \frac{x}{\sqrt{2}} + \frac{\sqrt{4-x^2}}{\sqrt{2}}$$

or $\quad \sqrt{2}y - x = \sqrt{4-x^2}$

or $\quad 2y^2 + x^2 - xy = 4 - x^2$

or $\quad 2y^2 + 2x^2 - xy = 4$

or $\quad x^2 + y^2 - \sqrt{2}xy = 2$

This is an equation of ellipse.

44. (2) $U = -ax^2 + bx^4$

$$\therefore \quad F = -\frac{dU}{dx} = -2ax + 4bx^3$$

At mean position $F = 0$

$\therefore \quad 2ax = 4bx^3$

or $\quad x^2 = \dfrac{2a}{4b} = \dfrac{a}{2b}$

$$\therefore \quad x_0 = -\sqrt{\frac{a}{2b}}$$

$$k_{\text{eff}} = \frac{d^2U}{dx^2} \text{ at } x_0 = 4a$$

$$ma' = +4ax$$

$$\therefore \quad a' = \frac{4a}{m}x$$

Comparing with $a' = \omega^2 x$

$$\omega = \sqrt{\frac{4a}{m}} = 2\sqrt{\frac{a}{m}}$$

45. (2) $y_1 = a\sin\omega t$ and $y_2 = \sin(\omega t + \phi)$

$$y_2 - y_1 = a\sqrt{2} = a\sin(\omega t + \phi) - a\sin\omega t$$

or $\quad \sqrt{2}\,a = 2a\cos\left(\dfrac{\omega t + \phi + \omega t}{2}\right)\sin\left(\dfrac{\omega t + \phi - \omega t}{2}\right)$

$$= 2a\cos\left(\omega t + \frac{\phi}{2}\right)\sin\frac{\phi}{2}$$

For maximum value, $\cos\left(\omega t + \dfrac{\phi}{2}\right) = 1$, therefore

$$2\sin\frac{\phi}{2} = \sqrt{2} \Rightarrow \sin\frac{\phi}{2} = \frac{1}{\sqrt{2}}$$

or $\quad \dfrac{\phi}{2} = \dfrac{\pi}{4}$ or $\phi = \dfrac{\pi}{2}$

46. (1) With mass m_2 alone, the extension of the spring l is given as

$$m_2 g = kl \qquad \text{...(i)}$$

With mass $(m_1 + m_2)$, the extension l' is given by

$$(m_1 + m_2)g = kl' = k(l + \Delta l) \qquad \text{...(ii)}$$

Hence Δl is the amplitude of vibration.

Subtracting Eq. (i) from Eq. (ii), we get $m_1 g = k\,\Delta l$

or $\quad \Delta l = \dfrac{m_1 g}{k}$

47. (3) Decrease in potential energy of the mass when the pan gets lowered by distance y (due to mass hitting on the pan) $= mg(h + y)$, where h is the height through which the mass falls on the pan. Increases in elastic potential of the spring $= 1/2\,ky^2$ (according to law of conservation of energy)

or $\quad mg(h + y) = \dfrac{1}{2}ky^2$

or $\quad ky^2 - 2mgy - 2mgh = 0$

$$\therefore \quad y = \frac{2mg \pm \sqrt{4m^2g^2 + 8mghk}}{2k} = \frac{mg}{k} \pm \frac{mg}{k}\sqrt{\left(1 + \frac{2hk}{mg}\right)}$$

Velocity of the pan will be maximum at the time of collision and will be zero at the lowest position.

Hence y should be the amplitude of oscillation.

So, amplitude of vibration $= \left[\dfrac{mg}{k} + \dfrac{mg}{k}\sqrt{\left(1 + \dfrac{2hk}{mg}\right)}\right]$

48. (4) Mean position of the particle is mg/k distance below the unstretched position of spring. Therefore, amplitude of oscillation is $A = \dfrac{mg}{k}$

$$\omega = \sqrt{\frac{k}{m}} = 2\pi f = 20\pi \;(f = 10 \text{ Hz})$$

$$\frac{m}{k} = \frac{1}{400\pi^2}$$

$$v_{\max} = A\omega = \frac{g}{400\pi^2} \times 20\pi = \frac{1}{2\pi} \text{ m/s}$$

49. (2) $E = \dfrac{1}{2}m\omega^2 A^2 = \dfrac{1}{2}m(2\pi f)^2 A^2$

$$\Rightarrow \quad A = \frac{1}{2\pi f}\sqrt{\frac{2E}{m}}$$

Putting $E = K + U$, we get

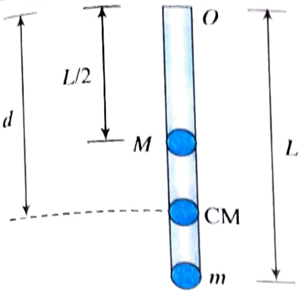
$$A = \frac{1}{2\pi(25/\pi)}\sqrt{\frac{2 \times (0.5 + 0.4)}{0.2}} = 0.06 \text{ m}$$

50. (2) When the rod is replaced by the string the simple pendulm will act as compound pendulum for which time period is given by

$$T = 2\pi\sqrt{\frac{I_0}{Mgd}} \qquad \text{...(i)}$$

I_0 = moment of inertia about point of suspension

$$= \frac{ML^2}{3} + mL^2 = \frac{(M + 3m)L^2}{3}$$



$M' =$ total mass $= (M + m)$

$d =$ separation between point of suspension and centre of mass of the pendulum

$$= \frac{M(L/2) + m(L)}{(M+m)} = \frac{(M+2m)L}{2(M+m)}$$

Substituting the values in Eq. (i), we get

$$\Rightarrow \quad T = 2\pi\sqrt{\frac{2(M+3m)L}{3(M+2m)g}}$$

Time interval is $T/6$

Let the equation of simple harmonic motion be

$$x = A\cos\omega t,\ v = -A\sin\omega t, \text{ where } \omega = 2\pi/T$$

Mean velocity over the required interval is $\langle v \rangle = \dfrac{\int_0^{T/6} v\,dt}{T/6}$

$$= \frac{\int_0^{T/6} -A\omega\sin\omega t\,dt}{T/6} = \frac{6A}{T}\cos\omega t\Big|_0^{T/6} = \frac{-3A}{T}$$

3) Time period of the system (object of mass 4 kg) before collision is $T_1 = 2\pi\sqrt{\dfrac{4}{100}}$

After collision, time period of the combined mass is

$$T_2 = 2\pi\sqrt{\frac{10}{100}}$$

We can apply momentum conservation for just before the collision and just after the collision in the horizontal direction.

$$4A_1\omega_1 = 10A_2\omega_2 \Rightarrow A\frac{4\times 2\times\sqrt{\frac{100}{4}}}{10\times\sqrt{\frac{100}{10}}} = 2\sqrt{\frac{2}{5}}\text{ m}$$

So, change in amplitude, $\Delta A = A_1 - A_2 = 2\left[1 - \sqrt{\dfrac{2}{5}}\right]\text{m}$

(2) $U = \dfrac{a}{x^2} - \dfrac{b}{x};\ F = -\dfrac{dU}{dx} = -\left[\dfrac{-2a}{x^3} + \dfrac{b}{x^2}\right]$

At equilibrium position, $F = 0$

$$x = x_0 = \frac{2a}{b}$$

At $x = x_0 + \Delta x$, i.e., at a displacement of Δx ($<< x_0$) from mean position,

$$F = -\left[\frac{bx - 2a}{x^3}\right] = -\left[\frac{b(x_0 + \Delta x) - 2a}{(x_0 + \Delta x)^3}\right] = -\frac{b\Delta x}{x_0^3} = -\frac{b^4}{8a^3}\Delta x$$

As $F \propto \Delta x$, so particle performs simple harmonic motion with time period

$$T = 2\pi\sqrt{\frac{8a^3 m}{b^4}}$$

54. (3) Let ω_1 and ω_2 be the angular frequencies of first and second particle, respectively. Then, the phase by which they will proceed in time t is $\omega_1 t$ and $\omega_2 t$, respectively.

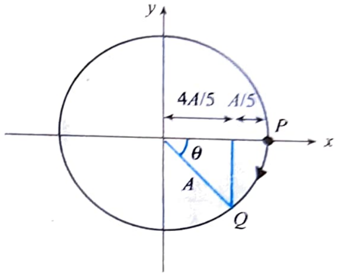
According to the given situation, $\omega_2 t - \omega_1 t = 3\times 2\pi$ for $t = 45$ s

$$\frac{2\pi}{T} - \frac{2\pi}{3} = \frac{3\times 2\pi}{45}$$

$$\frac{1}{T} = \frac{1}{3} + \frac{1}{15} = \frac{6}{15} \Rightarrow T = 2.5\text{ s}$$

55. (2) **Method 1:** Particle is starting from rest, i.e., from one of its extreme position.

As particle moves a distance $A/5$, we can represent it on a circle as shown.



$$\cos\theta = \frac{4A/5}{A} = \frac{4}{5}$$

$$\Rightarrow \quad \theta = \cos^{-1}\left(\frac{4}{5}\right)$$

$$\omega t = \cos^{-1}\left(\frac{4}{5}\right) \Rightarrow t = \frac{1}{\omega}\cos^{-1}\left(\frac{4}{5}\right) = \frac{T}{2\pi}\cos^{-1}\left(\frac{4}{5}\right)$$

Method 2: As the particle starts from rest, i.e., from extreme position $x = A\sin(\omega t - \phi)$

At $\quad t = 0;\quad x = A \quad \Rightarrow \phi = \dfrac{\pi}{2}$

$$A - \frac{A}{5} = A\cos\omega t$$

$$\frac{4}{5} = \cos\omega t \Rightarrow \omega t = \cos^{-1}\frac{4}{5}$$

$$t = \frac{T}{2\pi}\cos^{-1}\left(\frac{4}{5}\right)$$

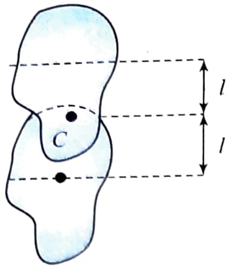
56. (2) $f_0 = \dfrac{1}{2\pi}\sqrt{\dfrac{mgl}{I}}$

where l is the distance between point of suspension and centre of mass of the body. Thus, for the stick of length L and mass m,

$$f_0' = \frac{1}{2\pi}\sqrt{\frac{mg\frac{L}{4}}{\frac{m}{2}\frac{(L/2)^2}{12}}} = \frac{1}{2\pi}\sqrt{\frac{12g}{L}} = \sqrt{2}f_0$$

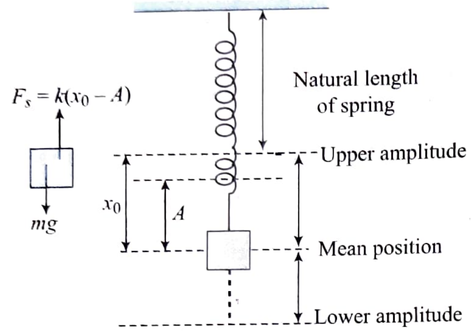
57. (1) $\dfrac{1}{2}I\omega^2 = mg(2l) \Rightarrow \dfrac{I}{mgl} = \dfrac{4}{\omega^2}$

$$T = 2\pi\sqrt{\frac{I}{mgl}} = 2\pi\sqrt{\frac{4}{\omega^2}} = \frac{4\pi}{\omega}$$



58. (1) The spring is never compressed. Hence spring shall exert least force on the block when the block is at topmost position.

At equilibrium position $kx_0 = mg \Rightarrow x_0 = mg/k$.



$$F_{\max} = k(x_0 - A) = k\left[\frac{mg}{k} - A\right]$$

$$= mg - 4\frac{\pi^2}{T^2}mA = mg - kA = mg - m\omega^2 A$$

59 (4) $T = 2\pi\sqrt{\frac{m}{K_1 + K_2}} = 2\pi\sqrt{\frac{10}{360}} = \frac{\pi}{3}$ s

The maximum velocity is always at equilibrium position since at any other point there will be a restoring force attempting to slow the mass.

$$V_{\max} = \frac{\text{impulse}}{\text{mass}} = \frac{50}{10} = 5 \text{ m/s} \Rightarrow \omega = \frac{2\pi}{T} = 6 \text{ rad/sec}$$

$$\Rightarrow \quad A = \text{amplitude} = \frac{V_{\max}}{\omega} = \frac{5}{6} = 0.83 \text{ m}$$

60. (3) Let the bar be rotated through a small angle θ.

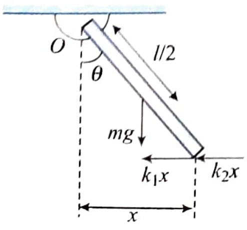
The restoring torque of the forces mg, k_1x and k_2x about O can be given as

$$\tau = -\left[mg\left(\frac{l}{2}\right)\sin\theta + k_1x(l\cos\theta) + k_2x(l\cos\theta)\right]$$

Since θ is small, $\sin\theta \cong \theta$, $x = l\,\theta$ and $\cos\theta \cong 1$

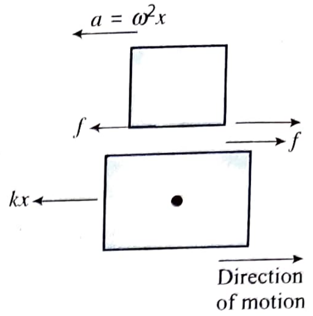
Putting $k_1 + k_2 = k$, we obtain $\tau = -\left[kl^2 + mg\left(\frac{l}{2}\right)\right]\theta$

or $\quad I\alpha = -\left[kl^2 + mg\left(\frac{l}{2}\right)\right]\theta$



$$\Rightarrow \quad \omega_{\text{osc.}} = \sqrt{\left(\frac{kl^2 + mg(l/2)}{(ml^2/3)}\right)} = \sqrt{\frac{3k}{m} + \frac{3g}{2l}}$$

61. (2) Let A be the maximum amplitude of oscillation for the required situation. Then $f = m\omega^2x$ (from free body diagram of blocks)

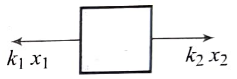


For required condition, $f = m\omega^2 A$ for no slipping $f < f_L$

i.e., $\quad m\omega^2 A < \mu_s mg$

$$\Rightarrow \quad A < \frac{0.6 \times g}{\frac{4\pi^2}{4}} = 0.6 \text{ m} \Rightarrow A_{\max} = 0.6 \text{ m}$$

62. (1) In equilibrium position, net force acting on the object (block of mass m) is zero. Let spring of spring constant k_1 is stretched by x_1 and spring of spring constant k_2 is stretched by x_2, then free body diagram of the block is as shown in the following figure.



Now, $\quad x_1 + x_2 = 20$ cm $\quad$ and $\quad k_1x_1 = k_2x_2$

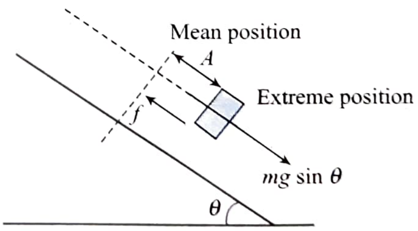
$x_1 = 15$ cm $\quad$ and $\quad x_2 = 5$ cm

So, new equilibrium position is at $x = (20 + 15)$ cm from P_1, and time period of oscillation of the block is given by

$$T = 2\pi\sqrt{\frac{m}{k_1 + k_2}} = 2\pi\sqrt{\frac{0.1}{4000}} = \frac{\pi}{100} \text{ s}$$

63. (3) Here $\omega = \sqrt{\frac{k}{3m}}$. The maximum static frictional force is

$$f_{\max} = \mu mg\cos\theta = 2\tan\theta\, mg\cos\theta = 2\, mg\sin\theta$$



Applying Newton's second law on the block at lower extreme position,

$$f - mg\sin\theta = m\omega^2 A \quad \Rightarrow f = m\omega^2 A + mg\sin\theta$$

As $f \le f_{\max}$, $\omega^2 A = g\sin\theta$ or $A = \frac{3mg\sin\theta}{k}$

64. (3) The centripetal acceleration on the bob as it oscillates (acting along the radius of circle) $= v^2/R$. This will act horizontally towards the centre of circular path.

The total acceleration acting on the pendulum bob is therefore

$$a = \sqrt{g^2 + \left(\frac{v^2}{R}\right)^2}$$

The frequency of oscillation will therefore be

$$n = \frac{1}{2\pi}\sqrt{\frac{a}{l}} = \frac{1}{2\pi}\sqrt{\frac{\left(g^2 + \frac{v^4}{R^2}\right)^{1/2}}{l}}$$

(4) When spring is compressed by $3x_0$. Amplitude, $A = 3x_0$. The time taken from extreme compressed position to mean position, $t_1 = T/4$.

If time taken (t_2) from mean position to $x = x_0$ is given by

$$x = A \sin \frac{2\pi t_2}{T} \Rightarrow x_0 = 3x_0 \sin \frac{2\pi t_2}{T}$$

$$\sin \frac{2\pi t_2}{T} = \frac{1}{3} \quad \Rightarrow \frac{2\pi t_2}{T} = \frac{\pi}{9}$$

$$\Rightarrow \quad t_2 = \frac{T}{18}$$

$$t_1 + t_2 = \frac{T}{4} + \frac{T}{18} = \frac{11}{18}T = \frac{11}{18}2\pi\sqrt{\frac{m}{K}} = \frac{11}{9}\pi\sqrt{\frac{m}{K}}$$

(1) $f = a - bx$

For maximum velocity, acceleration should be zero.

i.e., $a - bx = 0 \quad \Rightarrow \quad x = \frac{a}{b}$

$\therefore$ At $x = x = \frac{a}{b}$, the particle has its maximum velocity.

$$f = \frac{vdv}{dx} = a - bx$$

$$\Rightarrow \quad \frac{v^2}{2} = ax - \frac{bx^2}{2} + c$$

At $x = 0; v = 0 \quad \Rightarrow \quad c = 0$

Substituting $x = \frac{a}{b}$ gives $v_{\max} = \frac{a}{\sqrt{b}}$

Also, the velocity of the car should become zero at station B.

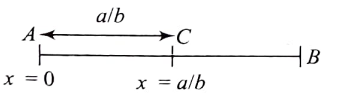
i.e., $ax - \frac{bx^2}{2} = 0 \quad \Rightarrow \quad x = 0; x = \left(\frac{2a}{b}\right)$

Therefore, distance between the two stations is $\frac{2a}{b}$.

Alternatively: $f = a - bx$ means particle will do SHM.

At mean position, $f = 0$

$$\Rightarrow \quad x = \frac{a}{b}$$



In the figure shown, C is the mean position and A ad B are extreme positions. Thus,

$$x_{\max} = \frac{2a}{b} \text{ and } V_{\max} = \omega A = \sqrt{b}\cdot\frac{a}{b} = \frac{a}{\sqrt{b}}.$$

7. (3) If the stick is rotated through a small angle θ, the spring is stretched by a distance $L\theta/2$. Therefore restoring force in spring = ($KL\theta/2$). Each spring causes a torque

$$T(= Fd) = \left(\frac{KL\theta}{2}\right)\frac{L}{2}$$

in the same direction. Therefore equation of rotational motion is $\tau = I\alpha$.

$$-2\left(\frac{KL\theta}{2}\right)\frac{L}{2} = I\alpha \quad \text{with} \quad I = \frac{ML^2}{12}$$

$$\alpha = -\left(\frac{6K}{M}\right)\theta$$

Standard equation of angular SHM is $\alpha = \pi\omega^2\theta$

$$\omega^2 = \frac{6K}{M} \Rightarrow f = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{6K}{M}}$$

68. (3) $\frac{d^2v}{dt} = -Kv^2$

This equation has standard solution $v = v_0 \sin(\sqrt{K}t + \theta)$, where $\omega = \sqrt{K}$

Hence the particle executes SHM with angular frequency

$$\omega = \sqrt{K} \text{ or frequency } f = \frac{\sqrt{K}}{2\pi}$$

69. (1) Let the cylinder be depressed a distance x metres into water from the position of rest.

Increase in tension of spring = $100x$ metres into water from the position of rest

Increase in tension of spring = $100x$ kg-wt

$$(A = 100 \text{ cm}^2 = 10^{-2} \text{ m}^2)$$

$$\text{Increase in buoyancy} = \frac{100}{100^2}x \times 1000 \text{ kg-wt}$$

$$K = 1 \text{ kg-wt/cm} = 100 \text{ kg-wt/m} = 10x \text{ kg-wt}$$

Total unbalanced vertical force on the cylinder

$$= 100x + 10x = 110x \text{ kg-wt}$$

$$\text{Acceleration of cylinder} = \frac{110x \times 10}{10} \text{ m/s}^2 = 110\,x \text{ m/s}^2$$

$$\text{Period of one oscillation} = \frac{2\pi}{\sqrt{110}} = 0.6 \text{ s}$$

70. (1) The oscillation of the block will be periodic but not simple harmonic. The springs are only compressed but not extended since the block loses contact with either spring just in its relaxed state. The minimum distance between the springs will therefore be when both the springs are relaxed, i.e., during the interval when the block moves between the springs.

Time taken by the block to move from one spring to the other is $t_1 = d/v$.

Time taken by the block in contact with either spring is

$$t_2 = \frac{1}{2}\left[2\pi\sqrt{\frac{m}{K}}\right] = \pi\sqrt{\frac{m}{K}}$$

Hence the periodic time T of the oscillation of the block is

$$T = 2(t_1 + t_2) = 2\left[\frac{d}{v} + \pi\sqrt{\frac{m}{K}}\right]$$

Also minimum distance between the springs = d.

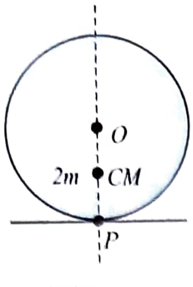
71. (3) Angular frequency of system, $\omega = \sqrt{\frac{K}{m+m}} = \sqrt{\frac{K}{2m}}$

Maximum acceleration, $a_{\max} = \omega^2 A$

Frictional force between P and Q = force exerted on lower block

$$= m\omega^2 A = m\left(\frac{K}{2m}\right)A = \frac{KA}{2}$$

72. (1) Rolling can be considered as pure rotation about point of contact. We can consider this system as a compound pendulum oscillating about P.



Time period $T = 2\pi\sqrt{\dfrac{I_P}{mgd}}$

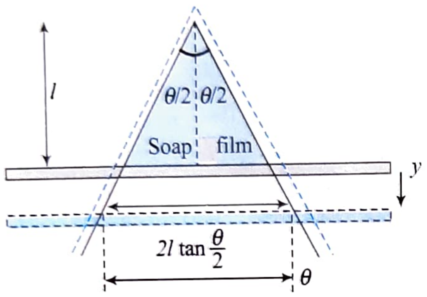
I_P = moment of inertia about rotation axis

$= I_0 + mR^2 = mR^2 + mR^2 = 2mR^2$

The centre of mass of tube and rod will be at a height $R/2$ from P, hence $d = R/2$

$$\Rightarrow \quad T = 2\pi\sqrt{\frac{2mR^2}{mgR/2}} = 2\pi\sqrt{\frac{4R}{g}}$$

73. (1) Let S be the surface tension of the soap film. For equilibrium of rod



$$2(l + y)\tan\frac{\phi}{2}$$

$$mg = (F_{\text{surface}})_1$$

$$mg = \left(2l\tan\frac{\theta}{2}\right)S \times 2; \quad mg = 4Sl\tan\frac{\theta}{2}$$

If the rod is displaced from its mean position by small displacement y, then restoring force on the rod is

$$F_{\text{rest}} = -\left[\left(F_{\text{surface}}\right)_2 - mg\right] - \left(F_{\text{surface}}\right)_1$$

$$= -\left[4S(l + y)\tan\frac{\theta}{2} - mg\right] - \left[4S\tan\frac{\theta}{2}y\right]$$

$$a = -\frac{4S\tan\frac{\theta}{2}}{m}y = \frac{-4S\tan\frac{\theta}{2}}{\left(\frac{4Sl\tan\frac{\theta}{2}}{g}\right)}y$$

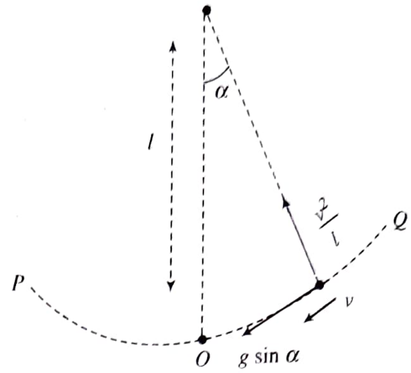
$$\frac{d^2y}{dt^2} = -\left(\frac{g}{l}\right)y$$

$$\therefore \quad T = 2\pi\sqrt{\frac{l}{g}}$$

Multiple Correct Answers Type

1. (1), (4)

Statement (1) is correct. At any position O and P or between O and Q, there are two accelerations—a tangential acceleration $g\sin\alpha$ and a centripetal acceleration v^2/l (because the pendulum moves along the arc of a circle or radius l), where l is the length of the pendulum and v its speed at that position. When the bob is at the mean position O, the angle $\alpha = 0$, therefore $\sin\alpha = 0$; hence, the tangential acceleration is zero. But at O, speed v is maximum and the centripetal acceleration v^2/l is directed radially towards the point of support. When the bob is at the end points P and Q, the speed v is zero, hence the centripetal acceleration is zero at the end points, but the tangential acceleration is maximum and is directed along the tangent to the curve at P and Q. The tension in the string is not constant throughout the oscillation. At any position between O and end point P or Q, the tension in the string is given by $T = mg\cos\alpha$.



At the end point P and Q, the value of α is the greatest, hence the tension is the least. At the mean position O, $\alpha = 0$ and $\alpha = 1$ which is the greatest; hence tension is greatest at the mean position.

2. (1), (2), (3)

The motion of the particle is simple harmonic. The displacement at time t is: $x = a\sin(bt + c)$

Therefore, displacement at time $(t + (2\pi/b))$ is

$$x \text{ at}\left(t + \frac{2\pi}{b}\right) = a\sin\left[b\left(t + \frac{2\pi}{b}\right) + c\right]$$

$$= a\sin[bt + c + 2\pi] = a\sin(bt + c) = x \text{ (at time } t)$$

Hence, statement (1) is correct.

Statement (2) is also correct since the same displacement is recovered after a time interval of $(2\pi/b)$. Statement (3) is correct ± because the velocity is zero when the displacement = ± the amplitude, i.e., at the extreme ends of the motion. Statement (4) is incorrect, the acceleration is maximum (in magnitude) at $x = \pm A$.

3. (1), (2)

When point of suspension of pendulum is moved upwards, $g_{\text{eff}} = g + a$. $g_{\text{eff}} > g$ and as $T \propto 1/\sqrt{g_{\text{eff}}}$, hence T decreases, i.e., choice (1) is correct.

When point of suspension of pendulum is moved downwards and $a > 2g$, then T decreases, i.e., choice (2) is also correct.

In case of horizontal acceleration $g_{\text{eff}} = \sqrt{g^2 + a^2}$, i.e., $g_{\text{eff}} > g$ i.e., again T decreases.

4. (1), (3), (4)

$$\vec{r} = A\cos\omega t\,\hat{i} + 2A\cos\omega t\,\hat{j}$$

$$|\vec{r}| = A\cos\omega t \cdot \sqrt{5} = \sqrt{5}A\cos\omega t$$

So motion is at straight line, periodic, SHM.

5. (1), (2), (3), (4)

For SHM

$$y = A\sin\omega t = A\sin\left(\frac{2\pi t}{T}\right)$$

$$v = \omega\sqrt{A^2 - y^2}; \quad v_{\max} = \omega A$$

(1) At $y = A/2$

$$v = \omega\sqrt{A^2 - A^2/4} = \sqrt{\frac{3\omega A}{2}} > \frac{\omega A}{2}$$

(2) For $v = v_0/2 \Rightarrow \omega\sqrt{A^2 - y^2} = \frac{\omega A}{2}$

$$\Rightarrow \quad A^2 - y^2 = \frac{A^2}{4}$$

(3) for $t = T/8$

$$y = A\sin\left(\frac{2\pi \cdot T}{8 \cdot T}\right) = A/\sqrt{2} > \frac{A}{2}$$

(4) for $y = A/2$, $\quad A/2 = A\sin\left(\frac{2\pi t}{T}\right)$

$$\Rightarrow \quad \frac{2\pi t}{T} = \frac{\pi}{6} \Rightarrow \quad t = \frac{T}{12} < \frac{T}{8}$$

(1), (2), (3), (4)

If a particle is performing SHM with amplitude A and angular frequency ω and if its initial phase is equal to zero, then its displacement from the mean position will be given by $x = A \sin \omega t$. Value of $\sin \omega t$ is same at two different values of the phase. If one is ωt, then the other is $(\pi - \omega t)$. Hence at these two instants, the phases are unequal. Therefore, option (3) is correct.

Velocity of the particle will be equal to $\omega\sqrt{(A^2 - x^2)}$. Since x is same at these two instants, therefore the magnitude of velocity will be same or the speeds are equal. However, velocities are unequal because at one instant direction of motion of the particle will be towards the extreme position or away from mean position and at the other instant, it will be towards the mean position. Hence (1) is also correct.

Since speeds are equal, kinetic energies are also equal. Hence, option (4) is also correct. Acceleration of the particle is $a = -\omega^2 x$. Since x is same at these two instants, accelerations are also equal. Therefore, option (2) is also correct.

(1), (2), (4)

Total mechanical energy of the oscillating system is $E = K_{\max} + U_{\min}$ $-K_{\min}$, $K_{\min} = 0$ at extreme position.

So, $\quad U_{\max} = E = 200$ J

$$K_{\max} = \frac{mv_{\max}^2}{2} = \frac{m \times A^2\omega^2}{2} = \frac{KA^2}{2} = 150 \text{ J}$$

So, $\quad U_{\min} = 50$ J

(1), (3)

Let O be the mean position and a be the acceleration at a displacement x from O.

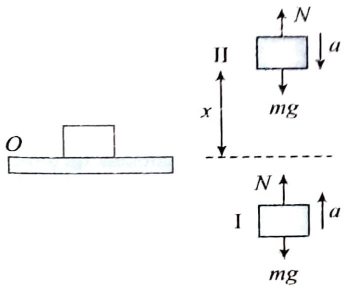
At position I, $N - mg = ma$

$\therefore \quad N \neq 0$

At position II, $mg - N = ma$

For $N = 0$ (loss of contact), $g = a = \omega^2 x$.

Loss of contact will occur for amplitude $x_{\max} = g/\omega^2$ at the highest point of the motion.



9. (1), (2), (4)

Description of motion is completely specified if we know the variation of x as a function of time. For simple harmonic motion, the general equation of motion is $x = A \sin(\omega t + \delta)$. As ω is given, to describe the motion completely, we need the values of A and δ.

From options (2) and (4), we can have the values of A and δ directly. For option (1), we can find A and δ if we know initial velocity and initial position. Option (3) cannot give the values of A and δ so it is not the correct condition.

10. (1), (3), (4)

$$U = x^2 - 4x + 3 \text{ and } F = -\frac{dU}{dx} = -(2x - 4)$$

At equilibrium position $F = 0$, so $x = 2$ m.

Let the particle is displaced by Δx from equilibrium position, i.e., from $x = 2$, then restoring force on body is

$$F = -2\,(2 + \Delta x) + 4 = -2\,\Delta x$$

i.e., $F \alpha -\Delta x$, so performs simple harmonic motion about $x = 2$ m.

$$\text{Time period, } T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{1}{2}} = \sqrt{2}\pi \text{ s}$$

$$\text{From energy conservation, } \frac{mv_{\max}^2}{2} + U_{\min} = U_{\max}$$

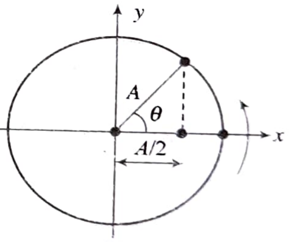
$$\frac{1 \times 4^2}{2} + (2^2 - 4 \times 2 + 3) = (A + 2)^2 - 4(A + 2) + 3$$

where A is amplitude.

Solving the above equation, we get $A = 2\sqrt{2}$ m.

11. (1), (3)

At $t = 0$ when particle is at extreme position, the situation is as shown in the figure.



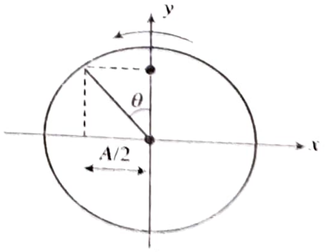
(1)

From the figure, $\cos\theta = \frac{A/2}{A} = \frac{1}{2}$

$$\theta = \frac{\pi}{3} \Rightarrow \frac{\pi}{3} = \frac{2\pi}{T} \times 1$$

$\Rightarrow \quad T = 6$ s

At $t = 0$ when particle is at mean position, the situation is as shown in the figure.



(2)

$\theta = \omega t$

From the figure, $\sin\theta = \frac{A/2}{A}$

$$\theta = \frac{\pi}{6}, \text{ but } \theta = \omega t \Rightarrow \frac{\pi}{6} = \frac{2\pi}{T} \times 1 \Rightarrow T = 12 \text{ s}$$

If initially the particle is located somewhere else, then time period comes out to be different. A reverse question can also be formed on the same concept.

12. (2), (3)

$$x = 3 \sin 100\pi t$$
$$y = 4 \sin 100\pi t$$

Equation of path is $\frac{y}{x} = \frac{4}{3}$ i.e., $y = \frac{4}{3}x$

which is equation of a straight line having slope 4/3.

Equation of resulting motion is

$$\vec{r} = x\hat{i} + y\hat{j} = (3\hat{i} + 4\hat{j})\sin 100\pi t$$

Amplitude $= \sqrt{3^2 + 4^2} = 5$

13. (2), (3)

$$v^2 = 108 - 9x^2 \quad \text{or} \quad v^2 = 9(12 - x^2)$$

We can compare the above expression with $v = \omega\sqrt{A^2 - x^2}$, which is the expression of velocity for SHM.

From this, we will get $\omega = 3$ and $A = \sqrt{12}$

SHM is not a uniformly accelerated motion,

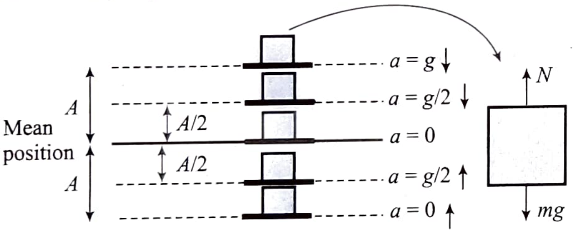
Acceleration at a distance 3 cm from the mean position,

$$a = \omega^2 (3 \text{ cm}) = 27 \text{ cm/s}^2$$

Maximum displacement from the mean position $= A = \sqrt{12}$ cm.

14. (1), (2), (3), (4), (5)

The position of momentary rest in SHM is the extreme position where velocity of particle is zero.



As the block loses contact with the plank at this position, i.e., normal force becomes zero, it has to be the upper extreme where acceleration of the block will be g downwards.

$$\omega^2 A = g$$

$$\Rightarrow \quad \omega^2 = \frac{10}{0.4} = 25$$

$$\Rightarrow \quad \omega = 5 \text{ rad/s} \quad \therefore \quad T = \frac{2\pi}{\omega} = \frac{2\pi}{5} \text{ s}$$

Acceleration in SHM is given as $a = \omega^2 x$.

From the figure, we can see that at lower extreme, acceleration is g upwards

$$N - mg = ma$$

or $\quad N = m(a + g) = 2mg$

At halfway up acceleration is $g/2$ downwards.

$$mg - N = ma \quad \text{or} \quad N = m(g - g/2) = \frac{1}{2}mg$$

At halfway down, acceleration is $g/2$ upwards.

$$N - mg = ma \quad \text{or} \quad N = m(g + g/2) = \frac{3}{2}mg$$

At mean position, velocity is maximum and acceleration is zero.

$\therefore \quad N = mg$

15. (1), (2), (3), (4)

At $t = 0$

Displacement, $x = x_1 + x_2 = 4\sin\frac{\pi}{3} = 2\sqrt{3}$ m

Resulting amplitude, $A = \sqrt{2^2 + 4^2 + 2(2)(4)\cos \pi/3} = 2\sqrt{7}$ m

Maximum speed, $A\omega = 20\sqrt{7}$ m/s

Maximum acceleration, $A\omega^2 = 200\sqrt{7}$ m/s^2

Energy of the motion $= \frac{1}{2}m\omega^2 A^2 = 28$ J

16. (2), (3), (4)

$$U = 5x(x - 4) = 5x^2 - 20x$$

$$F = -\frac{dU}{dx} = -10x + 20$$

i.e., force is not constant.

KE or speed of the particle will be maximum at the mean position where force becomes zero.

$$F = 0 \quad \text{or} \quad x = 2 \text{ m}$$

Acceleration experienced by the particle is

$$a = \frac{F}{m} = \frac{-10x + 20}{0.1} = -(100x - 200)$$

i.e., particle executes SHM.

As $\quad \omega^2 = 100$

$\therefore \quad \omega = 10$

Hence $T = \frac{2\pi}{\omega} = \frac{\pi}{5}$ s

17. (1), (3)

$$y = a \sin \omega t = a \sin \frac{2\pi t}{T}$$

$$v = \frac{dy}{dt} = \omega a \cos \frac{2\pi t}{T}$$

At $\quad t = \frac{T}{6}, \quad v = \omega a \cos\left(\frac{2\pi}{T}\frac{T}{6}\right) = \frac{1}{2}\omega a$

or, $\quad v = (v_{\max}/2)$

$$y = a\sin\frac{2\pi}{T} \times \frac{T}{6} = a \sin\frac{\pi}{3}$$

It is not half of a.

$$\text{Acceleration} = \frac{d^2y}{dt^2} = \frac{dv}{dt} = \omega^2 a \sin\frac{2\pi t}{T} = \omega^2 a \sin\frac{\pi}{3} = 0.86(AC)_{\max}$$

At this instant, $\text{KE} = \frac{1}{2}mv^2 = \frac{1}{2}m\left(\frac{v_{\max}}{2}\right)^2 = \frac{(\text{KE})_{\max}}{4} = \frac{1}{4}(\text{TE})$

$\therefore \quad \text{PE} = \text{TE} - \text{KE} = \frac{3}{4}(\text{TE})$

i.e., $\quad \text{KE} \neq \text{PE}$

18. (1), (2), (3), (4)

Period of oscillation changes as it depends on mass and becomes three times. The amplitude of oscillation does not change, because the new object is attached when the original object is at rest. Total energy does not change as at extreme position the energy is in the form of potential energy stored in spring which is independent of mass, and hence maximum; KE also does not change but as mass changes the maximum speed changes.

19. (2), (4)

The maximum extension x produced in the spring in case (a) is given by

$$F = kx \quad \text{or} \quad x = \frac{F}{k}$$

The time period of oscillation is $T = 2\pi\sqrt{\frac{\text{mass}}{\text{force constant}}} = 2\pi\sqrt{\frac{m}{k}}$

In case (a) one end A of the spring is fixed to the wall. When a force F is applied to the free end B in the direction, the spring is stretched exerting a force on the wall which in turn exerts an equal and opposite reaction force on the spring, as a result of which every coil of the spring is elongated producing a total extension x.

In case (b), both ends of the spring are free equal forces are applied at ends. By application of forces both the cases are same.

Thus, the maximum extension produced in the spring in cases (a) and (b) is the same. In case (b) the mid-point of spring will not move, we can say the blocks are connected with the springs whose lengths are half the original length of spring. Now, the force constant of half the spring is twice of complete spring. In case (b) the force constant = $2k$. Hence, the time period of oscillation will be

$$T' = 2\pi\sqrt{\frac{m}{2k}}$$

$$\frac{T'}{T} = \sqrt{2}$$

Hence, the correct choices are (2) and (4).

20. (1), (3), (4)

The only external horizontal force acting on the system of the two blocks and the spring is F. Therefore, acceleration of the centre of mass of the system is equal to $F/m_1 + m_2$.

Hence, centre of mass of the system moves with a constant acceleration. Initially, there is no tension in the spring, therefore at initial moment m_2 has an acceleration F/m_2 and it starts to move to the right. Due to its motion, the spring elongates and a tension is developed. Therefore, acceleration of m_2 decreases while that of m_1 increases from zero initial value.

The blocks start to perform SHM about their centre of mass and the centre of mass moves with the acceleration calculated above. Hence, option (2) is correct.

Since the blocks perform SHM about centre of mass, therefore the length of the spring varies periodically. Hence, option (1) is wrong.

Since magnitude of the force F remains constant, therefore amplitude of oscillations also remains constant. So, option (3) is also wrong.

Acceleration of m_2 is maximum at the instant when the spring is in its minimum possible length, which is equal to its natural length. Hence, at initial moments, acceleration of m_2 is maximum possible.

The spring is in its natural length, not only at initial moment but at time $t = T, 2T, 3T, \ldots$ also, where T is the period of oscillation. Hence, option (4) is wrong.

21. (1), (3), (4)

When the block is released suddenly, it starts to move down. During its downward motion the rubber cord elongates. Hence, a tension is developed in it but the block continues to accelerate downwards till tension becomes equal to weight mg of the block.

After this moment, the block continues to move down due to its velocity and rubber cord further elongates. Therefore, tension becomes greater than the weight; hence, the block now retards and comes to an instantaneous rest.

At lowest position of the block, strain energy in the cord equals loss of potential energy of the block. Suppose the block comes to an instantaneous rest when elongation of the rubber cord is equal to y. Then

$$\frac{1}{2}ky^2 = mgy \Rightarrow y = \frac{2mg}{k} \text{ and } 0$$

Hence, block will be instantaneously at rest, at $y = 0$ and at $y = 2mg/k$.

In fact, the block oscillates between these two values.

Since the rubber cord is elastic, tension in it is directly proportional to elongation. Therefore, the block will perform SHM.

Its amplitude will be equal to half of the distance between these extreme positions of the block or amplitude is

$$\frac{1}{2} \times \frac{2mg}{k} = \frac{mg}{k} = l$$

Hence, option (2) is correct.

The angular frequency of its SHM will be equal to $\omega = \sqrt{\frac{k}{m}}$

Since k and m are not given in the question, it cannot be calculated. Hence option (4) is not correct.

22. (1), (4)

Since liquid 2 is below liquid 1, liquid 2 is denser than liquid 1. Let area of cross section of the cylindrical block be A and it be displaced downward by y. Then volume of liquid 2 displaced will get increased by Ay and that of liquid 1 will get decreased by the same amount Ay. Hence, net increase in upthrust on the block will be equal to $(Ayd_2g - Ayd_1g)$. This additional upthrust tries to restore the block in original position.

It means, the block experiences a restoring force $Ay(d_2 - d_1)g$. Since this force is restoring and directly proportional to displacement y, it will execute SHM along a vertical line.

Hence, option (1) is correct and option (2) is wrong.

If mass of the block is equal to m, then its acceleration will be equal to

$$\frac{Ayg(d_2 - d_1)}{m}$$

Since its acceleration depends on mass m, frequency of oscillations will depend on size of the cylinder.

Hence option (3) is wrong.

If the cylinder is displaced upward through y from equilibrium position, then it will experience a net downward force, equal to calculated above. This shows that its motion will be symmetric about its equilibrium position.

23. (1), (2), (3), (4)

If in equilibrium position, elongation of the spring is equal to x_0, then

$$Kx_0 = mg \text{ or } x_0 = \frac{mg}{K} = 1 \text{ cm}$$

If the block is raised till spring becomes unstretched and then released, then during subsequent motion maximum elongation of the spring (y) will be calculated by energy conservation law.

$$\text{Loss of PE of block } (mgy) = \text{strain energy}\left(\frac{1}{2}Ky^2\right)$$

$$y = \frac{2mg}{K} = 2 \text{ cm}$$

Hence option (2) is correct.

Frequency of oscillations will be, $f = \frac{1}{2\pi}\sqrt{\frac{K}{m}} = 5 \text{ Hz}$

Hence option (3) is correct.

Since frequency, f, does not depend upon gravitational acceleration, therefore frequency will remain unchanged, even if the system is taken to moon. Hence option (4) is also correct.

24. (1), (3)

Energy stored $= \frac{1}{2}kx^2 = \frac{1}{2}k \times (0.25)^2 = 5$ J

$k = 160$ N/m

Period, $T = 2\pi\sqrt{\frac{m}{k}}$

$T = \frac{1}{5}$ s $\quad \therefore \quad T^2 = \frac{4\pi^2 m}{k}$

$\frac{1}{25} = \frac{4\pi^2 m}{160}$

$\therefore \quad m = \frac{4}{25}$ kg (As $\pi^2 = 10$), hence $m = 0.16$ kg

25. (2), (3)

Let the simple harmonic motions be given by

$$x_1 = a\sin\left(2\pi\frac{t}{T}\right) \quad \text{... (i)}$$

$$x_2 = a\sin\left(2\pi\frac{t}{T} + \frac{\pi}{4}\right) \quad \text{... (ii)}$$

and $$x_3 = a\sin\left(2\pi\frac{t}{T} + \frac{\pi}{2}\right) \quad \text{...(iii)}$$

Then the resultant periodic motion, by the principle of superposition is given by

$$x = x_1 + x_2 + x_3$$

$$= a\left(\sin\frac{2\pi t}{T}\right) + a\sin\left(\frac{2\pi t}{T} + \frac{\pi}{4}\right) + a\sin\left(\frac{2\pi t}{T} + \frac{\pi}{2}\right)$$

$$= a\left[\sin\left(\frac{2\pi t}{T}\right) + a\sin\left(\frac{2\pi t}{T} + \frac{\pi}{2}\right)\right] + a\sin\left(\frac{2\pi t}{T} + \frac{\pi}{4}\right)$$

$$= 2a\sin\left(\frac{2\pi t}{T} + \frac{\pi}{4}\right)\cos\frac{\pi}{4} + a\sin\left(\frac{2\pi t}{T} + \frac{\pi}{4}\right)$$

$$= a\left(\sqrt{2} + 1\right)\sin\left(\frac{2\pi t}{T} + \frac{\pi}{4}\right)$$

which is a simple harmonic motion with an amplitude $a\left(\sqrt{2} + 1\right)$ and phase angle $\pi/4$ and the same period; it has the same phase as second SHM.

The energy of resultant motion is proportional to $\left[a\left(\sqrt{2} + 1\right)\right]^2$ i.e., $\left[a^2\left(2 + 1 + 2\sqrt{2}\right)\right] = (3 + 2\sqrt{2})a^2$ which is greater than three times the energy of each separate SHM.

26. (2), (4)

Acceleration $a = \omega^2 x$

Maximum acceleration $a = \omega^2 A = \left(\frac{2\pi}{T}\right)^2 \frac{25}{100}$

When block and platform are separated, $a = g = 10$

$\frac{4\pi^2}{T^2} \cdot \frac{1}{40} \Rightarrow T^2 = \frac{\pi^2}{100} \quad T = \pi/10$ sec

Linked Comprehension Type

For Problems 1–2

1. (1) Let the equation of simple harmonic motion be

$x = A\sin(\omega t + \delta)$

Then, at $t = 1$ s, $x = 0$, so

$0 = A\sin(\delta + \omega) = \sin(\omega + \delta) = 0$

At $\quad t = 2$ s, $\quad v = +0.25$ m/s

so $\quad 0.25 = A\omega\cos(2\omega + \delta)$

Solving above equation, we get $A = \frac{3}{2\pi}$ m and $\delta = -\frac{\pi}{3}$

Hence equation of SHM will be $x = \frac{3}{2\pi}\sin\left(\frac{\pi}{3}t - \frac{\pi}{3}\right)$

For velocity at $t = 5$ s, i.e., after half a period of $t = 2$ s, the velocity is having same magnitude but opposite direction.

2. (3). The equation of simple harmonic motion is

$$x = \frac{3}{2\pi}\sin\frac{\pi}{3}(t - 1)$$

and velocity $v = \frac{1}{2}\cos\frac{\pi}{3}(t - 1)$

Hence at $t = 5$ s, $v = -\frac{1}{4}$ m/s

For Problems 3–4

3. (1) Using conservation of mechanical energy

$$\frac{1}{2}mv^2 = \frac{1}{2}kx^2 \Rightarrow x^2 = \frac{m}{k} \Rightarrow x = v\sqrt{\frac{m}{k}}$$

4. (3) The time taken by the mass to regain its original speed towards left wall is half the period of oscillation $= \pi\sqrt{m/k}$.

The ball strikes the left wall after a time $= L/v$ and since the collision is elastic, it again reaches the spring with same speed v and in a further time L/v.

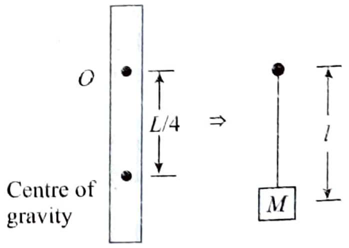
The block thus undergoes periodic motion with time period

$$\pi\sqrt{\frac{m}{k}} + \frac{2L}{v}$$

For Problems 5–7

5. (2) In the present case, $d = \frac{L}{4}$

$$I = \frac{ML^2}{12} + M\left(\frac{L}{4}\right)^2 = \frac{7}{48}ML^2$$



So, time period of the physical pendulum is

$$T = 2\pi\sqrt{\frac{I}{Mgd}} = 2\pi\sqrt{\frac{7/48\, ML^2}{Mg \times L/4}} = \pi\sqrt{\frac{7L}{3g}}$$

6. (4) Let the location of centre of oscillation is at a distance l from the point of pivot, then

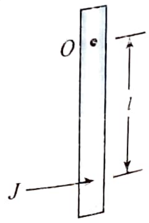
$$T = 2\pi\sqrt{\frac{l}{g}} = \pi\sqrt{\frac{7L}{3g}} \Rightarrow l = \frac{7L}{12}$$

7. (1) Due to the impulse, let us say the rod acquires an angular velocity ω.

Now, $J \times l = I\omega$

Here $l = \frac{7L}{12}$, $I = \frac{7}{48}ML^2$

Hence $\omega = \frac{4J}{ML}$



Problems 8–10

(2) The angular frequency of simple harmonic motion is given by

$$\omega = \sqrt{\frac{k}{m}}$$

The velocity of block when it is a displacement of y from mean position is given by $v = \omega\sqrt{A^2 - y^2}$,

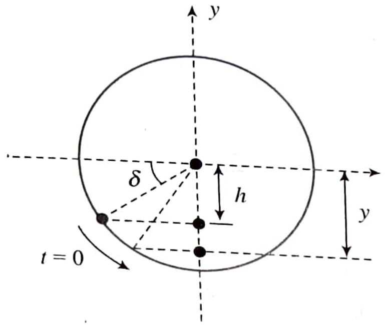
From given initial condition, $v_0 = \sqrt{\frac{k}{m}}\sqrt{A^2 - h^2}$

$$\Rightarrow \quad A^2 = \frac{mv_0^2}{k} + h^2 \quad \Rightarrow A = \sqrt{\frac{mv_0^2}{k} + h^2}$$

(1) To have the equilibrium of simple harmonic motion, it is best to represent simple harmonic motion as uniform circular motion. At $t = 0$, let particle is making an angle δ with –ve x-axis as shown, then $\sin\delta = \frac{h}{A}$

$$\Rightarrow \quad \delta = \sin^{-1}\left(\frac{h}{A}\right)$$

At time t, $y = -A \sin(\omega t + \delta)$



So the equation of simple harmonic motion is

$$y = -\sqrt{\frac{mv_0^2}{k} + h^2}\left\{\sin\left[\sqrt{\frac{k}{m}}t + \sin^{-1}\left(\frac{h}{A}\right)\right]\right\}$$

10. (3) To compute the time taken by the block to cross mean position for the first time, we can make use of circular motion representation.

$$t = \frac{\pi - \delta}{\omega} = \frac{\pi - \sin^{-1}\left(\frac{h}{A}\right)}{\sqrt{\frac{k}{m}}}$$

For Problems 11–15

11. (2) At position A, block is in positive region and slope is negative, so velocity is negative.

12. (3) At position B, block is in negative region and velocity is positive, so slope is positive.

13. (3) At position C, block is in positive region and velocity is zero, so slope is zero.

14. (3) $\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{25600}{100}} = 16$ rad/s

Using energy conservation, $\frac{1}{2}kx^2 + \frac{1}{2}mv^2 = \frac{1}{2}kA'^2$

$$A = \sqrt{x^2 + \frac{m}{k}v^2} = \sqrt{9+3} = 2\sqrt{3}$$

$$x = A\cos(\omega t + \phi) = 2\sqrt{3}\cos(16t + \phi)$$

From boundary conditions $t = 0$, ϕ can be obtained which is $\pi/6$.

$$x = 2\sqrt{3}\cos\left(16t + \frac{\pi}{6}\right)\text{cm}$$

15. (4) Differentiating previous result $v = -32\sqrt{3}\sin\left(16t + \frac{\pi}{6}\right)$cm/s

For Problems 16–18

16. (2) When speed of block is maximum, net force on block is zero. Hence at that instant spring exerts a force of magnitude 'mg' on the block.

17. (3) At the instant block is in equilibrium position, its speed is maximum and compression in spring is x given by

$$kx = mg \qquad \text{...(i)}$$

From conservation of energy,

$$mg(L + x) = \frac{1}{2}kx^2 + \frac{1}{2}mv_{\max}^2 \qquad \text{...(ii)}$$

From Eqs. (i) and (ii), we get $v_{\max} = \frac{3}{2}\sqrt{gL}$

18. (2) $V_{\max} = \frac{3}{2}\sqrt{gL}$ and $\omega = \sqrt{\frac{k}{m}} = 2\sqrt{\frac{g}{L}}$

$$A = \frac{V_{\max}}{\omega} = \frac{3}{4}L$$

Hence time taken, t, from start of compression till the block reaches mean position is given by

$x = A \sin \omega t_0$, where $x = L/4$

$$t_0 = \sqrt{\frac{L}{4g}}\sin^{-1}\frac{1}{3}$$

Time taken by the block to reach from mean position to bottom most position is $\frac{2\pi}{4\omega} = \frac{\pi}{4}\sqrt{\frac{L}{g}}$

Hence the required time $= \frac{\pi}{4}\sqrt{\frac{L}{g}} + \sqrt{\frac{L}{4g}}\sin^{-1}\frac{1}{3}$

For Problems 19–21

19. (2) $\omega = \sqrt{\frac{K}{m}}$, $K = \omega^2 m = (10)^2 = 100$ N/m

Angular frequency of oscillation of combined body is

$$\omega' = \sqrt{\frac{K}{m+M}} = \sqrt{\frac{100}{4}} = 5 \text{ rad/s}$$

20. (1) Conserving linear momentum.

$$(1 + 3)\,v = 1 \times 0.3 + 3\,(-0.3)$$

$$\Rightarrow \quad v = -0.15 \text{ m/s}$$

Negative sign indicates that combined body starts to move leftward. But at the instant of collision, spring is in its natural length or combined body is in equilibrium position. Hence, at $t = 0$, phase of combined body becomes equal to π.

Therefore, new amplitude of oscillation is

$$a' = \frac{|v|}{\omega} = \frac{0.15}{5} = 0.03 \text{ m} = 3 \text{ cm}$$

21. (2) Equation for position x of combined body is given by

$x = l_0 + a' \sin(\omega' t + \pi)$

or $\quad x = \{10 + 3\sin(5t + \pi)\}$ cm

or $\quad x = 10 - 3\sin(5t)$ cm

For Problems 22–24

22. (4) Let x be the maximum extension of the spring. The mass falls from rest through the vertical distance $(L + x)$ and so the energy given by it to the spring is $E_k = mg\,(L + x)$. This must be stored in the spring as elastic energy at its maximum extension.

Hence $E_K = \frac{1}{2}kx^2 = mg(L+x)$

This is quadratic, $x^2 - \frac{2mg}{k}x - \frac{2mgL}{k} = 0$

$$\text{Hence, } x = \frac{\frac{2mg}{k} \pm \sqrt{\left(\frac{2mg}{k}\right)^2 + \frac{8mgL}{k}}}{2}$$

$$\text{Maximum value of } \quad x = \frac{mg}{k} + \frac{mg}{k}\sqrt{1 + \frac{2kL}{mg}}$$

$$= \frac{mg}{k}\left(1 + \sqrt{1 + \frac{2kL}{mg}}\right)$$

23. (1) If the mass is pulled down by distance A from its equilibrium position and then released, it will perform SHM of amplitude A. Assuming that the string remains taut, the maximum acceleration will be at the amplitude points and given by

$$f_m = A\omega^2$$

Now $\omega^2 = \frac{k}{m}$; hence, $f_m = \frac{Ak}{m}$

24. (2) In the position of equilibrium, when the tension in the string $= mg$, the extension in the spring is $x_0 = mg/k$. If the amplitude exceeds this value, then in its upward motion, the body will rise to a height at which the string will become loose and $T = 0$. Hence maximum amplitude is $x_m = mg/k$.

It also follows from the fact that for $T \geq 0$, the maximum downward acceleration that the body can have is g. Since $f(t) = g - T(t)/m, f(t) \leq g$. Hence maximum amplitude is

$$A_m = \frac{g}{\omega^2} = \frac{mg}{k}$$

For Problems 25–27

25. (2), 26. (1), 27. (3)

As C collides with A and sticks to it, the combined mass moves rightward to compress the spring and hence B moves rightwards due to the spring force, i.e., B accelerates and the combined mass decelerates. The deformation in the spring is changing and the centre of mass of the system continues to move rightwards with constant speed, while both the blocks oscillate about centre of mass of the system.

The velocity of the combined mass just after collision is $mv_0 = 2mv$

$$v = \frac{v_0}{2} = 0.3 \text{ m/s}$$

Velocity of centre of mass of system,

$$v_{CM} = \frac{2mv + 0}{3m} = \frac{2v}{3} = \frac{v_0}{3} = 0.2 \text{ m/s}$$

Time period with which block B and the combined mass oscillate about centre of mass could be computed by using the reduced mass concept.

$$T = 2\pi\sqrt{\frac{2m}{3k}} = \frac{\pi}{5\sqrt{10}} \text{ s}$$

Oscillation energy of the system is $E = \frac{2mv^2}{2} = 0.27$ J

The translational kinetic energy of the centre of mass of system is

$$E_{CM} = \frac{3mv_{CM}^2}{2} = 0.18 \text{ J}$$

The remaining energy is oscillating between kinetic and potential energy during the motion of blocks.

For maximum compression in spring either we can use centre of mass approach or energy approach, here we are using the second method.

Oscillation energy = Maximum elastic potential energy

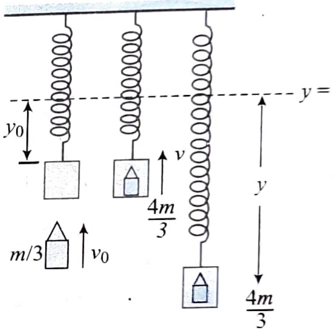
$$0.09 = \frac{1}{2}kx_m^2 \Rightarrow x_m = 3\sqrt{10} \text{ mm}$$

For Problems 28–30

28. (3), 29. (2), 30. (1)

Initially in equilibrium let the elongation in spring be y_0, then $mg = ky_0$.

$$y_0 = \frac{mg}{k}$$



As the bullet strikes the block with velocity v_0 and gets embedded into it, the velocity of the combined mass can be computed by using the principle of moment conservation.

$$\frac{m}{3}v_0 = \frac{4m}{3}v \quad \Rightarrow v = \frac{v_0}{4}$$

Let new mean position is at distance y from the origin, then

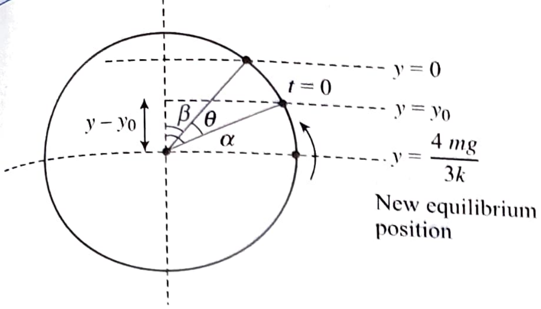
$$ky = \frac{4m}{3}g \quad \Rightarrow \quad y = \frac{4mg}{3k}$$

Now, the block executes SHM about mean position defined by $y = 4mg/3k$ with time period, $T = 2\pi\sqrt{4m/3k}$. At $t = 0$, the combined mass is at a displacement of $(y - y_0)$ from mean position and is moving with velocity v, then by using $v = \omega\sqrt{A^2 - x^2}$, we can find the amplitude of motion.

$$\left(\frac{v_0}{4}\right)^2 = \frac{3k}{4m}[A^2 - (y - y_0)^2] = \frac{3k}{4m}\left[A^2 - \left(\frac{mg}{3k}\right)^2\right]$$

$$\Rightarrow \quad A = \sqrt{\frac{mv_0^2}{12k} + \left(\frac{mg}{3k}\right)^2}$$

To compute the time taken by the combined mass from $y = mg/k$ to $y = 0$, we can either go for equation method or circular motion projection method.



Required time, $t = \frac{\theta}{\omega} = \frac{\alpha - \beta}{\omega}$

$$\cos\alpha = \frac{y - y_0}{A} = \left(\frac{\frac{4mg}{3k} - \frac{4mg}{k}}{A}\right) = \frac{mg}{3kA}$$

$$\cos\beta = \frac{y}{A} = \frac{4mg}{3kA}$$

So, $$t = \frac{\cos^{-1}\left(\frac{mg}{3kA}\right) - \cos^{-1}\left(\frac{4mg}{3kA}\right)}{\omega}$$

$$= \sqrt{\frac{4m}{3k}}\left[\cos^{-1}\left(\frac{mg}{3kA}\right) - \cos^{-1}\left(\frac{4mg}{3kA}\right)\right]$$

rix Match Type

i → b., c.; ii. → b., c.; iii. → a.; iv. → d.

The velocity–displacement relation is $x^2 + \frac{v^2}{\omega^2} = A^2$ which may be a circle if $\omega = 1$. and ellipse of $\omega \neq 1$.

The acceleration–velocity relation is

$+\frac{a^2}{\omega^2} = A^2$ which may be a circle if $\omega = 1$ and ellipse of $\omega \neq 1$.

Acceleration–displacement graph is straight and acceleration time graph is sinusoidal.

i → c.; ii. → a.; iii. → b.; iv. → d.

$$V_m = A\omega$$

$$\Rightarrow \quad A = \frac{V_m}{\omega} = \frac{2\pi}{2\pi} \times (0.2) = 0.20 \text{ m}$$

$$T = 2\pi\sqrt{\frac{m}{k}} \Rightarrow m = \frac{T^2 k}{4\pi^2} = 0.2 \text{ kg}$$

At $t = 0.1$, acceleration is maximum

$$\Rightarrow \quad = -\omega^2 A = -200 \text{ m/s}^2$$

Maximum energy $= \frac{1}{2} m v_m^2 = 4$ J

i. → a.; ii. → b.; iii. → a.; iv. → d.

(i) In frame of lift, effective acceleration due to gravity is

$$g + \frac{g}{2} = \frac{3g}{2} \text{ downwards}$$

$$\therefore \quad T = 2\pi\sqrt{\frac{2l}{3g}}$$

(ii) $Kl = mg \therefore \frac{k}{m} = \frac{g}{L}$

Constant acceleration of lift has no effect in time period of oscillation.

$$\therefore \quad T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{l}{g}}$$

(iii) $T = 2\pi\sqrt{\frac{I}{mgd}} = 2\pi\sqrt{\frac{\frac{ml^2}{3}}{mg\frac{l}{2}}} = 2\pi\sqrt{\frac{2l}{3g}}$

(iv) $T = 2\pi\sqrt{\frac{m}{\rho Ag}} = 2\pi\sqrt{\frac{\rho/2Al}{\rho Ag}} = 2\pi\sqrt{\frac{l}{2g}}$

4. i. → a., b.; ii. → a., b.; iii. → c.; iv. → d.

Position and velocity both can be positive and negative. If at mean position, PE is zero, then at any other position, PE is positive. Total energy is always constant.

5. i. → c., d.; ii. → c., d.; iii. → a., b., d.; iv. → c., d.

A constant force and a constant torque affect only the mean position.

In third case, as the block falls on mean position, the mean position is not affected.

In a car a constant pseudo force will act which will affect only the mean position.

6. i. → a., b., c.; ii. → a., b., c.; iii. → a.; iv. → d.

For A, $y = Ae^{i(\omega t + \phi)}$

$$\frac{dy}{dt} = A \times i\omega e^{i(\omega t + \phi)}$$

$$\frac{d^2y}{dt^2} = Ai\omega \times i\omega e^{i(\omega t + \phi)} = -\omega^2 \times Ae^{i(\omega t - \phi)} = -\omega^2 y$$

which represents the standard equation of motion of SHM.

For B, $y = B\sin\omega t + C\cos\omega t$ can be written as

$$y = \sqrt{B^2 + C^2}\sin(\omega t + \delta)$$

where $\tan\delta = \frac{C}{B}$

So, B also represents an SHM.

For C, it is a standard equation of harmonic travelling wave in which the particle performs an SHM.

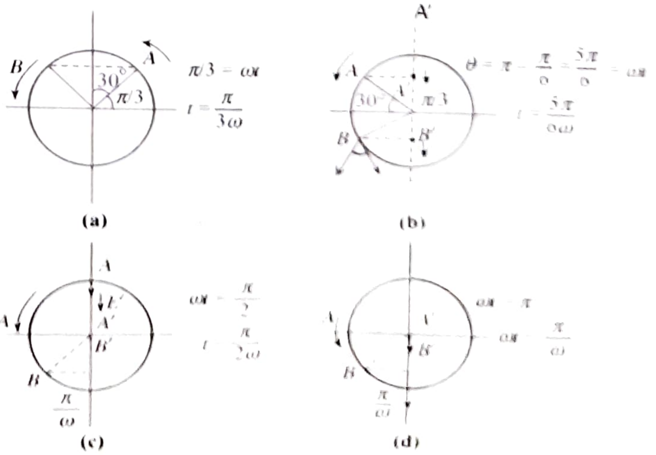
For D, y/x = constant, it represents the equation of the translatory motion.

7. i → c.; ii. → a.; iii. → a.; iv. → d.

Frequency depends only on the mass and the spring constant, so it will change. Due to increase in KE by a factor of 4, the amplitude gets doubled and hence the magnitude of maximum acceleration also gets doubled.

As maximum KE increases by a factor of 4, the maximum PE also increases by the same factor.

8 i. → b.; ii. → a.; iii. → d.; iv. → c.



9. i. → c.; ii. → e.; iii. → b.; iv. → c.; v. → b.

$$F = 8 - 2x = -2(x-4)$$

At equilibrium position, $F = 0$

$$x = 4 \text{ m}$$

As particle is released at rest from $x = 6$ m, i.e., it is one of the extreme position, amplitude $A = 2$ m.

Hence, force constant $k = 2$ N/m

$$m\omega^2 = 2 \quad \text{or} \quad \omega = 1$$

Time period $T = \dfrac{2\pi}{\omega} = 2\pi$

Time taken to go from $x = 2$ m to $x = 4$ m (i.e., from extreme position to mean position) is

$$\frac{T}{4} = \frac{\pi}{2}$$

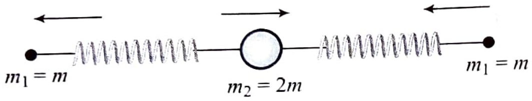
Energy of SHM $= \dfrac{1}{2}kA^2 = \dfrac{1}{2} \times 2 \times 4 \text{ N-m} = 4 \text{ J}$

As the particle has started its motion from positive extreme,

Phase constant $= \dfrac{\pi}{2}$

10. i. → b.; ii. → a.; iii. → d.; iv. → c.

(i) Motion is simple harmonic.



$$\frac{d^2x}{dt^2} = \frac{-k}{m_1 m_2}(m_2 + 2m_1)x$$

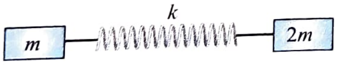
$$\omega^2 = k\left(\frac{1}{m_1} + \frac{2}{m_2}\right) = k\left(\frac{1}{m} + \frac{2}{2m}\right)x$$

$$\omega = \sqrt{\frac{2k}{m}}$$

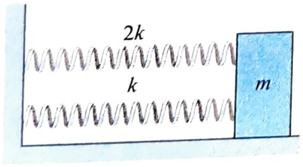
$\therefore$ Frequency $= \dfrac{1}{2\pi}\sqrt{\dfrac{2k}{m}}$

(ii) Angular frequency $\omega = \sqrt{\dfrac{k(m+2m)}{2m^2}} = \sqrt{\dfrac{3k}{2m}}$

$\therefore$ Frequency $= \dfrac{1}{2\pi}\sqrt{\dfrac{3k}{2m}}$

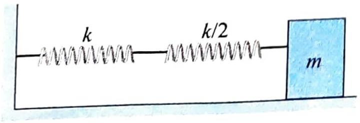


(iii) Here effective spring constant = $3k$.



$\therefore$ Frequency $= \dfrac{1}{2\pi}\sqrt{\dfrac{3k}{m}}$

(iv) Effective spring constant (K) is given by $\dfrac{1}{k} = \dfrac{1}{k} + \dfrac{1}{\frac{k}{2}} = \dfrac{3}{k}$



$$k = \frac{k}{3}$$

$\therefore$ Frequency $= \dfrac{1}{2\pi}\sqrt{\dfrac{k}{3m}}$

11. i. → b. c.; ii. → b., c., d.; iii. → a.; iv. → a.

(i) Linear combination of two SHMs will be an SHM if the individual SHMs have equal frequencies, their magnitudes may be different.

(ii) $y = A\sin\omega_1 t + A\sin(\omega_2 t + \phi)$

If $\omega_1 = \omega = \omega, \quad \phi = \pi/2$

$$y = A\sin\omega t + A\cos\omega t$$

Its amplitude will be $\sqrt{2}A$.

(iii) Time period of a pendulum of infinite length is $T = 2\pi\sqrt{\dfrac{R}{g}}$

(iv) The above will be the maximum value of time period of an oscillating pendulum.

Numerical Value Type

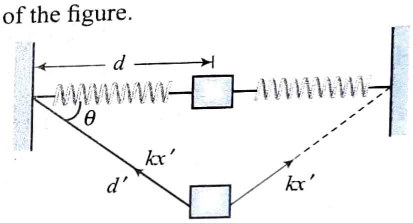
1. (2) Situation 1: When particle oscillates along the line of spring.

Initial stretch in both springs $= d - \dfrac{3d}{4} = \dfrac{d}{4}$

$$F_{\text{restering}} = k\left(\frac{d}{4} + x\right) - k\left(\frac{d}{4} - x\right) = 2kx$$

$$\Rightarrow \quad T_a = 2\pi\sqrt{\frac{m}{2k}}$$

Situation 2: When the particle oscillates perpendicular to the plane of the figure.



$$d' = d\sec\theta$$

$$x' = d\sec\theta - \frac{3d}{4} = d\left(\frac{1}{\cos\theta} - \frac{3}{4}\right)$$

Force towards equilibrium position ($kx'\sin\theta$)

$$= dk\left(\tan\theta - \frac{3\sin\theta}{4}\right)$$

due to one spring and net force $= 2dk\left(\tan\theta - \dfrac{3\sin\theta}{4}\right)$ for small θ,

$$= 2dk\left[\theta - \frac{3\theta}{4}\right] = k\left(\frac{d\theta}{2}\right)$$

$d\theta$ = displacement from mean position

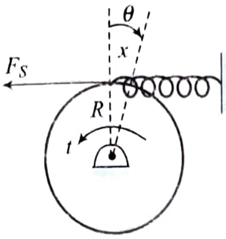
$$\Rightarrow \quad F = \frac{kx}{2} \Rightarrow T_b = 2\pi\sqrt{\frac{2m}{k}}$$

$$\Rightarrow \quad \frac{T_b}{T_a} = 2\pi\sqrt{\frac{m}{2k}} \Big/ 2\pi\sqrt{\frac{2m}{k}}$$

$$\Rightarrow \quad \frac{T_b}{T_a} = 2$$

2. (2) If we twist (rotate) the disc through a small clockwise angle θ, the spring will be deformed (compressed) by a distance $x = R\theta$.

Hence, the spring force $F_s = kx = k(R\theta)$ will produce a restoring torque.



Restoring torque: $\tau = -F_S R$ where $F_S = kR\theta$

This gives $\tau = -kR^2\theta$

It means after removing the external (applied) torque, the restoring torque rotates the disc with an angular acceleration α which will bring the spring–disc system back to its original state.

Newton's law of rotation (or torque equation): Applying Newton's second law of rotation, we have

$$\tau = I_C\alpha$$

where $\tau = -kR^2\theta$

This gives $\alpha = -\dfrac{kR^2\theta}{I_C}$ where $I_C = \dfrac{mR^2}{2}$

Then $a = -\dfrac{2k}{m}\theta$

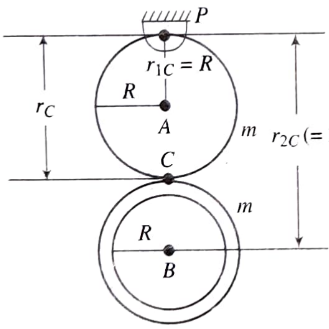
Comparing the above equation with $\alpha = -\omega^2\theta$, we have

$$\omega = \sqrt{\frac{2k}{m}}$$

After substituting the values we get $\omega = 2$ rad/s.

3. (2) The time period of a physical pendulum is $T = 2\pi\sqrt{\dfrac{I_P}{Mgr}}$

Here we have three quantities I_p, m and r.



Let us calculate the quantities one by one as follows:

Finding I_P: $I_P = (I_P)_{\text{disc}} + (I_P)_{\text{ring}}$

where $(I_P)_{\text{disc}} = I_A + m(PA)^2 = \dfrac{mR^2}{2} + mR^2 = \dfrac{3mR^2}{2}$

and $(I_P)_{\text{ring}} = I_B + m(PB)^2 = mR^2 + m(3R)^2 = 10mR^2$

Then, we have $I_P = \dfrac{3}{2}mR^2 = 10\,mR^2 = \dfrac{23}{2}mR^2$

Finding r: $r = \vec{r}_C = \dfrac{m_1\vec{r}_{1C} + m_2\vec{r}_{2C}}{m_1 + m_2}$,

where $m_1 = m$, $m_2 = m$

$\vec{r}_{1C} = -R$ and $r_{2C} = -3R$

This gives $r_c = 2R$

Finding M: $M = (m)_{\text{disc}} + (m)_{\text{ring}} = m + m = 2m$

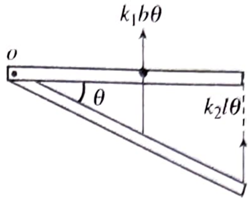
Substituting $I_P = 23/2\,mR^2$, $M = 2m$ and $r = 2R$ in the expression

$$T = 2\pi\sqrt{\frac{I_P}{Mgr}} \quad \text{we have} \quad T = \pi\sqrt{\frac{23R}{2g}}$$

After substituting the values we get $t = 2$ s.

4. (8) Applying torque equation about $\tau_0 = i_0\alpha$

$$K_1 b\theta \times b\cos\theta + K_2 l\theta \times l\cos\theta = -\frac{Id^2\theta}{dt^2}$$



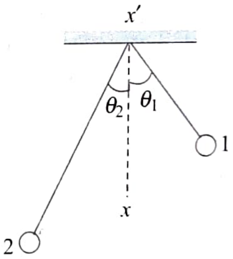
Here, $I = \dfrac{ml^2}{3}$, and as θ is small, $\cos\theta = 1$

$$\frac{ml^2 d^2\theta}{3dt^2} + (K_1 b^2 + K_2 l^2)\theta = 0$$

Hence, $\omega = \sqrt{\dfrac{3(K_1 b^2 + K_2 l^2)}{ml^2}}$

$$= \sqrt{\frac{3\left(16 \times \frac{1}{4^2} + 63 \times 1^2\right)}{3 \times 1^2}} = \sqrt{\frac{3 \times (1 + 63)}{3}} = 8 \text{ rad/s}$$

5. (1) The angular positions of pendulums 1 and 2 are (taking angles to the right of reference line xx' to be positive)



$$\theta_1 = \theta\cos\left(\frac{4\pi}{T}t\right) \quad \left(\text{where } T = 2\pi\sqrt{\frac{l}{g}}\right)$$

$$\theta_2 = -\theta\cos\left(\frac{2\pi}{T}t\right) = \cos\left(\frac{2\pi}{T}t + \pi\right)$$

For the strings to be parallel for the first time $\theta_1 = \theta_2$

$$\cos\left(\frac{4\pi}{T}t\right) = \cos\left(\frac{2\pi}{T}t + \pi\right)$$

$$\therefore \quad \frac{4\pi}{T}t = 2n\pi \pm \left(\frac{2\pi}{T}t + \pi\right)$$

for $n = 0$, $t = T/2$

for $n = 1$, $t = T/6$, $3T/2$

Both the pendulums are parallel to each other for the first time after

$$t = \frac{T}{6} = \frac{\pi}{3}\sqrt{\frac{l}{g}} = 1 \text{ s}$$

6. (1) Net increment in the tension in string connecting the block will provide acceleration to the block. We can write

$\Delta T = ma$ ...(i)

When the block m is displaced by a distance x beyond equilibrium position, the addition stretch of springs 1 and 2 are

x_1 and x_2, respectively.

We can write $x_1 = \dfrac{x - x_2}{2} \Rightarrow x = 2x_1 + x_2$ ...(ii)

As $\Delta T = k_2 x_2$ and $2\Delta T = k_1 x_1$ then $2(k_2 x_2) = k_1 x_1$

$$\Rightarrow \quad x_1 = \frac{2k_2 x_2}{k_1} \qquad \text{...(iii)}$$

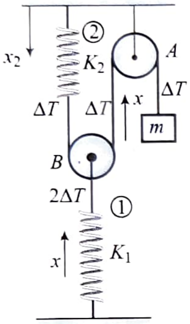
From Eqs. (ii) and (iii), $x = 2\left[\dfrac{2k_2 x_2}{k_1}\right] + x_2 = \dfrac{(4k_2 + k_1)}{k_1} x_2$

$$\Rightarrow \quad x_2 = \frac{k_1 x}{(k_1 + 4k_2)}$$

and $\vec{a} = \dfrac{-k_1 k_2}{(k_1 + 4k_2)m} \cdot x \quad \Rightarrow \quad \vec{a} = \dfrac{-k_1 k_2}{(k_1 + 4k_2)m} \cdot x$

$$\omega^2 = \frac{k_1 k_2}{(k_1 + 4k_2)m}$$

After substituting the values, we get $T = 1$ s.



7. (2) In static equilibrium of block, tension in the string is exactly equal to its weight. Let a vertically downward force F be applied on the block to pull it downward. Equilibrium is again restored when tension in the string is increased by the same amount F. Hence, total tension in the string becomes equal to $(mg + F)$.

Strings are further elongated due to the extra tension F. Due to this extra tension F in strings, tension in each spring increases by $2F$. Hence increase in elongation of springs is $2F/K_1$ $2F/K_2$ $2F/K_3$ and $2F/K_4$, respectively.

According to geometry of the arrangement, downward displacement of the block from its equilibrium position is

$$y = 2\left(\frac{2F}{K_1} + \frac{2F}{K_2} + \frac{2F}{K_3} + \frac{2F}{K_4}\right) \qquad \text{...(i)}$$

If the block is released now, it starts to accelerate upwards due to extra tension F in the strings. It means restoring force on the block is equal to F. From Eq. (i),

$$F = \frac{y}{4\left(\dfrac{1}{K_1} + \dfrac{1}{K_2} + \dfrac{1}{K_3} + \dfrac{1}{K_4}\right)}$$

Restoring acceleration of block

$$\frac{F}{m} = \frac{y}{4m\left(\dfrac{1}{K_1} + \dfrac{1}{K_2} + \dfrac{1}{K_3} + \dfrac{1}{K_4}\right)}$$

Since acceleration of block is restoring and is directly proportional to displacement y, the block performs SHM.

Its period, $T = 2\pi\sqrt{\dfrac{\text{displacement}}{\text{acceleration}}}$

$$T = 2\pi\sqrt{4m\left(\frac{1}{K_1} + \frac{1}{K_2} + \frac{1}{K_3} + \frac{1}{K_4}\right)}$$

$$T = 4\pi\sqrt{m\left(\frac{1}{K_1} + \frac{1}{K_2} + \frac{1}{K_3} + \frac{1}{K_4}\right)}$$

After substituting the values, we get $T = 2$ s.

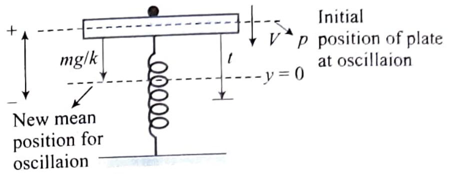
8. (5) Velocity of the particle just before collision,

$$u = \sqrt{2g \times \frac{4.5mg}{K}} \Rightarrow u = 3g\sqrt{\frac{m}{K}}$$

Now it collides with the plate.

Now just after collision velocity (V) of the system of 'plate + particle'

$$mu = 3mV \quad \Rightarrow V = \frac{u}{3} = g\sqrt{\frac{m}{K}}$$



Now the system performs SHM with time period $T = 2\pi\sqrt{3m / K}$ $\left(\omega = \sqrt{\dfrac{K}{3m}}\right)$ and mean position as mg/K distance below the point of collision.

Let the equation of motion is

$$y = A\sin(\omega t + \phi) \qquad \text{...(i)}$$

$$v = \frac{dy}{dt} = A\omega\cos(\omega t + \phi) \qquad \text{... (ii)}$$

At $t = 0$, $y = \dfrac{mg}{K}$ and $v = g\sqrt{\dfrac{m}{K}}$

From Eqs. (i) and (ii), $\dfrac{mg}{K} = A\sin\phi$ ... (iii)

$$A = \frac{2mg}{K} \qquad \text{... (iv)}$$

From Eqs. (iii) and (iv), $\phi = \dfrac{5\pi}{6}$

$$y = \frac{2mg}{K}\sin\left(\sqrt{\frac{K}{3m}}t + \frac{5\pi}{6}\right)$$

Hence equation of SHM should be

$$y = -A = -\frac{2mg}{K} = \frac{2mg}{K}\sin\left(\sqrt{\frac{K}{3m}}t + \frac{5\pi}{6}\right)$$

The plate will be at rest again when

$$= \frac{2mg}{K}\sin\left(\sqrt{\frac{K}{3m}}t + \frac{5\pi}{6}\right)$$

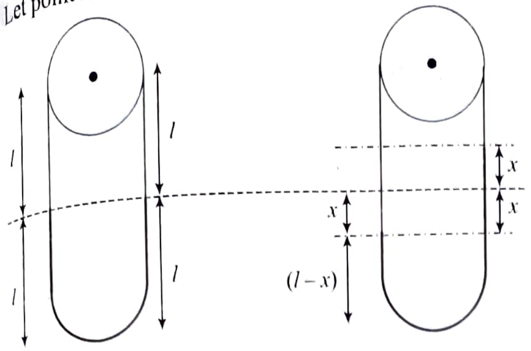
$$\Rightarrow \quad \sin\left(\sqrt{\frac{K}{3m}}t + \frac{5\pi}{6}\right) = -1 = \sin\frac{3\pi}{2}$$

$$\Rightarrow \quad \sqrt{\frac{K}{3m}}t + \frac{5\pi}{6} = \frac{3\pi}{2}$$

$$\Rightarrow \quad t = \frac{2\pi}{3}\sqrt{\frac{3m}{K}}$$

Using values, $t = \pi/5$ s.

(5) Let point B is displace in the downward direction by x.



Net imbalance forces = $m_{\text{total}} \cdot a$

$$[4mg(l+x)+mg(l-x)-\{4m(l-x)g+m(l+x)g\}]$$

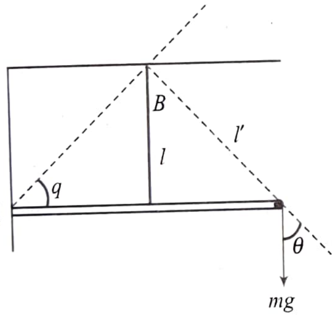
$$=(4m\times 2l+m\times 2l\,)a$$

$$a=-\left(\frac{3g}{5l}\right)x$$

$$\omega=\sqrt{\frac{3g}{5l}}\ \Rightarrow T=2\pi\sqrt{\frac{5l}{3g}}=5\text{ s}$$

(4) The bob will execute SHM about the stationary axis passing through AB. If its effective length is l' then

$$T=2\pi\sqrt{\frac{l'}{g'}}$$



$$l'=\frac{l}{\sin\theta}=\sqrt{2}\,l \qquad \text{[because } \theta=45°]$$

$$g'=g\cos\theta=\frac{g}{\sqrt{2}}$$

$$T=2\pi\sqrt{\frac{2l}{g}}=2\pi\sqrt{\frac{2\times 0.2}{10}}=\frac{2\pi}{5}\text{ s}$$

$$X=4$$

1. (6) $F=-4x+8=-4(x-2)=-4x_0$

It is equation of SHM.

Mean position is $x = 2$ m (as $F = 0$)

From $F = -4x_0$, $K = 4$ N/m

$$A=\left[\frac{2E}{K}\right]^{1/2}=\left[\frac{(2)(32)}{4}\right]^{1/2}=4\ m$$

The mean position is $x = 2$ m

So, extreme positions of the points are 6 m and −2 m. The answer is 6 m.

12. (6) $K=(2\pi f)^2\,\text{m}$

Energy of oscillations = 0.9 J

$$0.90=\frac{1}{2}KA^2$$

$$A=\left[\frac{1.8}{4\pi^2 f^2 m}\right]^{1/2}=\frac{1}{2\pi f}\left[\frac{1.8}{0.2}\right]^{1/2}=6\text{ cm}$$

13. (2) $U=2-20x+5x^2$

$$\frac{dU}{dx}=-20+10x$$

$$F=-\frac{dU}{dx}=20-10x=-10(x-2)$$

∴ The particle is executing SHM about mean position

$x-2=0$ or $x=2$

∴ $k=10$

⇒ $m\omega^2=10$

$$\omega^2=\frac{10}{m}=\frac{10}{0.1}=100$$

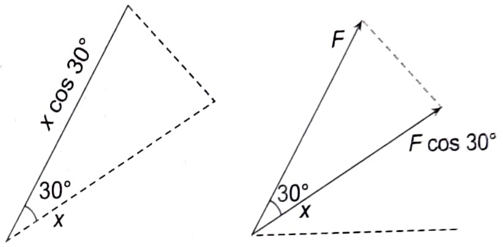
⇒ $\omega = 10\text{ rad s}^{-1}$

$$v_{\max}=A\omega=5(10)=50\text{ ms}^{-1}$$

∴ $\beta=2$

14. (1) If the sphere moves down along the plane by a distance x, extension in the spring = x cos 30°

∴ Restoring force = F cos 30°



$$=(kx\cos 30°)\cos 30°=kx\left(\frac{\sqrt{3}}{2}\right)^2=\frac{3}{4}kx$$

As it is restoring in nature, $F_R=-\frac{3}{4}kx$

$$\therefore\quad a=-\frac{3}{4}\cdot\frac{k}{M}x$$

$$\Rightarrow\quad =\sqrt{\frac{3}{4}\cdot\frac{k}{M}}=\sqrt{\frac{3}{4M}\cdot\frac{4M}{3}}=1\,\text{rads}$$

Archives

JEE Main

Single Correct Answer Type

1. (2) For a simple harmonic motion, acceleration, $a = \omega^2 x$ where ω is a constant $\omega=\frac{2\pi}{T}$.

$$\text{Hence, } a=-\frac{4\pi^2}{T^2}\cdot x\Rightarrow\frac{aT}{x}=-\frac{4\pi^2}{T}.$$

The period of oscillation T is a constant

$$\therefore\quad \frac{aT}{x}\text{ is a constant.}$$

2. (4) $T_1=2\pi\sqrt{\frac{M}{k}}$ ...(i)

When a mass m is placed on mass M, the new system is of mass = $(M + m)$ attached to the spring. New time period of oscillation

$$T_2=2\pi\sqrt{\frac{(m+M)}{k}} \qquad \text{...(ii)}$$

Consider v_1 is the velocity of mass M passing through mean position and v_2 velocity of mass $(m+M)$ passing through mean position.

Using, law of conservation of linear momentum $Mv_1 = (m+M)v_2$

$$M(A_1\omega_1) = (m+M)(A_2\omega_2)$$

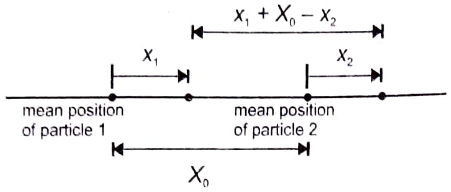
$$(\because \quad v_1 = A_1\omega_1 \text{ and } v_2 = A_2\omega_2)$$

or
$$\frac{A_1}{A_2} = \frac{(m+M)}{M}\frac{\omega_2}{\omega_1}$$

$$= \left(\frac{m+M}{M}\right)\times\frac{T_1}{T_2}\left(\because \omega_1 = \frac{2\pi}{T_1} \text{ and } \omega_2 = \frac{2\pi}{T_2}\right)$$

$$\frac{A_1}{A_2} = \sqrt{\frac{m+M}{M}} \qquad \text{[Using (i) and (ii)].}$$

3. (2) $x_1 = A\sin(\omega t + \phi_1)$, $x_2 = A\sin(\omega t + \phi_2)$



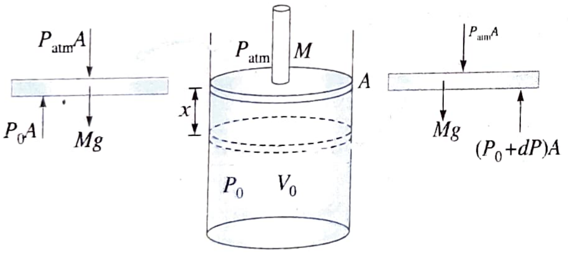
$$x_1 - x_2 = A\left[2\cos\left[\omega t + \frac{\phi_1+\phi_2}{2}\right]\sin\left[\frac{\phi_1-\phi_2}{2}\right]\right]$$

$$(x_1 - x_2 + X_0)_{\max} = X_0 + A$$

$$\Rightarrow \quad (x_1 - x_2)_{\max} = A \Rightarrow 2A\sin\left(\frac{\phi_1-\phi_2}{2}\right) = A$$

$$\Rightarrow \quad \frac{\phi_1-\phi_2}{2} = \frac{\pi}{6} \Rightarrow \phi_1 - \phi_2 = \frac{\pi}{3}$$

4. (2)



From FBD of piston at equilibrium

$$P_{atm}A + Mg = P_0A \qquad \text{...(i)}$$

From FBD of piston when piston is pushed down a distance x

$$(P_0 + dP)A - (P_{atm}A + Mg) = M\frac{d^2x}{dt^2} \qquad \text{...(ii)}$$

The system is completely isolated from its surrounding hence the change is adiabatic.

For an adiabatic process, PV^γ = constant

$$\therefore \quad V^\gamma dp + V^{\gamma-1}PdV = 0$$

or
$$dP = \frac{\gamma PdV}{V}$$

But $dV = Ax$

$$\therefore \quad dP = -\frac{\gamma P_0(Ax)}{V_0} \qquad \text{...(iii)}$$

Using (i) and (iii) in (ii), we get

$$M\frac{d^2x}{dt^2} = -\frac{\gamma P_0A^2}{V_0}x \text{ or } \frac{d^2x}{dt^2} = -\frac{\gamma P_0A^2}{MV_0}$$

Comparing it with standard equation of SHM, $\frac{d^2x}{dt^2} = -\omega^2x$

We get $\omega^2 = \frac{\gamma P_0A^2}{MV_0}$

or
$$\omega = \sqrt{\frac{\gamma P_0A^2}{MV_0}}$$

$$\text{Frequency, } f = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{\gamma P_0A^2}{MV_0}}$$

5. (2) The amplitude of a damped oscillator at a given instant of time t is given by

$$A = A_0e^{-bt/2m}$$

Where A_0 is its amplitude in the absence of damping, b is the damping constant.

As per question

After 5 s (i.e. t = 15 s), its amplitude becomes

$$0.9A_0 = A_0e^{-b(5)2m} = A_0e^{-5b/2m}$$

$$0.9 = e^{-5b/2m} \qquad \text{...(i)}$$

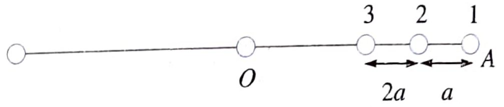
After 10 more second (i.e. t= 15 s), its amplitude becomes

$$\alpha A_0 = A_0e^{-b(15)/2m} = A_0e^{-15b/2m}$$

$$\alpha = (e^{-5b/2m})^3 = (0.9)^3 \quad \text{(Using (i))}$$

$$= 0.729$$

6. (2) As the particle starts from rest so we choose



$$x(t) = A\cos\omega t$$

At $\quad t = 0, x = A$

When $t = \tau, x = A - a$

When $t = 2\tau, x = A - 3a$

$$\therefore \quad A - a = A\cos\omega\tau \qquad \text{...(i)}$$

and $\quad A - 3a = A\cos 2\omega\tau = A(2\cos^2\omega t - 1)$

$$\Rightarrow \quad (A-3a) = A\left[2\left(\frac{A-a}{A}\right)^2 - 1\right]$$

$$\Rightarrow \quad \frac{A-3a}{A} = 2\left(\frac{A-a}{A}\right)^2 - 1$$

On solving, $A = 2a$

Now $A - a = A\cos\omega\tau$,

$$\Rightarrow \quad \cos\omega\tau = \frac{1}{2} = \cos\frac{\pi}{3} \Rightarrow \quad \omega\tau = \frac{\pi}{3}$$

$$\Rightarrow \quad \frac{2\pi}{T}\tau = \frac{\pi}{3} \Rightarrow T = 6\tau$$

7. (2) KE = $\frac{1}{2}m\omega^2(A^2 - d^2)$

PE = $\frac{1}{2}m\omega^2d^2$

At $d = \pm A$,

PE = maximum while KE = 0.

8. (4) Velocity at any position is given by, $v = \omega\sqrt{A^2 - x^2}$

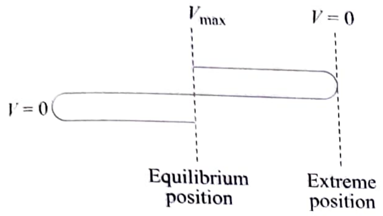
at $x = \frac{2A}{3}$, $v = \omega\sqrt{A^2 - \frac{4A^2}{9}} \Rightarrow v = \frac{\sqrt{5}}{3}A\omega$

If speed at $x = \frac{2A}{3}$ is tripled, i.e. $v' = \sqrt{5}\,A\omega$

So at new amplitude A' is $v' = \omega\sqrt{A'^2 - \left(\frac{2A}{3}\right)^2}$

$$\sqrt{5}\,A\omega = \omega\sqrt{A'^2 - \frac{4A^2}{9}} \Rightarrow A' = \frac{7A}{3}$$

(2) Velocity of a particle at equilibrium position should be maximum and amplitude position should be zero



Time taken to reach the extreme position from equilibrium position is $\frac{T}{4}$. Velocity is maximum at equilibrium position and zero at extreme position.

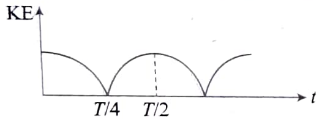
$$v = A\omega \cos \omega t$$

Hence kinetic energy, $\mathrm{KE} = \frac{1}{2}mv^2$

(m is the mass of particle and v is the velocity of particle

$$\Rightarrow \quad \mathrm{KE} = \frac{1}{2}mA^2\omega^2\cos^2\omega t$$

Hence graph of KE v/s time is square cos function



E Advanced

gle Correct Answer Type

1. (4) The given motion is represented by $\left(\omega = \frac{2\pi}{T} = \frac{2\pi}{8} = \frac{\pi}{4}\right)$

$$x = 1\sin\left(\frac{\pi}{4}t\right), \quad \frac{d^2x}{dt^2} = \frac{-\pi^2}{16}\sin(\pi/4)t$$

$$\text{At} \quad t = 4/3\text{ s}, \frac{d^2x}{dt^2} = -\frac{\sqrt{3}}{32}\pi^2\ \text{cm/s}^2$$

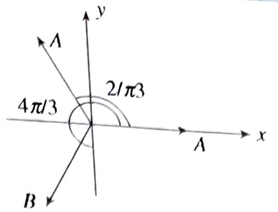
2. (3) Restoring torque $= -2\times k\left(\frac{l}{2}\theta\right)\frac{l}{2} = \frac{Id^2\theta}{dt^2}$

$$\frac{d^2\theta}{dt^2} = \frac{\frac{kl^2}{2}(-\theta)}{\frac{Ml^2}{12}}$$

$$\Rightarrow \quad \omega = \sqrt{\frac{6k}{M}}$$

$$f = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{6k}{m}}$$

3. (2) So, $B = A$, $\phi = 240° = \frac{4\pi}{3}$



4. (1) Time of flight for projectile

$$T = \frac{2u\sin\theta}{g} = 1\text{ s}$$

$$\frac{2u\sin 45}{g} = 1\text{ s}$$

$$u = \frac{g}{\sqrt{2}}$$

$$u = \sqrt{50}\ \text{m/s}$$

Multiple Correct Answers Type

1. (1), (4)

Torque is same for both the cases.

$$T = 2\pi\sqrt{\frac{I}{(M+m)gd}}$$

$$I_A = \frac{mL^2}{3} + \left(\frac{MR^2}{2} + ML^2\right)$$

When the disc is free to rotate about its centre, it will not rotate w.r.t. ground. It will act as a point mass, so moment of inertia is given by

$$I_B = \frac{mL^2}{3} + ML^2$$

$$I_A > I_B \Rightarrow T_A > T_B \Rightarrow \omega_A < \omega_B$$

2. (1), (4)

$v = u_0 \sin\omega t$ (suppose t_{01} is the time of collision)

$$\frac{u_0}{2} = u_0\cos\omega t_1 \Rightarrow t_1 = \frac{\pi}{3\omega}$$

Now the particle returns to equilibrium position at time $t_2 = 2t_1$, i.e., $\frac{2\pi}{3\omega}$ with the same mechanical energy, i.e., its speed will be u_0. Let t be the time at which the particle passes through the equilibrium position for the second time.

Energy of particle and spring remains conserved.

$$t_3 = \frac{T}{2} + 2t_1 = \frac{\pi}{\omega} + \frac{2\pi}{3\omega} = \frac{5\pi}{3\omega} = \frac{5\pi}{3}\sqrt{\frac{m}{k}}$$

3. (2), (4)

$$x = A\sin(\omega t)$$

$$V = A\omega\cos(\omega t)$$

$$p_1 = mA_1\omega_1 = b,\ A_1 = a\ \text{(From ellipse)}$$

$$p_2 = mA_2\omega_2 = R,\ A_2 = R\ \text{(From circle)}$$

Also, $\frac{a}{b} = n^2$ and $\frac{a}{R} = n \Rightarrow \frac{R}{b} = n$

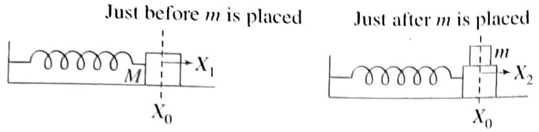
$$E_1 = \frac{p_1^2}{2m} \Rightarrow E_2 = \frac{p_2^2}{2m}$$

$$\frac{E_1}{E_2} = \left(\frac{b}{R}\right)^2 = \frac{1}{n^2} \Rightarrow \frac{m\omega_1^2A_1^2}{m\omega_2^2A_2^2} = \frac{1}{n^2}$$

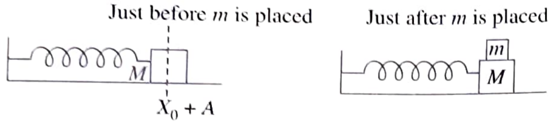
$$\Rightarrow \quad \frac{\omega_1}{\omega_2} = \frac{1}{n^2} \Rightarrow \frac{E_1}{\omega_1} = \frac{E_2}{\omega_2}$$

4. (1), (2), (4)

Case 1


Just before m is placed
Just after m is placed

Case 2


Just before m is placed
Just after m is placed

In case 1

$MV_1 = (M+m)V_2$

$$V_2 = \left(\frac{M}{M+m}\right)V_1$$

$$\sqrt{\frac{k}{M+m}}A_2 = \left(\frac{M}{M+m}\right)\sqrt{\frac{k}{M}}A_1$$

$$A_2 = \sqrt{\frac{M}{M+m}}A_1$$

In case 2

$A_2 = A_1$

$$T = 2\pi\sqrt{\frac{M+m}{k}} \text{ in both cases.}$$

Total energy decreases in first case whereas remain same in 2nd case. Instantaneous speed at x_0 decreases in both cases.

Linked Comprehension Type

For Problems 1–3

1. **(3)** Energy must be less than V_0, so that KE becomes zero before PE becomes maximum and particle returns back.
2. **(2)** Dimension of α can be found as

$[\alpha] = ML^{-2}T^{-2}$

Only option (2) has dimension of time

Alternatively:

From conservation of energy:

$$\frac{1}{2}m\left(\frac{dx}{dt}\right)^2 + \alpha x^4 = \alpha A^4$$

$$\left(\frac{dx}{dt}\right)^2 = \frac{2\alpha}{m}(A^4 - x^4)$$

$$\int dt = \sqrt{\frac{m}{2\alpha}}\int_0^A \frac{dx}{\sqrt{A^4 - x^4}}$$

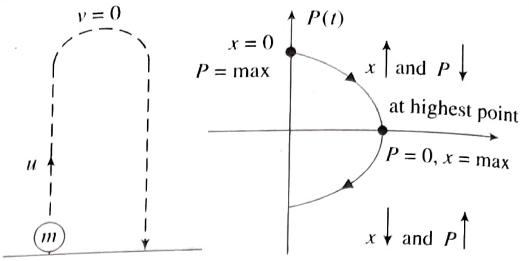
$$\Rightarrow t = \frac{1}{A}\sqrt{\frac{m}{2\alpha}}\int_0^1 \frac{du}{\sqrt{1-u^4}} \qquad [\text{Substitute } x = Au]$$

$$\Rightarrow t \propto \frac{1}{A}\sqrt{\frac{m}{2\alpha}}$$

3. **(4)** As potential energy is constant for $|x| > X_0$, the force on the particle is zero hence acceleration is zero.

For Problems 4–6

4. **(4)**



5. **(3)** In the 1st case amplitude of SHM is a.
In the 2nd case amplitude of SHM is $2a$.
Total energy = 1/2 k (amplitude)^2

$$E_1 = \frac{1}{2}k(2a)^2 \quad \Rightarrow \quad E_2 = \frac{1}{2}k(a)^2$$

$$E_1 = 4E_2$$

6. **(2)** Amplitude of oscillation inside liquid will decrease due to viscous force, So radius of circular arcs will decrease as position change. Let initially mass is taken to maximum upward position and released, then its initial momentum is zero and then afterwards momentum will become negative first which is correctly shown in option (2).

Numerical Value Type

1. **(4)** When a wire of length L, area of cross-section A, Young's modulus Y is stretched by suspending a mass m, then the mass performs simple harmonic motion with angular frequency

$$\omega = \sqrt{\frac{YA}{mL}}$$

Substituting the given values, we get

$$140 = \sqrt{\frac{n \times 10^9 \times 4.9 \times 10^{-7}}{0.1 \times 1}}$$

or $$140 \times 140 = \frac{n \times 10^9 \times 4.9 \times 10^{-7}}{0.1 \times 1}$$

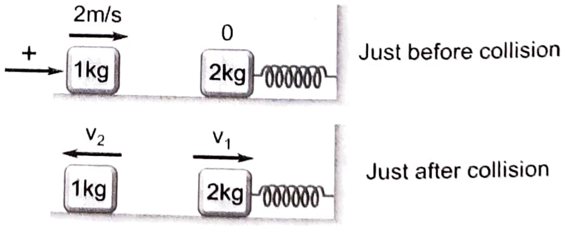
$$\Rightarrow \quad n = \frac{14 \times 14 \times 10^2}{49 \times 10^2} = 4$$

2. **(2.09)**

Time period of oscillation of spring pendulum

$$T = 2\pi\sqrt{\frac{m}{k}} = 2\pi \text{ sec}$$

Block connected with spring returns to original position in $T/2 = \pi$ sec



Conservation of linear momentum $1.2 + 2.0 = -1 \cdot v_2 + 2 \cdot v_1$

or $\quad 2v_1 - v_2 = 2$ ...(i)

Coefficient of restitution equation $e = 1 = 2 - 0$

or $\quad v_1 + v_2 = 2$ ...(ii)

From (i) and (ii) $v_1 = \frac{4}{3}$ m/sec and $v_2 = \frac{2}{3}$ m/sec

$$d = \frac{2}{3}(\pi) = \frac{2}{3}(3.14) = 2.0933 \text{ m}$$

$$d = 2.09 \text{ m}$$

Chapter 6

Concept Application Exercises

Exercise 6.1

1. Transverse waves require 'rigidity' in the medium. Solids have rigidity but fluids do not. However, in a liquid surface, the surface tension provides some rigidity and hence, partially transverse waves are possible on a liquid surface, e.g., ripples.
2. The wavefront at a large distance from any source is a plane wavefront, so one can create a plane wavefront by keeping a source at a large distance. One can produced a spherical wavefront by using a point source.
3. No, an oscillation is not a wave. The term 'wave' implies the transfer of energy through successive vibration of the particles of the medium. So the oscillations of a body do not constitute a 'wave'.
4. ABC represent compression (negative slope region) and CDE represents rarefaction (positive slope region).
5. A longitudinal wave can be set up in a stretched spring by compressing the coils in a small region, and releasing the compressed region. The disturbance will proceed to propagate as a longitudinal pulse. It is quite possible to set up a transverse wave in a spring, simply by displacing a section of the spring in a direction perpendicular to its length and releasing it.
6. Given, speed of wave, $v = 30$ cm/s

   And frequency of wave, $f = 20$ Hz

   $$\therefore \quad \text{Wavelength of wave, } \lambda = \frac{v}{f} = \frac{30}{20} = 1.5 \text{ cm}$$

   Thus the separation between the consecutive compression = 1.5 cm.
7. The frequency of the note produced by the whistle is not equal to 1/20 or 0.05 Hz, it is only the frequency of pulse repetition.
8. (a) On the same wavefront phase difference = 0.

   Linear distance between the points does not matter.

   (b) Between two successive crests, path difference = λ.

   $\therefore$ Phase difference = 2π radian

   (c) Again, path difference between two successive troughs = λ

   $\therefore$ Phase difference between A and $B = 2\pi$ radian.
9. Transverse waves travel in the form of crests and troughs involving change in shape of the medium. As liquid and gases do not possess the elasticity of shape, therefore mechanical transverse waves cannot be produced in liquids and gases.

Exercise 6.2

1. The amplitude of wave velocity is given by the maximum particle displacement y_{max} which, evidently, occurs when

   $$(x - ct)^2 = 0, \quad \text{i.e.,} \quad y_{\text{max}} = \text{amplitude} = b/a\sqrt{a^2} = b.$$

   The combination $(x - ct)$, appearing in the wave equation, when compared with the expression $x - vt$, reveals that, the wave velocity is $v = c$.

   Differentiating the displacement equation with respect to time t, we get,

   $$\frac{dy}{dt} = \frac{b}{2a}\frac{1}{\sqrt{a^2 - (x - ct)^2}}[-2(x - ct)](-c) = \frac{bc(x - ct)}{a\sqrt{a^2 - (x - ct)^2}}$$

   Putting $x = a/2$ and $t = 0$; we get

   $$\text{Initial particle velocity } \frac{dy}{dt} = \frac{bc(a/2)}{a\sqrt{a^2 - (a^2/4)}} = \frac{bc\sqrt{3}}{3a}$$

2. (a) Differentiating expression (i) twice w.r.t. t we have $\frac{\partial^2 y}{\partial t^2} = 2v^2$

   And differentiating expression (i) twice w.r.t. x we have,

   $$\frac{\partial^2 y}{\partial x^2} = 2$$

   $$\text{Clearly, } \frac{\partial^2 y}{\partial t^2} = v^2\left(\frac{\partial^2 y}{\partial x^2}\right)$$

   Thus expression (a) is a solution of the one dimensional wave equation.

   Similarly, treatment can be done for (b), (c) and (d).

   Note: The expression (d) satisfies differential equation of wave, but it does not represent progressive wave. It represents stationary wave.

   (e) Differentiating expression (e) twice w.r.t. t we have

   $$\frac{\partial^2 y}{\partial t^2} = -2v^2$$

   And differentiating expression (e) twice wrt x, we have

   $$\frac{\partial^2 y}{\partial x^2} = 2$$

   Clearly $\frac{\partial^2 y}{\partial t^2} \neq v^2 \frac{\partial^2 y}{\partial x^2}$, so the expression (e) is not a solution of the one dimensional wave equation.

   (f) Differentiating expression (f) twice w.r.t. t, we have

   $$\frac{\partial^2 y}{\partial t^2} = -v^2 \sin 2x \cos vt = -v^2 y$$

   $$\frac{\partial^2 y}{\partial x^2} = -4 \sin 2x \cos vt = -4y$$

   Clearly $\frac{\partial^2 y}{\partial t^2} \neq v^2 \frac{\partial^2 y}{\partial x^2}$ and therefore the expression is not

   solution of the one dimensional wave equation.

3. (a) The wave equation may be written as $y = y_0 e^{-\frac{1}{T^2}\left(t - \frac{x}{\lambda/T}\right)^2}$.

   Comparing with the general equation $y = f(t - x/v)$, we see tha

   $$v = \frac{\lambda}{T} = \frac{4 \text{ cm}}{1.0 \text{ s}} = 4 \text{cm/s}$$

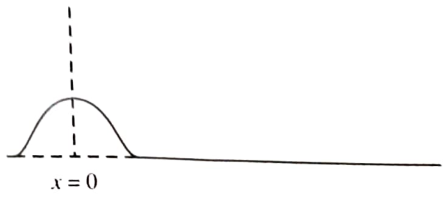
   (b) Putting $x = 0$ in the given equation,

   $$f(t) = y_0 e^{-(t/T)^2} \qquad \ldots$$

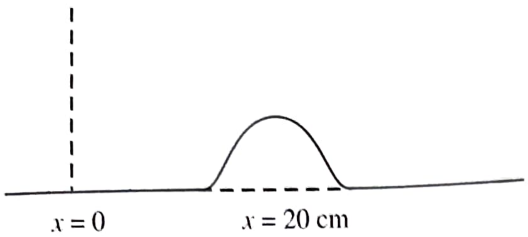
   (c) Putting $t = 0$ in the given equation

   $$g(x) = y_0 e^{-(x/\lambda)^2} \qquad \ldots$$

   (d)

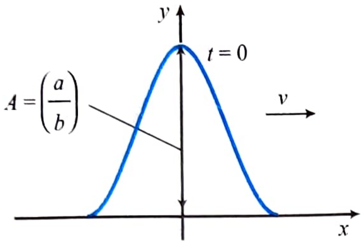


(e)



4. A wave pulse is a disturbance localized only in a small part of space at a given instant (as shown in figure) and its shape does not change during propagation. Though a pulse can be represented by exponential or trigonometric functions also, it is usually expressed by the form

$$y = \frac{a}{b + (x \pm vt)^2}$$



Comparing the above with the given pulse we find that

$$f(x \pm vt) = (x + 2t)^2$$

i.e., the pulse is travelling along negative x-axis with velocity 2 m/s. Further amplitude is the maximum value of wave function which will be when

$$(x + 2t)^2 = 0$$

So, $A = y_{\max} = \frac{10}{5} = 2$

5. $y = \frac{0.8}{(3x^2 + 12xt + 12t^2 + 4)} = \frac{0.8}{3[x^2 + 4xt + (2t)^2] + 4}$

$$= \frac{0.8}{[4 + 3(x + 2t)^2]}$$

On solving, we get

$$x + 2t = x + vt$$

$$v = 2 \text{ m/s}$$

Amplitude is $y_{\max}$: when $x + 2t = 0$

Therefore, $y_{\max} = \frac{0.8}{4} = 0.2\,\text{m}$

6. Particle speed $v_p = \left|\frac{\partial y}{\partial t}\right|$

$$v_p = \left|5 \times \frac{1}{0.04} \cos\left(\frac{t}{0.04} - \frac{x}{4}\right)\right| \text{ms}^{-1}$$

The maximum particle speed is, thus,

$$\left|\frac{\partial y}{\partial t}\right|_{\max} = \frac{5}{0.04} \text{ms}^{-1} = 125 \text{ ms}^{-1}$$

7. By taking partial derivatives of this function w.r.t. x and t

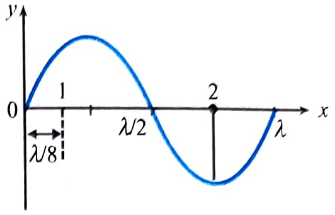
$$\frac{\partial^2 y}{\partial x^2} = \frac{12(x - 3t)^2 - 4}{[(x - 3t)^2 + 1]^3}, \text{ and } \frac{\partial^2 y}{\partial t^2} = \frac{108(x - 3t)^2 - 36}{[(x - 3t)^2 + 1]^3}$$

or $$\frac{\partial^2 y}{\partial x^2} = \frac{1}{9}\frac{\partial^2 y}{\partial t^2}$$

Comparing with linear wave equation, we see that the wave function is a solution to the linear wave equation if the speed at which the pulse moves is 3 cm/s. It is apparent from wave function therefore it is a solution to the linear wave equation.

Exercise 6.3

1. The distance between the particles



$$\Delta x = \left(\frac{\lambda}{2} - \frac{\lambda}{8}\right) + \frac{\lambda}{4} = \frac{5\lambda}{8}$$

$$\Delta\phi = \frac{2\pi}{\lambda}\Delta x = \frac{2\pi}{\lambda} \times \frac{5\lambda}{8} = \frac{5\pi}{4}$$

2. Given $y = 2.0 \cos(10t - 0.0080x + 0.35)$.

The standard equation of travelling harmonic wave can be written as $y = A\cos(\omega t - kx + \phi)$.

On comparing two equations, we have $\omega = 10$ rad/s

and $k = 0.0080 \text{ cm}^{-1} = 0.80 \text{ m}^{-1}$

$$\lambda = \frac{2\pi}{k} = \frac{2}{0.80}\text{m}$$

Phase difference $\Delta\phi = \frac{2\pi}{\lambda} \times \Delta x$

(a) When $\Delta x = 4$ m, $\Delta\phi = \frac{2\pi}{(2\pi/0.80)} \times 4 = 3.2$ rad

(b) When $\Delta x = 0.5$ m, $\Delta\phi = \frac{2\pi}{(2\pi/0.80)} \times 0.5 = 0.40$ rad

(c) When $\Delta x = \frac{\lambda}{2}$, $\Delta\phi = \frac{2\pi}{\lambda} \times \frac{\lambda}{2} = \pi$ rad

(d) When $\Delta x = \frac{3\lambda}{4}$, $\Delta\phi = \frac{2\pi}{\lambda} \times \frac{3\lambda}{4} = \frac{3\pi}{2}$ rad

3. The standard equation of a plane progressive wave is $y = A\sin(kx - \omega t)$, the phase of the wave is $(kx - \omega t)$.

(a) Comparing $4x - 8t$ with $(kx - \omega t)$, we get

$$\omega = 8 \quad \text{and} \quad k = 4$$

$$\therefore \quad \lambda_1 = \frac{2\pi}{k} = \frac{2\pi}{4} = \frac{\pi}{2}$$

(b) Comparing $8x - 16t$ with $(kx - \omega t)$, we get $\omega = 16$ and $k = 8$

$$\therefore \quad \lambda_2 = \frac{2\pi}{k} = \frac{2\pi}{8} = \frac{\pi}{4}$$

Clearly $\lambda_1 = 2\lambda_2$

Therefore, snapshots 1 and 2 correspond to a and b respectively.

4 The given equation can be written as

$$y = (2.15 \times 10^{-3}\text{cm})\sin 2\pi\left[50(t) - \frac{\pi}{600}\right]$$

Comparing the above equation with the standard equation,

$$y = A\sin 2\pi\left(\frac{t}{T} - \frac{x}{\lambda}\right)$$

We get amplitude $A = 2.15 \times 10^{-3}$ cm

Frequency $f = \frac{1}{T} = 50\ \text{s}^{-1}$

[Coefficient of t in the displacement equation after removing 2π]

Wavelength λ = 600 cm and wave velocity

$$v = f\lambda = 600 \times 50\ \text{cm}^{-1} = 300\ \text{ms}^{-1}$$

The equation of the harmonic wave is $y = A\sin(\omega t - kx + \phi)$

Where $t = 0,\ x = 0$ and $y = -A$

Or $\sin\phi = -1$ or $\phi = -\frac{\pi}{2}$

Then, $y = -A\cos(\omega t - kx)$,

Where, $A = 0.02\ m,\ \omega = v \times k = v \times \frac{(2\pi)}{\lambda} = 5 \times \frac{2\pi}{1.25}$

$$= 8\pi, \text{and } k = \frac{2\pi}{\lambda} = \frac{2\pi}{1.25} = 1.6\pi$$

Hence, $y = 0.02\cos(8\pi t - 1.6\pi x)$.

The wavelength of a wave is given by $\lambda = \frac{v}{T} = \frac{360}{500} = 0.72\,\text{m}$

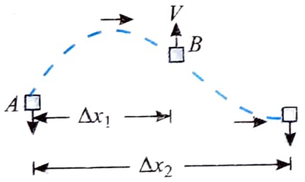
The phase difference is given by $\Delta\phi = \frac{2\pi}{\lambda}\Delta x$

Here $\Delta x = 12\ \text{cm} = 0.12\,\text{m}$

$$\Delta\phi = \frac{2\pi}{(0.72)}(0.12) = \frac{\pi}{3}$$

(a) If the corks have equal KE, the speed of the points A and B must be same (they both may move in up or down or one may move up and other may move down, does not matter).

Then, $\Delta x_1 = \left(n - \frac{1}{2}\right)\frac{\lambda}{2}$ of the points A and B move opposite.



or $\Delta x_1 = \frac{\lambda}{2}, \frac{3\lambda}{2}, \frac{5\lambda}{2}$ etc

$\Delta x_2 = n\lambda$ if the points A and B move in same direction

or $\Delta x_2 = \lambda, 2\lambda, 3\lambda$ etc

Combining, $\Delta x = \frac{\lambda}{2}, \lambda, \frac{3\lambda}{2}, 2\lambda, \frac{5\lambda}{2}, 3\lambda$ etc

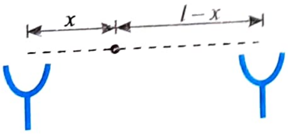
For the corks to move with same KE.

(b) For same momenta, the velocities (but not the speeds) must be same. Then the corks at A at B must move in same direction.

Hence, $\Delta x = n\lambda$

or $\Delta x = \lambda, 2\lambda, 3\lambda$ etc

8. The phase difference is $\Delta\phi = \phi_1 - \phi_2$



$$\Delta\phi = \omega\left(t - \frac{x}{v}\right) - \omega\left(t + \frac{l - x}{v}\right) = \frac{\omega(l - 2x)}{v}$$

$$= (l - 2x)\frac{2\pi f}{v} = \left(l - 2 \times \frac{l}{4}\right)\left(\frac{2\pi \times 150}{10}\right) = 15\pi l$$

9. A travelling wave in the positive x-direction is represented by

$$y(x,t) = A\sin(\omega t - kx) \qquad \text{...(i)}$$

The phase of the wave is given by, $\phi = (\omega t - kx)$

The phase difference is $\Delta\phi = \omega\Delta t - k\Delta x$

(a) The phase difference is $\Delta\phi = \omega\Delta t - k\Delta x = k\Delta x$

$$\Delta\phi = \frac{2\pi}{\lambda}\Delta x = \frac{2\pi}{1} \times \frac{1}{4} = \frac{\pi}{2}\text{rad}$$

(b) $\Delta\phi = \omega\Delta t - k\Delta x$

$$\Delta\phi = \omega\Delta t - k\Delta x = \frac{2\pi}{10}(1 - 0) - \frac{2\pi}{1}\left(-\frac{1}{4}\right) = \frac{\pi}{5} + \frac{\pi}{2} = +\frac{7\pi}{10}\text{rad}$$

10. The given equation $y = 0.20\sin 2\pi\left(60t - \frac{x}{5}\right)$

when compared with the standard one, $y = A\sin 2\pi\left(ft - \frac{x}{\lambda}\right)$

where the symbols have their usual meaning, we get the frequency as 60 s^{-1}, so that the time period $T = 1/60$ s.

(a) From, phase difference $\Delta\theta = \left(\frac{2\pi}{\lambda}\right)\Delta x$

$$\Delta\theta = \left(\frac{125\ \text{cm}}{500\,\text{cm}}\right)2\pi = \frac{\pi}{2}$$

(b) From, phase difference $\Delta\theta = \left(\frac{2\pi}{T}\right)\Delta t$

$$\Delta\theta = \frac{\left(\frac{1}{40} - \frac{1}{120}\right)\text{s}}{\frac{1}{60}\text{s}} \times 2\pi = 2\pi$$

(or effectively, no phase difference)

(c) The phase of a vibrating particle is given by

$$\theta = 2\pi\left(60t - \frac{x}{5}\right) \qquad \text{...(i)}$$

$\therefore$ For $x = 10$ m; $\theta_1 = 2\pi\left(60t - \frac{10}{5}\right)$ ...(ii)

$\therefore$ The phase difference between any particle at position x and at $x = 10$ m will be $\theta - \theta_1 = \frac{2\pi}{5}(10 - x)$

From the problem $\theta - \theta_1 = \frac{\pi}{3} + 2n\pi$ where $n \in I$

$$\frac{2\pi}{5}(10 - x) = \left(\frac{\pi}{3} + 2n\pi\right)$$

$$10 - x = \pm\left(\frac{5}{6} + 5n\right)$$

$$x = 10 \pm \left(5n + \frac{5}{6}\right) \text{;where, n = 0, 1, 2, etc,....}$$

Exercise 6.4

1. (a) Given equation is $y = 5\sin(4.0t - 0.02x)$

Comparing this with standard equation $y = A\sin(\omega t - kx)$, we get

Amplitude $A = 5$cm

$$\omega = 4.0\ \text{s}^{-1} \quad \text{and} \quad k = 0.02\ \text{cm}^{-1}$$

$$\therefore \quad \text{Frequency } f = \frac{\omega}{2\pi} = \frac{4.0}{2\pi} = \frac{2}{\pi} = 0.637\ \text{s}^{-1}$$

Velocity $v = \frac{\omega}{k} = \frac{4.0}{0.02} = 200$ cm/s

Wavelength $\lambda = \frac{v}{f} = \frac{200}{2/\pi} = 100\pi$ cm

(b) The maximum velocity, $u_{\max} = \omega A = 4 \times 5 = 20$ cm/s.

2. Comparing the equation with the standard form of the wave equation

$y = a\sin(\omega t - kx)$, where $\omega = 2\pi f$ and $k = 2\pi/\lambda$

Given that

(a) $\omega = 500\pi$ or $f = 250$ Hz

and $\frac{2\pi}{\lambda} = \frac{500\pi}{30}$ or $\lambda = \frac{3}{25}\,m = 0.12$ m

(b) $\therefore\ C = f\lambda$ or $C = 250 \times \frac{3}{25} = 30$ m/s

(c) v (particle velocity) $= \frac{dy}{dt} = a\omega\cos(\omega t - kx)$

$\therefore\ v_{\max} = a\omega = 0.01 \times 500\pi = 15.7$ m/s

(d) f (particle acceleration)

$$= \frac{dv}{dt} = -a\omega^2\sin(\omega t - kx)$$

$\therefore\ f_{\max} = a\omega^2 = 0.01 \times (500\pi)^2 = 2.47 \times 10^4$ m/s^2

3. Let equation of wave is $y = A\sin(\omega t - kx + \phi)$

$$v_p = \frac{dy}{dt} = A\omega\cos(\omega t - kx + \phi)$$

at $x = 0, t = 0$, we have $y = 0$, $v_p > 0$

$\Rightarrow$ $0 = A\sin\phi \Rightarrow \phi = 0$ or π

But $v_p > 0$, so $\phi = 0$

at $t = 0, x = 0.090$ m

$y = A = 4$ mm $\rightarrow$ amplitude

$A = A\sin(-k(0.090))$

$\Rightarrow$ $-k(0.090) = 2n\pi + \frac{\pi}{2}$

$\Rightarrow$ $-\frac{2\pi}{\lambda}(0.090) = 2n\pi + \frac{\pi}{2}$

$\Rightarrow$ $\lambda = \frac{-0.36}{4n+1}$

for $n = -1$, $\lambda = 0.12$ m

from graph, time period $= T = 0.04$ s

Wave speed: $v = \frac{\lambda}{T} = \frac{0.12}{0.04} = 3$ m/s

4. $\phi = \pi/6$

$$v = \sqrt{T/\mu} = \sqrt{\frac{900}{0.01}} = 300 \text{ m/s}$$

frequency = 150 Hz

$\therefore$ $\lambda = \frac{v}{f} = \frac{300}{150} = 2$m

Then equation will be

$$y = A\sin\left\{2\pi\left(\frac{t}{T} - \frac{x}{\lambda}\right) + \phi\right\} = 5\sin\left\{2\pi\left(150t - \frac{x}{2}\right) + \frac{\pi}{6}\right\}$$

$$= 5\sin\left\{\pi(300t - x) + \frac{\pi}{6}\right\}$$

5. As elevator accelerates up at 2 m/s^2, tension in the string is $T = m(g + a) = 4(10 + 2) = 48$ N.

Linear mass density of string is $\mu = 192 \times 10^{-3}$ kg/m

Speed of transverse waves on string is

$$v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{48}{19.2 \times 10^{-3}}} = 50 \text{ m/s}$$

6. Tension in string AB is $T_{AB} = 6.4$ g $= 64$ N

Thus speed of transverse waves in string AB is

$$v_{AB} = \sqrt{\frac{T_{AB}}{\mu_{AB}}} = \sqrt{\frac{64}{10 \times 10^{-3}}} = \sqrt{6400} = 80 \text{ m/s}$$

Tension in string CD is $T = 3.2$ g $= 32$ N

Thus speed of transverse waves in string CD is

$$v_{CD} = \sqrt{\frac{T_{CD}}{\mu_{DC}}} = \sqrt{\frac{32}{8 \times 10^{-3}}} = \sqrt{4000} = 63.24 \text{ m/s}$$

7. (a) From given curve: Amplitude of the wave $A = 10$ cm and wavelength $\lambda = 12$ cm

$$y = A\sin(kx + \omega t + \phi) = A\sin\left(\frac{2\pi}{\lambda}x + \frac{2\pi t}{T} + \phi\right) \quad \text{...(i)}$$

At $t = 0$, $y = A\sin(kx + \phi) = 10\sin\left(\frac{\pi}{6}x + \phi\right)$

At $x = 0$, $y = -5$ cm $= 10\sin\phi$

$$\sin\phi = -\frac{1}{2} \Rightarrow \phi = -\frac{\pi}{6}$$

Hence $y = 0.10(\text{m})\sin\left(\frac{\pi}{6}x - \frac{\pi}{6}\right)$

(b) $\tan\theta = \frac{dy}{dx} = 0.10 \times \frac{\pi}{6}\cos\left(\frac{\pi}{6}x - \frac{\pi}{6}\right)$

At $x = 6$ m

$$\frac{dy}{dx} = \tan\theta = \frac{\pi}{60}\cos\left(\pi - \frac{\pi}{6}\right) = \frac{\pi}{60}\left(-\cos\frac{\pi}{6}\right)$$

$$= -\frac{\pi}{60}\cdot\frac{\sqrt{3}}{2} = \frac{-\pi\sqrt{3}}{120}$$

$$\theta = \tan^{-1}\left(-\frac{\pi\sqrt{3}}{120}\right)$$

(c) Particle velocity $v_P = -v\left(\frac{dy}{dx}\right)$

$$-v_P = -v\left(-\frac{\pi\sqrt{3}}{120}\right)$$

$$v = \frac{-120}{\sqrt{3}\pi}v_p = \frac{-40\sqrt{3}}{\pi}v_p$$

Along negative x-axis.

(d) Wave velocity $v = f\lambda$

$$\Rightarrow \quad f = \frac{v}{\lambda} = \frac{40\sqrt{3}\,v_p}{\pi \times 12} = \frac{10\sqrt{3}}{3\pi}v_p\,\text{s}^{-1}$$

(e) Hence displacement equation of medium particle at origin at $x = 0$, from Eq. (ii)

$$y = 10(\text{cm})\sin\left[\frac{20\sqrt{3}}{3}v_p t - \frac{\pi}{6}\right]$$

(f) As wave is travelling towards negative x-direction hence positive sign should be used between kx and ωt in Eq. (i)

The equation of the wave,

$$y = 0.1(\text{m}) \sin\left[\frac{\pi}{6}x(\text{m}) + \omega t - \frac{\pi}{6}\right]$$

Here $\omega = 2\pi f = 2\pi \frac{10\sqrt{3}}{3\pi} v_p = \frac{20\sqrt{3}}{3} v_p$

Hence displacement equation of the wave

$$y = 0.1(\text{m})\sin\left[\frac{\pi}{6}x(\text{m}) + \frac{20\sqrt{3}}{3} v_p t - \frac{\pi}{6}\right] \qquad \text{...(ii)}$$

The displacement equation for a particle is given by

$$y = 10 \sin \pi(0.01\,x - 2.00\,t) \text{ cm} \qquad \text{...(i)}$$

(a) Particle velocity is given by

$$\frac{dy}{dt} = -20.0\pi \cos(0.01x - 2.00t) \text{ cm/s} \qquad \text{...(ii)}$$

Putting $x = 200$ cm and $t = 5/6$ s, we get

$$\frac{dy}{dt} = -20.0\pi \cos \pi\left(2 - \frac{5}{3}\right) \text{cm/s} = -10.0\pi \text{ cm/s}$$

Also, the velocity amplitude $\left.\frac{dy}{dt}\right]_{\max} = 20.0\,\pi$ cm/s

(corresponding to $|\cos \pi\,(0.01\,x - 2.00\,t)| = 1$

(b) Differentiating Eq. (ii) w.r.t. time t, we get the particle acceleration as

$$\frac{d^2y}{dt^2} = -40.0\pi^2 \sin \pi\ (0.01\pi - 2.00t)\text{cm/s}^2$$

Putting $x = 100$ cm and $t = 1/4$ s, we get

$$\frac{d^2y}{dt^2} = -40.0\pi^2 \sin \pi\left(1 - \frac{1}{2}\right)\text{cm/s}^2 = -40.0\ \pi^2 \text{ cm/s}^2$$

Also, the acceleration amplitude $\left.\frac{d^2y}{dt^2}\right]_{\max} = 40.0\pi^2$ cm/s^2

(corresponding to $|\sin \pi\ (0.01\,x - 2.00\,t)| = 1$

Given $A = 5$ cm, $\lambda = 2$ m and $v = 330$ ms^{-1}

Since frequency $f = \frac{v}{\lambda} \Rightarrow f = \frac{330\,\text{ms}^{-1}}{2\,\text{ms}^{-1}} = 165$ Hz

Now, the equation of a plane progressive, wave in its standard form can be written as,

$$y = A\sin 2\pi\left(\frac{t}{T} - \frac{x}{\lambda}\right) = A\sin 2\pi\left(ft - \frac{x}{\lambda}\right)$$

$$y = 5\text{cm} \sin 2\pi\left[165t - \frac{x}{200}\right]$$

where t is in second and x in cm.

. Given $A = 7$ cm; $v = 8$ cms^{-1} and $f = 50$ s^{-1}

$$\lambda = \frac{v}{f} = \frac{80\,\text{ms}^{-1}}{50\text{s}^{-1}} = \frac{8}{5}\text{cm}$$

The equation of wave can be written as $y = A\sin\frac{2\pi}{\lambda}(vt - x)$

$$\Rightarrow y = (7cm)\sin\frac{5\pi}{4}(80\,\text{cms}^{-1}t - x)$$

For the particle under consideration $x = 399$ cm, and the time $t = 5$ s.

$\therefore$ Displacement

$$y = (7\text{cm})\sin\frac{5\pi}{4}\left[(80\,\text{cms}^{-1})5\text{s}^{-1} - 399\,\text{cm}\right]$$

$$\Rightarrow \qquad y = (7\,\text{cm})\sin\frac{5\pi}{4} = \frac{-7\sqrt{2}\,\text{cm}}{2}$$

Differentiating equation (i) w.r.t. to time t, we get the particle velocity as,

$$\frac{dy}{dt} = (700\,\pi\,\text{cm}^{-1})\cos\frac{5\pi}{4}(80\,\text{cms}^{-1}t - x) \qquad \text{...(ii)}$$

Putting $t = 5$ s and $x = 399$ cm, we get the required particle velocity as

$$\frac{dy}{dt} = -350\pi\sqrt{2}\ \text{cms}^{-1}$$

Differentiating equation (ii) again w.r.t. time t, we get, the particle acceleration is 342

$$\frac{d^2y}{dt^2} = -(70000\,\pi^2\,\text{cms}^{-2})\sin\frac{5\pi}{4}(80\,\text{cms}^{-1}\,t - x)$$

Putting $t = 5$ s and $x = 399$ cm, the required particle acceleration can be obtained as

$$\frac{d^2y}{dt^2} = 35000\pi^2\sqrt{2}\,\text{cms}^{-2}$$

Exercise 6.5

1. Let P be the power of the source.

Then $I = \frac{P}{4\pi r^2}$

$\therefore \quad I \propto \frac{1}{r^2} \quad$ But $\quad I \propto a^2$

$\therefore \quad a \propto \frac{1}{r}$

The equation in the standard form is

$y = a \sin k\,(r - vt)$

Therefore, $y = a_0/r \sin k(r - vt)$, simply replacing a by a_0/r where a_0 is a constant.

Comparing with the usual form $y = a \sin\,(kr - \omega t)$

$\omega = kv \quad$ or $\quad f = kv/2\pi$

and $\quad k = 2\pi/\lambda \quad$ or $\quad \lambda = 2\pi/k$

$$\therefore \quad c = f\lambda = \frac{kv}{2\pi} \times \frac{2\pi}{k} = v$$

and $\quad T = 1/f = 2\pi/kv$

$$I = \frac{1}{2}\rho a^2\omega^2 v = \frac{1}{2}\rho\frac{a_0^2}{r^2}k^2v^2v = \frac{1}{2}\rho a_0^2k^2v^3/r^2$$

2. $v_{\max}$ (velocity amplitude) $= a\omega$

$$\therefore \quad v_{\max} = 2 \times 10^{-8} \times (2\pi \times 1000) = 1.3 \times 10^{-4} \text{ m/s}$$

$$I = \frac{1}{2}\rho a^2\omega^2 c = \frac{1}{2}\rho v_{\max}^2 c$$

$$\Rightarrow \quad I = \frac{1}{2} \times 1.3\left(1.3 \times 10^{-4}\right)^2 \times 340 = 3.7 \times 10^{-6}\ \text{W/m}^2$$

3. (a) As we know $P_{\text{ave}} = \frac{1}{2}\left(\sqrt{\mu F}\right)\omega^2 A^2$

$$= \frac{1}{2}\left[\sqrt{\left(\frac{3.00 \times 10^{-3}\ \text{kg}}{0.80\ \text{m}}\right)(25.0\ \text{N})}\right][(2\pi(120.0\ \text{Hz})]^2(1.6 \times 10^{-3})^2$$

$= 0.223$ W

(b) Halving the amplitude quarters the average power, so 0.056 W.

4. $$v = \sqrt{\frac{450}{50 \times 10^{-3}}} = 30\sqrt{10} \text{ m/s}$$

(a) The average power that can be carried by the transverse wave in wire is

$$P_{av} = 2\pi^2 \mu V A^2 f^2$$

If the amplitude A equals to 20% of wavelength λ, then

$$A = 0.2\,\lambda$$

The maximum power

$$P_A = 2\pi^2 \mu V (0.2\lambda)^2 f^2 = 0.8\,\mu v^3 = 1080\sqrt{10} \text{ W}$$

(b) $$\rho \alpha \mu v = \mu \sqrt{\frac{T}{\mu}} = \sqrt{\mu T}$$

$$\Rightarrow \quad \rho \propto \sqrt{\mu}$$

So if μ is doubled, power becomes $\sqrt{2}$ times.

5. $y_1 = A_1 \sin(\omega t - kx)$ and $y_2 = A_2 \sin(\omega t - kx + \phi)$

Then, $y = y_1 + y_2 = A \sin(\omega t - kx + \beta)$

where $A = \sqrt{A_1^2 + A_2^2 + 2A_1A_2 \cos\phi}$

The average power of the wave in the string is

$$P_{av} = \frac{1}{2}\mu A^2 \omega^2 v = \frac{1}{2}\mu (A_1^2 + A_2^2 + 2A_1A_2\cos\phi)\omega^2\left(\frac{\omega}{k}\right)$$

$$= \frac{1}{2}m(A^2 + A^2 + 2AA\cos\phi)\frac{\omega^3}{k} = \mu(1+\cos\phi)\frac{A^2\omega^3}{k}$$

Putting $\frac{\omega}{k} = v = \sqrt{\frac{T}{\mu}}$ and $\omega = 2\pi f$, we have

$$P_{av} = 4\pi^2 f^2 (1+\cos\theta) A^2 \sqrt{T\mu}$$

Exercises

Single Correct Answer Type

1. (4) $y = 4\sin\left(4\pi t - \frac{\pi}{16}x\right)$

$$\omega = 4\pi,\ k = \pi/16$$

$$v = \frac{\omega}{k} = \frac{4\pi}{\pi/16} = 64 \text{ cm/s}$$

in positive x-direction.

2. (3) At $t = 2$ s

$$y = \frac{1}{[1 + (x-1)^2]}$$

or $x - vt = x - 1 \Rightarrow 1 = vt$

$\Rightarrow$ $1 = v \times 2$

$\Rightarrow$ $v = 0.5$ m/s

3. (3) Power for a travelling wave on a string is given by

$$P = pv A^2 \omega^2 \cos^2 (kx - \omega t)$$

For the displacement wave,

$$y = A \sin (kx - \omega t)$$

Power delivered is maximum when $\cos^2 (kx - \omega t)$ is maximum, which would be the case when $\sin (kx - \omega t)$ is the least, i.e., displacement is minimum (acceleration is minimum). Power delivered is minimum when $\cos^2 (kx - \omega t)$ is minimum, which would be when $\sin (kx - \omega t)$ is maximum, i.e., displacement is maximum (acceleration is maximum).

4. (2) $$\frac{I_1}{I_2} = \frac{a_1^2 f_1^2}{a_2^2 f_2^2} = \frac{(3)^2(8)^2}{(2)^2(12)^2} = 1$$

5. (1) Slope at any point on the string in wave motion represents the ratio of particle speed to wave speed.

Therefore, slope $B <$ slope A

Hence $R_A > R_B$.

6. (4) $v = 960$ m/s; $n = \frac{3600}{60}$ Hz.

So $$\lambda = \frac{v}{n} = \frac{960}{60} = 16 \text{ m}$$

7. (2) Maximum particle velocity = 4 wave velocity

$$A\omega = 4 f\lambda$$

$$y_0 2\pi f = 4f\lambda$$

$$\lambda = \frac{\pi y_0}{2}$$

8. (4) $y = 8\sin 2\pi\left(\frac{x}{10} - 2t\right)$ given by comparing with standard equation.

$$y = a\sin 2\pi\left(\frac{t}{T} - \frac{x}{\lambda}\right)$$

$\lambda = 10$ cm

So phase difference = $(2\pi/\lambda) \times$ path difference

$$= \frac{2\pi}{10} \times 2 = \frac{2}{5} \times 180° = 72°$$

9. (1) Standard equation of travelling wave $y = A \sin (kx - \omega t)$

By comparing with the given equation

$$y = 10 \sin (0.01\,\pi x - 2\pi t)$$

$$A = 10 \text{ cm},\ \omega = 2\pi$$

Maximum particle velocity $= A\omega = 2\pi \times 10 = 63$ cm/s

10. (2) Maximum particle velocity $= \alpha_0 \omega = 2\pi a_0 v$

Wave velocity $= v\lambda$

Given that

$$2\pi a_0 v = 3\,v\lambda$$

or $\lambda = (2\pi\, a_0/3)$

11. (4) Maximum particle velocity $= \omega A$

Wave velocity $= \frac{\omega}{K}$

Therefore, the required ratio

$$= \frac{\omega A}{\omega/K}$$

$$= AK$$

$$= 60 \times 10^{-6} \times 6$$

$$= 3.6 \times 10^{-4}$$

12. (2) $\omega = \frac{2\pi}{0.01}$ and $k = \frac{2\pi}{0.30}$

$$v = \frac{\omega}{k} = \frac{2\pi}{0.01} \times \frac{0.30}{2\pi} = 30 \text{ m/s}$$

13. (3) $$\Delta\phi = \frac{2\pi}{\lambda}\Delta x$$

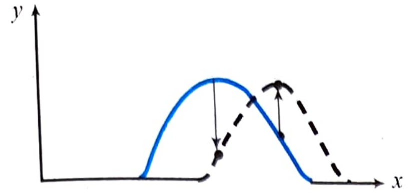
$$\frac{\pi}{4} = \frac{2\pi}{\lambda} \times \frac{1.25}{100}$$

On solving, we get

$$\lambda = \frac{1}{10} \text{ m/s}$$

$$u = n\lambda = 1000 \times \frac{1}{10} = 100 \text{ m/s}$$

(2)



Dotted shape shows pulse position after a short time interval. Direction of the velocities are decided according to direction of displacements of the particle.

15. (2) $v = 1$ m/s, $\nu = 100$ Hz

$$\lambda = \frac{v}{\nu} = \frac{1}{100} \text{ m} = 1 \text{ cm}$$

$$\Delta\phi = \frac{2\pi}{\lambda} \times 2.75 \text{ rad} = 5.5\ \pi \text{ rad} = \frac{11\pi}{2} \text{ rad}$$

16. (1) $y = 0.02 \sin(x + 30t)$

Comparing with standard equation

$y = A \sin(Kx + \omega t)$, $\omega = 30$, $K = 1$

Velocity of wave, $v = \frac{\omega}{K} = \frac{30}{1} = 30$ m/s

Expression $v = \sqrt{\frac{T}{m}}$ gives

Tension $T = v^2 m = (30)^2 \times 10^{-4} = 0.09$ N

17. (4) Assume $1/\sqrt{a} = A\cos\theta$ (i)

$$\frac{1}{\sqrt{b}} = A\sin\theta \quad \text{(ii)}$$

On simplifying, we get $y = A\sin(\omega t + \theta)$

Squaring and adding Eqs. (i) and (ii) $A = \sqrt{\frac{a+b}{ab}}$

18. (3) Suppose at an instant t, the x-coordinate of a point with reference to moving frame is x_0. Since, at this moment, origin of moving frame is at distance vt from origin of the fixed reference frame, therefore, putting this value of x in the given equation, we get

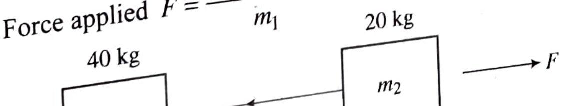
$$y = a \cos[\omega t - k(vt + x_0)]$$

$$y = a \cos[(\omega - kv)t - kx_0)]$$

Hence, option (3) is correct.

19. (2) Tension T in the wire $= v^2\rho = (400)^2 \times 10^{-3} = 160$ N

Force applied $F = \frac{T(m_1 + m_2)}{m_1}$



$$= 160 \times \frac{(40 + 20)}{40} = 240 \text{ N}$$

20. (2) $y(x, t) = f(x - vt)$

$$y = (x, 0) = \frac{4 \times 10^{-3}}{8 - x^2}$$

For a travelling wave in the x-direction

$$y(x, t) = \frac{4 \times 10^{-3}}{8 - (x - 5t)^2}$$

21. (2) Given $\omega = 3\pi$

$$f = \frac{\omega}{2\pi} = 1.5$$

Also $\Delta x = 1.0$ cm

Given, $\phi = \frac{2\pi}{\lambda}\Delta x \Rightarrow \frac{\pi}{8} = \frac{2\pi}{\lambda} \times 1$

$\lambda = 16$ cm

$v = f\lambda = 16 \times 1.5 = 24$ cm/s

22. (2) $y(x, t = 0) = \frac{6}{x^2}$ then $y(x, t) = \frac{6}{(x - 2t)^2}$

$$\frac{\partial y}{\partial t} = \frac{24}{(x - 2t)^3} \quad \text{at} \quad x = 2,\ t = 2$$

$$V_y = \frac{24}{(-2)^3} = -3 \text{ m/s}$$

23. (3) $P = \frac{1}{2}\mu\omega^2 A^2 V$ using $V = \sqrt{\frac{T}{\mu}}$

$$P = \frac{1}{2}\omega^2 A^2 \sqrt{T\mu}$$

$$\omega = \sqrt{\frac{2P}{A^2\sqrt{T\mu}}},\ f = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{2P}{A^2\sqrt{T\mu}}}$$

Using the given data, we get $f = 30$ Hz.

24. (2) In the figure, 'C' reaches the position where 'A' already reaches after $\omega t = \pi/2$ and 'A' reaches the position where 'B' already reaches after $\omega t = \pi/2$.

25. (2) The equation for wave A can be rewritten as

$$y = A \sin[kx - \omega t - \phi] = A \sin[k(x - \phi/k) - \omega t]$$

$$= A\sin\left[kx - \omega\left(t + \frac{\phi}{\omega}\right)\right]$$

While equation of wave B is $y = A \sin(kx - \omega t)$.

Comparing above equations, we can easily conclude that A is at a distance ahead of ϕ/k from B or wave A is ahead of B by a time difference of ϕ/ω. So, (2) is the correct option.

Remember! In y versus t 'ahead of means to the left of' while in y versus x 'ahead of means to the right of' if the wave travels in positive x-direction and vice versa.

26. (4) $y(x, t) = \frac{a}{(x \pm vt)^2 + b}$

is another form of progressive wave equation propagating with a speed v.

Negative sign to be taken for propagation along $+x$-axis and positive sign to be taken to propagation along $-x$-axis.

27. (1) $y = \frac{1}{1 + x^2}$ at $t = 0$ and $y = \frac{1}{1 + (x - 2)^2}$ at $t = 4$ s

$$v = \frac{\Delta x}{\Delta t} = \frac{x - (x - 2)}{4 - 0} = \frac{2}{4} = 0.5 \text{ m/s}$$

28. (1) $V_P(\max) = \left(\frac{dy}{dt}\right)_{\max} = 50$ units

$$V_\omega = \frac{\omega}{k} = \frac{100}{25} = 4 \text{ units}$$

$$\frac{V_P(\max)}{V_\omega} = 12.5$$

29. (1) At any instant t, the wave equation will express the variation of y with x which is equal to the shape of the string at an instant t.

30. (3) For the wave $y = A \sin(\omega t - kx)$, $v_0 = A\omega$ where A is, the maximum displacement.

For the given condition,

$$\frac{A}{2} = A\sin(\omega t - kx)$$

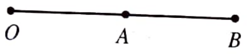
$$\sin(\omega t - kx) = \frac{1}{2}$$

And $\dfrac{\partial y}{\partial t} = A\omega\cos(\omega t - kx) = A\omega\dfrac{\sqrt{3}}{2} = \dfrac{\sqrt{3}v_0}{2}$

31. (3) $V_{\max} = A\omega = 5 \Rightarrow A\dfrac{2\pi}{4} = 5$

$$\Rightarrow \quad A = \frac{10}{\pi} \text{ cm}$$

32. (4) Let the power of source be P and it is placed at O. Then intensity at A and B would be given by

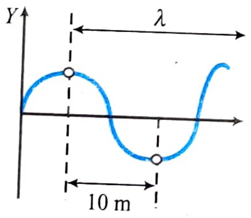


$$I_A = \frac{P}{4\pi \times 1^2} \text{ and } I_B = \frac{P}{4\pi \times 2^2}$$

Since, intensity ∝ (Amplitude)^2 × (Frequency)^2 (here, amplitude means displacement amplitudes), the frequency is same at both the points.

$$\Rightarrow \quad \frac{(\text{Amp})_A}{(\text{Amp})_B} = \sqrt{\frac{I_A}{I_B}} = \sqrt{\frac{2^2}{1^2}} = 2:1$$

33. (2)



Frequency of wave = 1/4 Hz

Wavelength of wave = $\lambda = 2 \times 10 = 20$ m

Velocity of wave = $f\lambda = 5$ m/s

34. (4) For the wave, $y = A\sin(kx - \omega t)$, the wave speed is ω/k and the maximum transverse string speed is $A\omega$.

35. (2) The wave is travelling along the length of a string, while particles constituting the string are oscillation in a direction perpendicular to the length to string. In one time period (cycle), the wave moves forward by one wavelength while the particle on string travels a distance of 4 times the amplitude.

Here, $T = 1/f = 0.02$ s

Wave speed, $v = f\lambda = 25$ m/s

Time taken by wave to travel a distance of 8 m,

$$t_1 = 8/25 \text{ s} = 0.32 \text{ s}.$$

Time taken by particle on string to travel a distance of 8 m,

$$t_2 = \frac{8 \times T}{4 \text{ times amplitude}} = \frac{8}{4 \times 0.01} \times 0.02 = 4 \text{ s}$$

36. (1) Negative sign with 'ω' indicates that wave is propagating along positive x-axis.

$$\lambda = \frac{2\pi}{k} = \frac{2\pi}{10\pi} = 0.2 \text{ m}$$

And $\quad v = \dfrac{\omega}{k} = \dfrac{15\pi}{10\pi} = 1.5$ m/s

37. (1) Amplitude of wave,

$$A = \frac{2.0\text{cm}}{2} = 1\text{cm}$$

Frequency of wave, $f = 125$ Hz

Wavelength of wave, $\lambda = 15.6$ cm $= 0.156$ m

Let equation of wave be, $y = A\sin(kx - \omega t + \phi)$ where $k = 2\pi/\lambda = 40.3$ rad/m and $\omega = 2\pi f = 786$ rad/s

Using initial conditions, $y(0, 0) = 0 = A\sin\phi$

and $\quad \dfrac{\partial y}{\partial t}(0,0) = -A\omega\cos\phi < 0$

We get, $\phi = 0$

So, the equation of wave is

$$y = (1 \text{ cm}) \sin[(40.3 \text{ rad/m})x - (786 \text{ rad/s})t]$$

38. (1) $\quad y = (0.2 \text{ m}) \sin[kx \pm \omega t]$

For $\quad x = 0, y = 0.1$ m

$0.1 = 0.2 \sin(\omega t)$

$\Rightarrow \quad \omega t = \pi/6$ or $5\pi/6$

So, $\quad t_2 = 5\pi/6\omega$ and $t_1 = \pi/6\omega$

$t_2 - t_1 = 2\pi/3\omega = 1/3f = 1.9$ ms

39. (2) Let the speed of wave be v, for crossing one wave crests to the other while travelling in the same direction, the surfing speed has to be greater than speed of the wave, i.e., $v < 15$ m/s.

Let wavelength of wave be λ m

While surfing in the same direction, $\lambda = (15 - v) \times 0.8$

While surfing in the direction opposite to the wave motion,

$\lambda = (15 + v) \times 0.6$

$(15 - v)\,0.8 = (15 + v)\,0.6$

$v = 15/7$ m/s $= 2.143$ m/s

So, $\quad \lambda = (15 - 2.143) \times 0.8 = 10.3$ m

40. (3) At the moment shown in the figure, particle at 1 is moving in the downward direction.

We have, $T = 1/0.1$ s $= 10$ s.

In one complete cycle, particle travels a distance, 4 times the amplitude. So, in time 10 min 15 s, i.e., 615 s which means 61 full + 1 half cycles, the distance travelled

$= (4 \times 3) \times 61 + (2 \times 3) \times 1 = 732 + 6 = 738$ cm

At time instant, the particle is moving in the upward direction.

41. (2) Wave velocity: $v = \sqrt{\dfrac{F}{\mu}} = \sqrt{\dfrac{25}{1}} = 5$m/s

Average power: $P_{av} = \dfrac{1}{2}\mu v(\omega A)^2 = \dfrac{1}{2}\mu v(v_{p-\max})^2$

$$= \frac{1}{2} \times 1 \times 5(1 \times 10^{-2})^2 = 2.5 \times 10^{-4} \text{ W}$$

42. (4) $v = \sqrt{\dfrac{T}{\mu}}$

T can be calculated by using Hooke's law and on stretching μ also changes.

43. (3) $y = a\left[\dfrac{1 + \cos(2\omega t - 2kx)}{2}\right]$

$$y=\frac{a}{2}+\frac{a}{2}\cos(2\omega t-2kx)$$

1) Let equation of the wave is $y = A\sin(\omega t - kx + \phi_0)$
The particle at origin is moving downwards (from graph) at $t = 0$, so $\phi_0 = \pi$. Also $A = 2$ m from graph.

From graph, $\lambda = 8\text{m}$, so $k=\frac{2\pi}{\lambda}=\frac{\pi}{4}$

$$\omega = vk = \frac{4\times\pi}{4}=\pi$$

Putting all the values

$$y=2\sin\left[\pi t-\frac{\pi}{4}x+\pi\right]\Rightarrow y=2\sin\left[\frac{\pi}{4}x-\pi t\right]$$

1) $$v_p=-v\left(\frac{dy}{dx}\right)$$

here v is negative and dy/dx is positive at point P. Hence v_p is positive. So P is moving upwards.

2) Given $y = a_0 \sin(\omega t - kx)$

Put $t = 0$ and $x=\frac{\pi}{2k}$

$$y = a_0\sin(\omega\times 0 - k\pi/2k)\Rightarrow y=-a_0$$

This corresponds to point Q.

3) We start with a general form for a rightward moving wave,

$$y(x, t) = A\sin(kx - \omega t + \phi)$$

The amplitude given is $A = 2.0\text{ cm} = 0.02\text{ m}$.
The wavelength is given as, $\lambda = 1.0$ m
Wave number $= k = 2\pi/\lambda = 2\pi\text{ m}^{-1}$
Angular frequency, $\omega = vk = 10\pi$ rad/s

$$y(x, t) = (0.02)\sin[2\pi(x - 5.0t) + \phi]$$

We are told that for $x = 0$, $t = 0$,

$$y = 0 \quad\text{and}\quad \frac{\partial y}{\partial t}<0$$

i.e., $0.02\sin\phi = 0$ (as $y = 0$)
and $-0.2\pi\cos\phi < 0$

From these conditions, we may conclude that

$$\phi = 2n\pi \quad\text{where}\quad n = 0, 2, 4, 6, \ldots$$

Therefore, $y(x, t) = (0.02\text{ m})\sin[(2\pi\text{ m}^{-1})x - (10\pi\text{ s}^{-1})t]$ m

2) The amplitude, $A = 0.06$ m

$$\frac{5}{2}\lambda = 0.2\text{ m}$$

$$\therefore\quad \lambda = 0.08\text{ m}$$

$$f=\frac{v}{\lambda}=\frac{300}{0.08}=3750\text{ Hz}$$

$$k=\frac{2\pi}{\lambda}=78.5\text{ m}^{-1}\quad\text{and}\quad \omega=2\pi f=23562\text{ rad/s}$$

At $t = 0$, $x = 0$, $\frac{dy}{dx}=$ positive

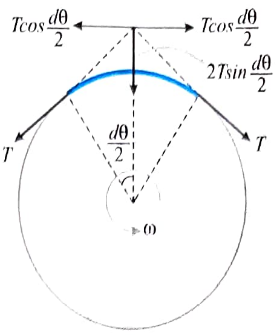
and the given curve is a sine curve.
Hence, equation of wave travelling in positive x-direction should have the form,

$$y(x, t) = A\sin(kx - \omega t)$$

Substituting the values, we have

$$y = (0.06\text{ m})\sin[(78.5\text{ m}^{-1})x - (23562\text{ s}^{-1})t]\text{ m}$$

49. (2)



$$2T\sin\frac{d\theta}{2}=dm\cdot\omega^2R$$

$$2T\frac{d\theta}{2}=\frac{m}{l}Rdq\cdot\omega^2R \qquad T=\frac{m\omega^2R^2}{l}$$

$$\therefore\quad V=\sqrt{\frac{T}{\mu}}=\sqrt{\frac{T}{m/l}}=\omega R^2 \qquad \therefore\quad V=\frac{\omega L}{2\pi}$$

50. (3) $$V_{P/R}=\frac{\omega L}{2\pi}$$

$$V_{P/G}-V_{R/G}=\frac{\omega L}{2\pi}\Rightarrow V_{P/G}-\frac{\omega L}{2\pi}=\frac{\omega L}{2\pi}$$

$$\therefore\quad V_{P/G}=\left(\frac{wL}{2\pi}\right)2=\frac{\omega L}{\pi}$$

51. (4) $$V_{P/R}=\frac{\omega L}{\pi} \qquad V_{P/G}-\left(-\frac{wL}{2\pi}\right)=\frac{wL}{2\pi}$$

$$\therefore\quad V_{P/G}=0$$

Multiple Correct Answers Type

1. (1), (2), (4)
It is a known fact as well as experimentally and analytically verified that wave speed depends on the properties of the medium and is same for the entire wave. The particle velocity is given by

$$v_P=\frac{\partial y}{\partial t}=-A\omega\cos(kx-\omega t)$$

where symbols have their usual meanings. It is clear from above expression that v_p depends upon amplitude and frequency of wave which are wave properties and are having different values for different particles at a particular instant.

2. (1), (2), (3), (4)

$v_p = v\times\partial y/\partial x$ for $y = A\sin(\omega t - kx)$, we have
$v_p = kv\times A\cos(\omega t - kx) = \omega A\cos(\omega t - kx)$

i.e., it is varying
Also, $v_p\propto\omega$ and $v_p\propto A$

3. (1), (2), (3)
Wavelength of a wave is a property of source and medium both. So, wavelength can change if either frequency or speed of wave or both change. Here, medium property (like tension in string) can change freq. may change which causes the change in the speed of wave, or source frequency may change.

4. (1), (2), (3), (4)
For a point source $I\propto\frac{1}{r^2}$

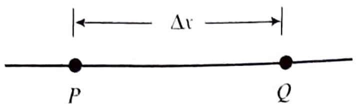
For a line source $I\propto\frac{1}{r}$

For a plane wave, intensity remains same because there is no spreading of wave.

5. (2), (3), (4)

$$\frac{\Delta\phi}{2\pi} = \frac{\Delta x}{\lambda}$$

$$\Delta x = \frac{\lambda}{2\pi}\Delta\phi = \frac{\lambda}{2\pi}\pi = \frac{\lambda}{2}$$



Let for P: $y_1 = A \sin \omega t$, then

For Q: $y_2 = A \sin(\omega t - \pi) = -A \sin \omega t$

We see that $y_1 = -y_2$

and $|y_1| = |y_2|$

Frequency of both particles should be same because the same wave passes through them.

6. (2), (3), (4)

$$y(x,t) = \frac{0.8}{16\left[\left(x + \frac{5}{4}t\right)^2 + \frac{5}{16}\right]}$$

$$y(x,t) = \frac{0.05}{\left(x + \frac{5}{4}t\right)^2 + \frac{5}{16}}$$

where wave velocity = 5/4 m/s

Distance travelled by the wave with this velocity in 2 s is

5/4 (2) = 2.5 m

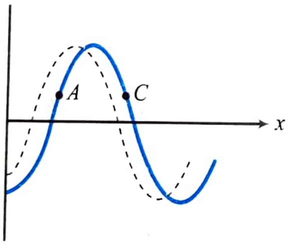
$y(x, t)$ will be maximum when $4x + 5t = 0$

$$y_{\max} = \frac{0.8}{5} = 0.16 \text{ m}$$

7. (2), (4)

Since A is moving upwards, after an elemental time interval, the wave will be as shown dotted in figure. It means, the wave is travelling leftwards. Therefore, option (1) is wrong.

Displacement amplitude of the wave means maximum possible displacement of medium particles, due to propagation of the wave which is equal to the displacement at B at the instant shown in the figure. Hence, option (2) is correct.



From the figure, it is clear that C is moving downwards at this instant. Hence, option (3) is wrong.

The phase difference between two points will be equal to $\pi/2$ if distance between them is equal to $\lambda/4$. Between A and C, the distance is less than $\lambda/2$. It may be equal to $\lambda/4$. Hence, phase difference between these two points may be equal to $\pi/2$. Therefore, option (4) is correct.

8. (1), (2), (3), (4)

$$y = 10^{-4} \sin(60t + 2x)$$

$$y = a \sin(\omega t + kx)$$

Now, $k = 2$, $2\pi/\lambda = 2$ or $\lambda = \pi$ metre

Again, $\omega = 60$

or $2\pi f = 60$ or $f = \frac{60}{2\pi}$ or $\nu = \frac{30}{\pi}$ Hz

Again, $v = \frac{\omega}{k} = \frac{60}{2}$ m/s = 30 m/s

9. (2), (4)

Comparing with $y = a \cos(\omega t - kx)$

$\omega = 500$, $k = 70$

Speed of wave $= \frac{\omega}{k} = \frac{500}{70} = \frac{50}{7}$ m/s

$k = 70$

$$\Rightarrow \quad \frac{2\pi}{\lambda} = 70$$

$$\text{or} \quad \lambda = \frac{2\pi}{70}\text{m} = \frac{2\pi}{70} \times 100 \text{ cm} = \frac{20\pi}{7}\text{cm}$$

10. (2), (3)

Let the equation to the wave be

$$y = A\sin\left[2\pi\left(\frac{t}{T} - \frac{x}{\lambda}\right) + \phi\right] \quad \text{...(i)}$$

where A is the amplitude of the wave and ϕ, phase angle.

It is given that $y = a$, when $x = 0$ and $t = T/4$ and also that $y = v$ when $x = \lambda/4$ and $t = T/4$

Substituting in (i), $y = a = A\sin\left(\frac{\pi}{2} + \phi\right)$

$$\dot{y} = v = \frac{2\pi A}{T}\cos\left[2\pi\left(\frac{1}{4} - \frac{1}{4}\right) + \phi\right]$$

$$v = \frac{2\pi A}{T}\cos\phi$$

Putting $\phi = 0$, $y = a = A$, so that amplitude $A = a$

Also, $v = \frac{2\pi A}{T}[\cos 0] = \frac{2\pi a}{T}$

$$\frac{2\pi}{T} = \frac{v}{a}$$

Hence the equation to the wave is

$$y = a\sin\frac{v}{a}\left[t - \frac{Tx}{\lambda}\right]$$

$$y = a\sin\frac{v}{a}\left[t - \frac{x}{V}\right]$$

where $V = \frac{\lambda}{T}$ is the velocity of the wave in the gas.

11. (1), (2), (3)

Comparing the given equation of travelling wave with

$$y = a\sin\left(\omega t - \frac{2\pi x}{\lambda}\right)$$

Amplitude $= a$, angular frequency $\omega = b$ and $2\pi/\lambda = c$ or wavelength $\lambda = 2\pi/c$

$$\frac{a}{\lambda} = \frac{ac}{2\pi}$$

Therefore, option (1) is correct.

Velocity oscillation amplitude of medium particles is $a\omega = ab$.

Wave propagation velocity $v = \omega\lambda/2\pi = b/c$

$$\frac{a\omega}{v} = ac$$

Hence, option (2) is also correct.

Relative deformation or strain produced in the medium is $\varepsilon = u/v$, where u is particle's velocity and v is wave velocity. Since v is property of the medium, therefore, ε is directly proportional to velocity of the medium particles (u).

ε will be maximum possible when u is maximum. Hence, option (3) is also correct.

12. (1), (2)

$$y = 0.01\sin[2\pi(10t - x)]$$

$$\Rightarrow \quad y = 0.01\sin[20\pi t - 2\pi x]$$

$$\omega = 20\pi,\ K = 2\pi$$

Wave velocity: $v = \dfrac{\omega}{k} = \dfrac{20\pi}{2\pi} = 10\,\text{m/s}$

$$\Delta\phi = K\,\Delta x, \text{ given } \Delta\phi = 60^\circ = \frac{\pi}{3}$$

$$\frac{\pi}{3} = 2\pi\,\Delta x \Rightarrow \Delta x = \frac{1}{6}\,\text{m}$$

$$v_{p-\max} = \omega A = 20\pi \times 0.01 = \frac{\pi}{5}\,\text{m/s}$$

$$\Delta\phi_t = \omega\,\Delta t = 20\pi \times \frac{1}{20} = \pi$$

So in time $\dfrac{1}{20}$ s, phase changes by π but not by 120°.

13. (1), (2), (4)

Given wave is $y = A\cos\dfrac{\pi}{2}[kx - \omega t - \alpha]$

Here wave number, $k \times \dfrac{\pi}{2} = \dfrac{2\pi}{\lambda}$ giving $\lambda = \dfrac{4}{k}$

Here $k = 0.005$ cm^{-1}. Hence $\lambda = \dfrac{4}{0{:}005}\,\text{cm} = 8\,\text{m}$

Maximum velocity $V_m = A \times$ angular velocity.

Here angular velocity $= \dfrac{\pi\omega}{2} = \dfrac{3.14 \times 8}{2} = 12.56$ rad/s

Hence $V_m = 0.6 \times 12.56$ m/s $= 7.53$ m/s

Also, to produce stationary waves, the two waves should travel in opposite directions and have same frequency. The wave given by $y = A\cos\dfrac{\pi}{2}(kx + \omega t - \alpha)$ fulfils this condition.

14. (2), (3), (4)

$$y = 4\sin\left[\frac{\pi}{16}(16t + x)\right]$$

Compared with

$$y = a\sin\left[\frac{2\pi}{\lambda}(vt + x)\right]$$

Also, $\Delta\phi = \dfrac{2\pi}{\lambda}\Delta x = \dfrac{2\pi}{\lambda}(v\Delta t)$

15. (1), (4)

$$v = \sqrt{\frac{T}{\mu}}$$

For equilibrium $Mg = mg\sin 30 = T$

$M = m/2$

$$100 = \sqrt{\frac{Mg}{9.8 \times 10^{-3}}} = \sqrt{\frac{M(9.8)}{9.8 \times 10^{-3}}}$$

$$100 = \sqrt{M(1000)}$$

$M = 10$ kg and $m = 20$ kg

16. (1), (3)

$$v_{\max} = a\omega = a(2\pi f)$$

Given that $a(2\pi f) = \dfrac{v}{10} = \dfrac{10}{10} = 1$

$$f = \frac{1}{2\pi a} = \frac{10^3}{2\pi}\,\text{Hz}$$

As $\quad v = f\lambda$

or $\quad \lambda = \dfrac{v}{f} = \dfrac{10}{10^3/2\pi} = 2\pi \times 10^{-2}\,\text{m}$

17. (1), (3), (4)

$$y = f(x - vt)$$

Particle velocity.

$$v_P = \frac{dy}{dt} = -v f'(x - vt)$$

To find velocity of wave $\dfrac{d}{dt}(x - vt) = 0$

$$\frac{dx}{dt} = v$$

Linked Comprehension Type

For Problems 1–4

1. (4), 2. (2), 3. (2), 4. (2)

$$\lambda = 2(4.5 - 2.5) = 4\ \text{cm};\quad v = n\lambda$$

$$\Rightarrow \quad n = \frac{40}{4} = 10\,\text{Hz}$$

$$\phi = \frac{2\pi}{\lambda}\Delta x = \frac{5\pi}{4}\,\text{rad} \Rightarrow \phi = \frac{2\pi}{T}\Delta t$$

$$\Delta t = \frac{\phi}{2\pi n} = \frac{\pi/3}{2\pi 10} = \frac{1}{60}\,\text{s}$$

Velocity of p should be maximum, as it is at mean position.

$$v_p = \omega A = 2\pi f A = 2\pi \times 10 \times \frac{2}{100}$$

$$= 1.26\,\text{m/s}$$

This velocity should be –ve, because slope at p is +ve.

For Problems 5–7

5. (1) The equation of wave moving in negative x-direction, assuming origin of position at $x = 2$ and origin of time (i.e., initial time) at $t = 1$ s.

$$y = 0.1\sin(4\pi t + 8x)$$

Shifting the origin of position to left by 2 m, to $x = 0$. Also shifting the origin of time backwards by 1 s, that is to $t = 0$ s.

$$y = 0.1\sin[4\pi(t - 1) + 8(x - 2)]$$

6. (3) As given the particle at $x = 2$ is at mean position at $t = 1$ s.

$\therefore$ Its velocity $v = \omega A = 4\pi \times 0.1 = 0.4\pi$ m/s.

7. (4) Time period of oscillation $T = \dfrac{2\pi}{\omega} = \dfrac{2\pi}{4\pi} = \dfrac{1}{2}\,\text{s}$

Hence at $t = 1.125$ s, that is, at $T/4$ seconds after $t = 1$ s, the particle is at rest at extreme position. Hence, instantaneous power at $x = 2$ at $t = 1.125$ s is zero.

For Problems 8–9

8. (3), 9. (1)

Let general wave equation be $y = A \sin(\omega t - kx + \phi)$

$$v = \frac{dy}{dt} = A\omega \cos(\omega t - kx + \phi)$$

For curve (1), $x = 0$

at $\quad t = 0, x = 0$, we have $y = 0$

$\Rightarrow \quad 0 = A \sin[\phi] \Rightarrow \sin\phi = 0$

$\Rightarrow \quad \phi = 0$ or π

here $\phi = \pi$ (because particle velocity $\frac{dy}{dt}$ is negative)

for curve (2), $x = 7$ cm

at $\quad t = 0, x = 7$ cm, $y = -1$

$-1 = \sin(-k \times 7 + \pi)$

$\Rightarrow \quad \sin(-7k + \pi) = -1/2$

$$\Rightarrow \quad -7k + \pi = 2n\pi + \frac{7\pi}{6} \text{ or } 2n\pi + \frac{11\pi}{6}$$

here

$$\Rightarrow \quad -7k + \pi = 2n\pi + \frac{11\pi}{6}$$

(because at $t = 0$, particle velocity $\frac{dy}{dt}$ is positive)

$$\Rightarrow \quad -7\left(\frac{2\pi}{\lambda}\right) = 2n\pi + \frac{5\pi}{6}$$

$$\Rightarrow \quad \lambda = \frac{-14\pi}{\frac{5\pi}{6} + 2n\pi}$$

$$\Rightarrow \quad \lambda = \frac{-84}{12n + 5}$$

for $n = -1$, $\lambda = 12$ cm

for $n = -2$, $\lambda = \frac{84}{19}$ cm (not possible)

because $\lambda > 7$ cm

$$v = f\lambda = 100 \times \frac{12}{100} = 12 \text{ m/s}$$

For Problems 10–11

10. (4), 11. (3)

From the graph it is clear

$$\text{Wave velocity } v = \frac{(8-3)}{0.006}$$

$$v = \frac{5 \times 10^3}{6} \text{ m/s}$$

And wave velocity $v = f\lambda$

$$\frac{5 \times 10^3}{6} = f(9-1) \Rightarrow f = \frac{5 \times 10^3}{6 \times 8} = 104 \text{ s}^{-1}$$

Alternate method: Let $y = A \sin(\omega t - kx + \phi)$

For curve (1): $-\frac{1}{\sqrt{2}} = 1 \sin(\omega \times 0.002 + \phi)$

$$\Rightarrow \quad \sin(0.002\omega + \phi) = -\frac{1}{\sqrt{2}}$$

$$\Rightarrow \quad 0.002\omega + \phi = \frac{5\pi}{4} \text{ or } \frac{7\pi}{4}$$

but as the velocity is downward

$$\text{So} \quad 0.002\omega + \phi = \frac{5\pi}{4} \quad \text{...(i)}$$

For curve (2): $1 = 1 \sin(\omega \times 0.008 + \phi)$

$$\Rightarrow \quad 0.008\omega + \phi = 2\pi + \frac{\pi}{2} \quad \text{..(ii)}$$

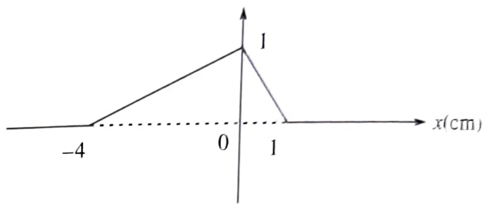
From Eqs. (i) and (ii), $6\omega = 1250\pi$

$$\Rightarrow \quad f = \frac{1250\pi}{12\pi} = 104 \text{ Hz}$$

$$v = f\lambda = \frac{1250}{12} \times 8 = \frac{2500}{3} \text{ m/s}$$

For Problems 12–14

12. (2) Shape of the pulse at $t = 0$



That is a triangular pulse.

$$\text{Area of the pulse} = \frac{1}{2}[(4 \times 1) + (1 \times 1)] = \frac{5}{2} \text{ cm}^2$$

13. (3) $v = \sqrt{\frac{T}{\mu}} = 10$ m/s

Solution of the wave equation that gives displacement of any piece of the string at any time

$$y = f(x,t) = \begin{cases} \frac{(x - vt)}{4} + 1 & \text{for } vt - 4 < x \le vt \\ -(x - vt) + 1 & \text{for } vt < x < vt + 1 \\ 0 & \text{otherwise} \end{cases}$$

Using $v = 1000$ cm/s, $t = 0.01$ s

$vt = 10$ cm

as $(vt - 4) < (x = 7 \text{ cm}) < vt$

$$y = \frac{1}{4}(7 - 10) + 1 = \frac{1}{4} \text{ cm} = 0.25 \text{ cm}$$

14. (1) Transverse velocity $= \frac{\partial y}{\partial t}$ at $t = 0.015$ s, $vt = 15$ cm

As for $x = 13$ cm

$(vt - 4) < x < vt$

$$\text{Therefore, } \frac{\partial y}{\partial t} = -\frac{v}{4} = -250 \text{ cm/s}$$

Matrix Match Type

1. i. → d.; ii. → a.; iii. → b.; iv. → c.

As it is clear from the equation that speed of each wave is same and equal to 2 m/s.

From the graphs it is clear that wavelength is maximum for graph in (a) and the least for (c) As wavelength is the property of both source and medium and here medium is the same, so we can conclude that the relation among three wavelengths, is determined by source property, i.e., frequency. From equation it is clear that frequency is maximum for y_3 and least for y_1. From $\lambda = v/f$, we can conclude that wavelength is maximum for the wave having least frequency.

2. **i. → c.; ii. → a.; iii. → d.; iv. → b.**

Use $x = 0$; $t = 0$ for y and particle velocity $\frac{\partial y}{\partial x}$. For case (i)

$y = 0$ at $x = 0$ and $t = 0$. $\frac{\partial y}{\partial t} > 0$, i.e., positive, therefore, it matches with (c).

3. **i. → b., d.; ii. → a., d.; iii. → a., b.; iv. → a., b., d.**

If we change the speed of hand, then particle speed changes. If the amount by which hand movement changes, then amplitude of pulse changes. If time in which hand comes to its original position changes, the width of pulse changes.

4. **i. → b., c.; ii. → a., d.; iii. → b., c.; iv. → a., d.**

Particle velocity, $v_P = \frac{\partial y}{\partial t} = (4x)\text{ cm/s}\cos[\pi t + 2\pi x]$

Particle acceleration, $a_P = \frac{\partial^2 y}{\partial t^2} = (-4\pi^2)\text{cm/s}^2 \sin[\pi t + 2\pi x]$

Now substitute the values of t and x given in column II.

5. **i. → d.; ii. → a.; iii. → d.; iv. → c.**

The relation between particle velocity and wave velocity, $v_P = -v\frac{\partial y}{\partial x}$

Here, $\frac{\partial y}{\partial x} =$ Slope of y-x graph.

As the sign of v is not given in the problem. Hence, direction of v_P cannot be determined.

For particle acceleration, $a_p \propto -y$.

i.e., a_p and y are away in opposite directions.

If y = 0, then $a_p = y$

6. **i. → a., c.; ii. → a.; iii. → b., d.; iv. → a., d.**

Particle velocity, $v_P = \frac{\partial y}{\partial t}$

Particle acceleration, $a_P = \frac{\partial^2 y}{\partial t^2}$

If ωt and kx are of same sign, then wave travel in negative x-direction.

If they are of opposite signs, then wave travels in positive direction.

Numerical Value Type

1. **(0.25)**

Let v_1 = velocity in the 1st string

$$v_1 = \sqrt{\left(\frac{T}{\mu}\right)} = \sqrt{\left(\frac{T}{\rho_1 a}\right)} \quad \text{...(i)}$$

Where a = area of cross-section

Let v_2 = velocity in the second string

$$v_2 = \sqrt{\left(\frac{T}{\mu_2}\right)} = \sqrt{\left(\frac{T}{\rho_2 a}\right)} \quad \text{...(ii)}$$

Given that $v_1 = 2v_2$

$$\sqrt{\left(\frac{T}{a\rho_1}\right)} = 2\sqrt{\left(\frac{T}{a\rho_2}\right)} \Rightarrow \frac{\rho_1}{\rho_2} = \frac{1}{4} = 0.25$$

2. **(0.050)**

Mass per unit length

$$\mu = \frac{10}{40} = \frac{1}{4}\text{ (gm/cm) Spring constant } k = 160\text{ N/m}$$

Deflection $= x = 1$ cm $= 0.01$ m

$$T = kx = 160 \times 0.01 = 1.6\,\text{N} = 16 \times 10^4\text{ dyne}$$

$$\text{Again } v = \sqrt{\left(\frac{T}{\mu}\right)} = \sqrt{\left(\frac{(16\times10^4}{\frac{1}{4}}\right)} = 8\times10^2\text{ cm/s} = 800\text{ cm/s}$$

Time taken by the pulse to reach the spring

$$t = \frac{40}{800} = \frac{1}{20} = 0.05\,\text{s}$$

3. **(50)** $r = 0.5$ mm $= 0.5 \times 10^{-3}$ m and, $f = 100$ Hz

$T = 100$ N, $v = 100$ m/s

$$v = \sqrt{\left(\frac{T}{\mu}\right)} \Rightarrow v^2 = \left(\frac{T}{\mu}\right)$$

$$\mu = \frac{T}{v^2} = \frac{100}{(100)^2} = 0.01\text{ kg/m}$$

$$P_{ave} = 2\pi^2 mvr^2 f^2$$

$$= 2(10)(0.01)\times100\times(0.5\times10^{-3})^2\times(100)$$

$$= 0.050\text{ watt} = 50\text{ mW}$$

4. **(4)** $y = \frac{0.8}{(3x^2 + 24xt + 48t^2 + 4)} = \frac{0.8}{3[x^2 + 8xt + 16t^2] + 4}$

$$= \frac{0.8}{3(x+4t)^2 + 4}$$

$\therefore \quad x + 4t = x + vt \qquad \therefore\ v = 4$ m/s

5. **(2)** Velocity of the wave,

$$V = \sqrt{\left(\frac{T}{\mu}\right)} = \sqrt{\frac{(16\times10^5)}{0.4}} = 2000\text{ cm/s}$$

Time taken to reach to the other end $= \frac{20}{200} = 0.01$ s.

Time taken to see the pulse again in the original position $= 0.01 \times 2 = 0.02$ s.

6. **(4)** The linear mass density is

$$\mu = \frac{5\times10^{-3}\text{kg}}{50\times10^{-3}\text{m}} = 1.0\times10^{-2}\frac{\text{kg}}{\text{m}}$$

The wave speed is $v = \sqrt{F/\mu}$

Thus, the tension is $F = \mu v^2$

$$= \left(1.0\times10^{-2}\frac{\text{kg}}{\text{m}}\right)\times6400\frac{\text{m}^2}{\text{s}^2} = 64\,\text{N}$$

Young's modulus is given by $Y = \frac{F/A}{\Delta L/L}$.

The extension is, therefore,

$$\Delta L = \frac{FL}{AY} = \frac{64\times0.50}{(1.0\times10^{-6})\times(8\times10^{11})} = 0.04\text{ mm}$$

7. (3) Time period $T = 4 \times 5\text{ ms} = 20 \times 10^{-3} = 2 \times 10^{-2}\text{ s}$

Frequency, $f = \frac{1}{T} = \frac{1}{(2 \times 10^{-2})} = 50\text{ Hz}$

$\lambda = 2 \times 3\text{ cm} = 6\text{ cm}$

Wave speed: $v = \lambda f = 0.06 \times 50 = 3\text{ m/s}$

8. (2) Given that $x = 40\cos(50\pi t - 0.02\pi y)$

$\therefore$ particle velocity

$$v_p = \frac{dx}{dt} = (40 \times 50\pi)\{-\sin(50\pi t - 0.02\pi y)\}$$

Putting $x = 25$ and $t = \frac{1}{200}$ s.

$$v_p = -(2000\pi\text{ cm/s})\sin\left[50\pi\left(\frac{1}{200}\right) - 0.02\pi(25)\right]$$

$$= 10\pi\sqrt{2}\text{ m/s}$$

9. (2) $a_{\max} = \omega^2 A = g$

$$\omega = \frac{2\pi v}{\lambda}, \quad v = \sqrt{\frac{F}{\mu}}$$

$$A_{\min} = \frac{g\lambda^2\mu}{4\pi^2 F} = \frac{\lambda^2\mu}{4F} = 2 \times 10^{-3}\text{ m} = 2\text{ mm}$$

10. (5) As elevator accelerates up at 2 m/s², tension in the string is $T = m(g + a) = 4(10 + 2) = 48$ N.

Linear mass density of string is $\mu = 19.2 \times 10^{-3}$ kg/m

Speed of transverse waves on string is

$$v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{48}{19.2 \times 10^{-3}}} = 50\text{ m/s}$$

Archives

JEE Main

Single Correct Answer Type

1. (4) The given equation of a wave is

$$y = 0.02\sin\left[2\pi\left(\frac{t}{0.04} - \frac{x}{0.50}\right)\right]$$

Compare it with the standard wave equation

$$y = A\sin(\omega t - kx)$$

we get $\omega = \frac{2\pi}{0.04}\text{ rad s}^{-1}$; $k = \frac{2\pi}{0.5}\text{ rad m}^{-1}$

$$\text{Wave velocity, } v = \frac{\omega}{k} = \frac{(2\pi/0.04)}{(2\pi/0.5)}\text{ ms}^{-1} \quad \ldots\text{(i)}$$

$$\text{Also} \quad v = \sqrt{\frac{T}{\mu}} \quad \ldots\text{(ii)}$$

where T is the tension in the string and μ is the linear mass density Here, linear mass density $\mu = 0.04\text{ kg m}^{-1}$

Equating equations (i) and (ii), we get

$$\frac{\omega}{k} = \sqrt{\frac{T}{\mu}} \text{ or } T = \frac{\mu\omega^2}{k^2}$$

$$T = \frac{0.04 \times \left(\frac{2\pi}{0.04}\right)^2}{\left(\frac{2\pi}{0.05}\right)^2} = 6.25\text{ N}$$

2. (2) $y(x,t) = e^{-(ax^2 + bt^2 + 2\sqrt{ab}\,xt)} = e^{-(\sqrt{a}x + \sqrt{b}t)^2}$

Comparing equation (i) with standard equation

$y(x, t) = f(ax + bt)$

As there is positive sign between x and t terms, hence wave travel in $-x$ direction.

$$\text{Wave speed} = \frac{\text{Coefficient of } t}{\text{Coefficient of } x} = \sqrt{\frac{b}{a}}$$

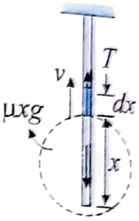
3. (3) Let linear mass density (mass/length) is μ. Consider x length of the string, the tension at this position is T.

$$T = (x\mu)g$$

The velocity of wave pulse $v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{(x\mu)g}{\mu}} = \sqrt{gx}$

$$\frac{dx}{dt} = \sqrt{gx} \Rightarrow \frac{dx}{\sqrt{x}} = \sqrt{g}\,dt$$

$$\Rightarrow \int_0^{20}\frac{dx}{\sqrt{x}} = \sqrt{g}\int_0^t dt$$



On solving we get, $t = 2\sqrt{2}$ s

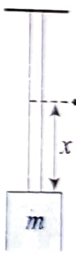
JEE Advanced

Multiple Correct Answers Type

1. (2), (4)

Velocity is independent of frequency and wavelength. Option (2) is correct.

Velocity of the wave depends upon the tension in the rope and mass per unit length of the rope.



$$V_A = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{(M + \mu x)g}{g}};$$

μ = mass per unit length

$\Rightarrow$ v depends on x

Since tension at the mid-point is same, therefore speed at that point will also be same. But the direction of motion of pulses is opposite, velocities will be equal and opposite. With this point of view, Option (1) will be incorrect.

Since the velocity at position is dependent only on tension at every point (μ = constant) therefore time taken for the wave to reach A from O and O from A will be same.

$T_{OA} = T_{AO}$, Option (4) is correct.

We define wave speed as, $v = f\lambda$

Since frequency (f) depends on source

$\Rightarrow \lambda \propto v \Rightarrow \lambda \propto \sqrt{T}$

Tension decreases as we move from O to A, therefore λ becomes shorter. Option (3) is incorrect.

Chapter 7

Concept Application Exercises

ercise 7.1

1. Given $A_1 = 4$ cm and $A_2 = 3$ cm

(a) When two vectors act along the same direction, their resultant is simply th sum of the magnitudes of the two vectors, so

$$A = A_1 + A_2 = 4 + 3 \text{ cm} = 7 \text{ cm}$$

(b) Using $A = \sqrt{A_1^2 + A_2^2 + 2A_1A_2\cos\delta}$,

we have $A = \sqrt{4^2 + 3^2 + 2\times4\times3\times\cos\frac{\pi}{3}} = \sqrt{16+9+12} = \sqrt{37}$

∴ The resultant amplitude is $\sqrt{37}$ cm.

(c) The resultant of two perpendicular vectors A_1 and A_2 is given by $\sqrt{A_1^2 + A_2^2}$, [∵ $\cos(\pi/2) = 0$]

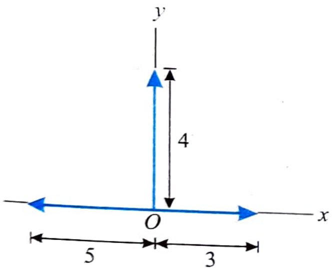
$$A = \sqrt{4^2 + 3^2} \text{ cm} = 5 \text{ cm}$$

(d) The resultant of two vectors in opposite direction is simply the difference in magnitudes between the two vectors, so,

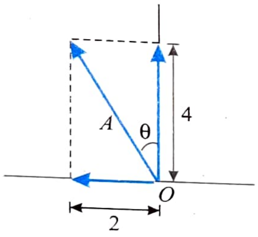
$$A = A_1 - A_2 = 4 - 3 \text{ cm} = 1 \text{ cm}$$

2. The amplitudes can be added vectorially, with angle between them as their phase difference.

All these are shown in Fig. (a). The resultant amplitude 'A' due to the interference of the three waves is evidently from Fig. (b) given by



(a)



(b)

$$A = \sqrt{4^2 + 2^2} = 2\sqrt{5} \text{ cm}$$

Also, $\tan\theta = \frac{2}{4} = \frac{1}{2}$

$$\theta = \tan^{-1}\left(\frac{1}{2}\right)$$

Therefore, the resultant wave leads the first wave by an angle

$$\frac{\pi}{2} + \tan^{-1}\left(\frac{1}{2}\right)$$

Phase difference due to path difference $\Delta\phi_0 = \frac{2\pi}{\lambda}\Delta x = \frac{2\pi}{\lambda}\frac{\lambda}{2} = \pi$

(a) When sources are coherent and in phase, the phase difference at P will be π.

$$\therefore \quad I = I_0 + 2I_0 + 2\sqrt{I_0 2I_0}\cos\pi = 3I_0 - 2\sqrt{2}\,I_0 = 0.2\,I_0$$

(b) When sources are 180° out of phase, the resulting phase difference at P will be zero or 2 π

$$\therefore \quad I = I_0 + 2I_0 + 2\sqrt{I_0(2I_0)}\cos 0 = 3(3 + 2\sqrt{2})I_0$$

(c) When sources are incoherent the intensities add,

$$\therefore \quad I = 3I_0$$

4. $2A_0\cos\left(\frac{\phi}{2}\right) = A_0$

So $\frac{\phi}{2} = \cos^{-1}\left(\frac{1}{2}\right) = 60° = \frac{\pi}{3}$

Thus, the phase difference is $\phi = 120° = \frac{2\pi}{3}$

This phase difference results if the time delay is, $\frac{T}{3} = \frac{1}{3f} = \frac{\lambda}{3v}$

Time delay $= \frac{3\,\text{m}}{3(2\,\text{m/s})} = 0.500\,\text{s}$

5. $AQ - BQ = \lambda$ for reinforcement of waves at Q

or $\sqrt{1.00^2 + 2.40^2} - 2.40 = \lambda$ or $\lambda = 0.2$ m

⇒ $c = n\lambda \Rightarrow c = 1700 \times 0.2 = 340$ m/s

6. $(SA + AD) - SD = n\lambda$ for reinforcement at D

Again $(SB + BD) - SD = n\lambda + \frac{\lambda}{2}$ for total annulment at D

Subtracting, $(SB + BD) - (SA + AD) = \frac{\lambda}{2}$

$$2(SB - SA) = \frac{\lambda}{2}$$

or $\lambda = 2\sqrt{4(H+h)^2 + d^2} - 2\sqrt{4H^2 + d^2}$

7. Since intensity ∝ (amplitude)², the waves may be expressed as

$$y_1 = a\sin(\theta) \quad \text{and} \quad y_2 = 2a\sin(\theta - \pi/2)$$

where $\theta = \omega t - kx$ and $I_0 = \text{constant} \times a^2$ ...(i)

The resultant wave is $y = y_1 + y_2 = a\sin\theta - 2a\cos\theta$

Let $a = A\cos\varphi$, $2a = A\sin\varphi$ ...(ii)

Then $y = A\sin(\theta - \varphi) = A\sin(\omega t - kx - \varphi)$

This is a wave of amplitude A, and lags behind the y_1 wave by phase angle φ, where $\tan\varphi = 2$.

Resultant intensity:

$$I_R = I_0 + 4I_0 + 2\sqrt{I_0 4I_0}\cos\left(\frac{\pi}{2}\right) = 5I_0$$

8. Given $v_1 = 50$ m/s and $v_2 = 200$ m/s

$$A_r = \left(\frac{v_2 - v_1}{v_2 + v_1}\right)A_i \quad \text{and} \quad A_t = \left(\frac{2v_2}{v_2 + v_1}\right)A_i$$

$$\therefore \quad \frac{A_r}{A_t} = \frac{v_2 - v_1}{2v_2} = \frac{(200 - 50)\text{ m/s}}{2 \times 200 \text{ m/s}} = \frac{3}{8}$$

9. At point D, the distance of the ship from point A is

$$d_1 = \sqrt{d_2^2 + (800\,\text{m})^2} = \sqrt{(600\,\text{m})^2 + (800\,\text{m})^2} = 1000\,\text{m}$$

Since destructive interference occurs for the first time when the ship reaches D, it is necessary that $d_1 - d_2 = \frac{\lambda}{2}$

or $\lambda = 2(d_1 - d_1) = 2(1000\,\text{m} - 600\,\text{m}) = 800\,\text{m}$

10. By conservation of energy, the incident power equals the transmitted power plus the reflected power

$$P_{in} = P_t + P_r$$

$$\frac{1}{2}\mu_1\omega^2 A_{in}^2 v_1 = \frac{1}{2}\mu_2\omega^2 A_t^2 v_2 + \frac{1}{2}\mu_1\omega^2 A_r^2 v_1 \quad \text{...(i)}$$

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$$\mu_1 = F/v_1^2 \quad \text{and} \quad \mu_2 = F/v_2^2 \qquad \text{...(ii)}$$

From Eqs. (i) and (ii)

$$\frac{A_{in}^2}{v_1} = \frac{A_t^2}{v_2} + \frac{A_r^2}{v_1}$$

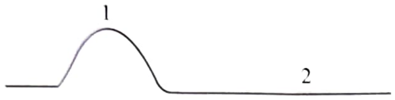
Given, $v_1 = 3v_2$

$$A_{in}^2 = 3A_t^2 + A_r^2 \qquad \text{...(iii)}$$

$$A_t = \left(\frac{2v_2}{v_1 + v_2}\right) A_{in} = \frac{A_{in}}{2}$$

$$A_r = \left(\frac{v_2 - v_1}{v_2 + v_1}\right) A_{in} = \frac{-A_{in}}{2} = \frac{\sqrt{13}}{2} \text{ cm}$$

11. Since $v = \sqrt{T/\mu}$, $T_2 = T_1$



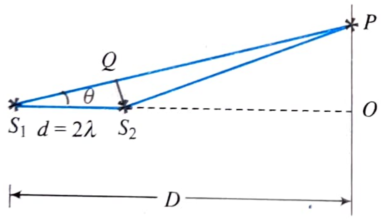
$$\mu_2 = 1/2\mu_1, \ v_2 = \sqrt{2}\, v_1$$

$$A_t = \left(\frac{2v_2}{v_1 + v_2}\right) A_i$$

$$\frac{A_t}{A_i} = \frac{2\sqrt{2}v_1}{v_1 + \sqrt{2}v_1} = \frac{2\sqrt{2}}{(1+\sqrt{2})}$$

12. When detector is at O, we can see that the path difference in the two waves reaching O is $d = 2\lambda$ thus at O detector receives a maximum sound. When it reaches p and again there is a maximum sound detected at P the path difference between two waves must be $\Delta = \lambda$. Thus from the figure the path difference at P can be given as

$$\Delta = S_1P - S_2P \simeq S_1Q = 2\lambda \cos\theta$$



And we have at point P, path difference $\Delta = \lambda$, thus

$$\Delta = 2\lambda \cos\theta = \lambda$$

$$\cos\theta = \frac{1}{2}$$

$$\theta = \frac{\pi}{3}$$

Thus the value of x can be written as $x = D\tan\theta = D\tan\left(\frac{\pi}{3}\right) = \sqrt{3}\,D$

13. The path difference between the sounds going through the tubes

$$\Delta x = ABCD - AED = (10 + 20 + 10) - (\sqrt{500} + 10) = 7.64 \text{ cm}$$

The wavelength of the sound $\lambda = \dfrac{v}{f} = \dfrac{340}{f}$ m

For maximum intensity, $\Delta x = n\lambda$

$$7.64 \times 10^{-2} = n \times \frac{340}{f}$$

$$f = n\frac{340}{7.64 \times 10^{-2}} = n\,4450 \text{ Hz}$$

Here $n \geq 1$. For $n = 1, f = 4450$ Hz and for $n = 2, f = 8900$ Hz. Thus the frequency within the specified range is 4450 Hz.

14. (a) Let d_1 be the length of the fixed path and d_2 be the length of other path that can be varied. Then for minimum sound $d_2 - d_1 = (n + 1/2)\lambda$. When the adjustable tube is moved out by x, the length of the path becomes $d_2 + 2x$. For the next maximum sound $(d_2 + 2x) - d_1 = \left(n + \frac{1}{2}\right)\lambda + \frac{\lambda}{2} = n\lambda + \lambda$

Subtracting $2x = \dfrac{\lambda}{2}$ or $x = \dfrac{\lambda}{4}$

Here $x = 1.65$ cm.

$$\therefore \quad \lambda = 4 \times 1.65 = 6.6 \text{ cm} = 6.6 \times 10^{-2} \text{ m}$$

$$C = n\lambda \quad \text{or} \quad n = \frac{c}{\lambda} = \frac{340}{6.6 \times 10^{-2}} = 5152 \text{ Hz}$$

(b) If a and b are the amplitudes of the waves, then minimum amplitude $= a - b$ and maximum amplitude $= a + b$.

$$\therefore \quad \frac{I_{min}}{I_{max}} = \frac{(a-b)^2}{(a+b)^2} \quad \text{or} \quad \frac{100}{900} = \frac{(a-b)^2}{(a+b)^2}$$

$$\text{or} \quad \frac{a}{b} = \frac{2}{1}$$

Exercise 7.2

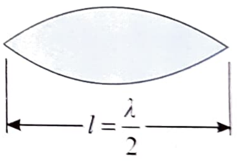
1. Equation of wave is $y = 2\sin\pi t \cos\pi x$

$$\frac{2\pi}{\lambda} = \pi \Rightarrow \lambda = 2\text{m}$$

$x = 0$ m is an antinode and the next antinode is at $x = 1.0$ m. In between node is at $x = 0.5$ m. Two given points are on two sides of a node. Hence they differ in phase by π

2. The fundamental frequency $f_0 = \dfrac{1}{2l}\sqrt{\dfrac{T}{\mu}}$

The wavelength for fundamental frequency is $\lambda = 2l$



$$\text{Or} \quad \lambda = \frac{1}{f_0}\sqrt{\frac{T}{\mu}} \qquad \text{...(i)}$$

Since $f_3 = 3f_0$, we have $f_0 = \dfrac{f_3}{3} = \dfrac{600}{3} = 200$ Hz

$$\text{Then,} \quad \lambda = \frac{1}{200} \times \sqrt{\frac{500}{0.01}} = \frac{\sqrt{5}}{2}\text{m} = 1.118 \text{ m}$$

3. The fundamental frequency of oscillation is

$$f = \frac{1}{2l}\sqrt{\frac{Tl}{m}} = \frac{1}{2}\sqrt{\frac{T}{ml}}$$

$$\frac{f_1}{f_2} = \sqrt{\frac{T_1}{T_2}} \times \sqrt{\frac{m_2}{m_1}} \times \sqrt{\frac{l_2}{l_1}}$$

$$\text{Or} \quad \frac{m_1}{m_2} = \left\{\sqrt{\frac{T_1}{T_2}} \times \sqrt{\frac{l_2}{l_1}} \times \left(\frac{f_2}{f_1}\right)\right\}^2$$

$$= \frac{T_1}{T_2}\frac{l_2}{l_1}\left(\frac{f_2}{f_1}\right)^2 = \frac{3}{1} \times \frac{2}{1} \times \left(\frac{1}{2}\right)^2 = 3:2$$

4. Let f_1 and f_2 be the frequencies in me two cases and L_1, L_2 and T_1, T_2 be respectively the corresponding lengths of the string and tensions under which it is vibrating.

$$f_1 = \frac{1}{2L_1}\sqrt{\frac{T_1}{\mu}} \text{ and } f_2 = \frac{1}{2L_2}\sqrt{\frac{T_2}{\mu}}$$

Thus, $\frac{f_1}{f_2} = \frac{L_2}{L_1}\sqrt{\frac{T_1}{T_2}}$ or $\frac{200}{300} = 2\sqrt{\frac{T_1}{T_2}}$

$(f_1 = 200, f_2 = 300 \text{ and } L_2 = 2L_1 \text{ or } L_2 / L_1 = 2)$

or $\frac{2}{3} = 2\sqrt{\frac{T_1}{T_2}}$ or $\frac{4}{9} = 4\frac{T_1}{T_2}$ or $\frac{T_2}{T_1} = 9$

Thus, the new tensions is 9 times the original tension.

5. The natural frequencies of vibration of a string fixed at both ends are given by

$$f_n = \frac{n}{2L}\sqrt{\frac{T}{\mu}} = \frac{n}{2(2.00)}\sqrt{\frac{20.0}{\left(\frac{0.100}{2.00}\right)}} = n(5.00\,\text{Hz})$$

where $n = 1, 2, 3, \ldots$

(a) $f_1 = 5.0\,\text{Hz}, f_2 = 10.0\,\text{Hz}, f_3 = 15.0\,\text{Hz}$

(b) This could be any mode that has a node 0.40 m from an end. If $D = 0.40$ m is the distance between adjacent nodes (the distance across a pair of nodes), $d_{NN} = D = \frac{\lambda}{2}$ and its wavelength is 0.800 m:

$$\frac{\lambda}{2} = D \Rightarrow \lambda = 2D = 2(0.400\,\text{m})$$

$$\lambda = \frac{2L}{D} \Rightarrow \quad n = \frac{2L}{\lambda} = \frac{2L}{2D} = \frac{L}{D} = \frac{2.0}{0.40} = 5$$

This mode corresponds to the 5th harmonic: f_5 = 5(5.0 Hz) = 25.0 Hz. But D could be the distance across two pairs of nodes (from node to node to node), $d_{NNN} = D = 2\left(\frac{\lambda}{2}\right)$, or three pairs, d_{NNN}, or across N pairs of nodes:

$$N\frac{\lambda}{2} = D \Rightarrow \quad \lambda = \frac{2D}{N}$$

then, $n = \frac{2L}{\lambda} = \frac{2L}{(2D/N)} = N\frac{L}{D} = N\frac{2.0}{0.40} = 5N$

and so on, corresponding to the 10th, or the 15th harmonic, etc. The frequency could be the fifth state at 25.0 Hz or any integer multiple, such as the tenth state at 50.0 Hz, the fifteenth state at 75.0 Hz, and so on.

6. Let A be the area of cross-section of wire and T be the tension applied.

The lowest frequency of transverse vibration is given by

$$n_0 = \frac{1}{2l}\sqrt{\left(\frac{T}{\mu}\right)}$$

Where μ = mass per unit length of wire

= Volume of unit length × density $= A \times 1 \times \rho = A\rho$

We know that Young's modulus $Y = \frac{\text{stress}}{\text{strain}} = \frac{T/A}{\Delta l/l} \Rightarrow T = \frac{YA\Delta l}{l}$

$$n_o = \frac{1}{2l}\sqrt{\left(\frac{YA\Delta l}{lA\rho}\right)} = \frac{1}{2l}\sqrt{\left(\frac{Y\Delta l}{l\rho}\right)}$$

Substituting the given values

$$n_0 = \frac{1}{2\times1}\sqrt{\left(\frac{(9\times10^{10})\times(4.9\times10^{-4})}{1\times(9\times10^3)}\right)} = 35\,\text{Hz}$$

7. Using the relation $2\sin C\cos D = \sin(C + D) + \sin(C - D)$

$$y = 5\sin\frac{\pi x}{3}\cos 40\pi t = \frac{5}{2}\times 2\sin\frac{\pi x}{3}\cos 40\pi t$$

$$y = \frac{5}{2}\left[\sin\left(\frac{\pi x}{3} + 40\pi t\right) + \sin\left(\frac{\pi x}{3} - 40\pi t\right)\right]$$

$$= \frac{5}{2}\sin\left(40\pi t + \frac{\pi x}{3}\right) - \frac{5}{2}\sin\left(40\pi t - \frac{\pi x}{3}\right)$$

Thus, the given stationary wave is formed by the superposition of the progressive waves

$$y_1 = \frac{5}{2}\sin\left(40\pi t + \frac{\pi x}{3}\right)$$

and $y_2 = \frac{5}{2}\sin\left(40\pi t - \frac{\pi x}{3} + \pi\right)$

(a) Comparing each wave with the standard form of the progressive wave

$$y = a\sin\left(\omega t - \frac{2\pi}{\lambda}x + \alpha\right)$$

$a = 5/2 = 2.5$ cm

$\omega = 40\pi \Rightarrow 2\pi f = 40\pi \Rightarrow f = 20\ \text{s}^{-1}$

and $\frac{2\pi}{\lambda} = \frac{\pi}{3}$ or $\lambda = 6$ cm $= 0.06$ m

$\therefore \quad v = f\lambda = 20 \times 0.06 = 1.2$ m/s

(b) Distance between the nodes $= \lambda/2 = 0.06/2 = 0.03$ m

(c) $y = 5\sin\frac{\pi x}{3}\cos 40\pi t$

$$v = \frac{dy}{dt} = -5\times 40\pi\sin\frac{\pi x}{3}\sin 40\pi t$$

$\Rightarrow \quad v = -200\pi\sin\frac{\pi x}{3}\sin 40\pi t$

$\therefore \quad$ At $x = 1.5$ cm and $t = 9/8$ s

$v = -200\pi\sin(\pi/2)\sin 45\pi = 0$

8. $f = \frac{1}{2l}\sqrt{\frac{T}{m}} = \frac{1}{2\times0.25}\sqrt{\frac{10\times9.8}{0.5\times10^{-3}}\times\frac{1}{0.25}} = 1770.8\,\text{Hz}$

A string emits the full series of harmonics. The highest frequency that a man of normal hearing can detect is 20,000 Hz. Let n be the highest harmonic that can be detected by a man of normal hearing. Then

$n \times 1770.8 = 20000$ or $n = 11.29$

$\therefore \quad$ Acceptable value = 11

$\therefore \quad$ Highest overtone = 11 – 1 = 10

9. Stress = $Y \times \text{strain} = Y \times \frac{1}{100} = \frac{2\times10^{11}}{100} = 2\times10^9$

$T(\text{tension}) = \pi r^2 \times \text{stress} = \pi r^2 \times 2 \times 10^9$

$m = \pi r^2\rho = \pi r^2 \times 7700$

$$\therefore \quad n_0 = \frac{1}{2l}\sqrt{\frac{T}{m}} = \frac{1}{2\times1.5}\sqrt{\frac{\pi r^2\times2\times10^9}{\pi r^2\times7700}} = 170\,\text{Hz}$$

10. $n = \frac{1}{2l}\sqrt{T/m}$ for the lowest frequency. Here $m = \pi r^2\rho$

$T\,(\text{tension}) = \pi r^2 \times (\gamma \times \text{strain}) = \pi r^2\gamma\varepsilon$

$$\therefore \quad n = \frac{1}{2l}\sqrt{\frac{\pi r^2\gamma\varepsilon}{\pi r^2\rho}} = \frac{1}{2l}\sqrt{\frac{\gamma\varepsilon}{\rho}} = \frac{1}{2\times1}\sqrt{\frac{9\times10^{10}\times\frac{0.05}{100}}{9000}} = 35.4\,\text{Hz}$$

11. n_0 (fundamental frequency) $= \dfrac{1}{2l}\sqrt{T/m}$

Here $m = \pi r^2 \rho = \pi(0.02 \times 10^{-2})^2 \times 8000$

$= 32\pi \times 10^{-5}$ kg/m

$$\therefore \quad n_0 = \frac{1}{2 \times 0.5}\sqrt{\frac{80}{32\pi \times 10^{-5}}} = 280 \text{ Hz}$$

Obviously 840 Hz is its 3rd harmonic. The third harmonic needs a node at 1/3 point. So strike the wire at 1/6 point and touch gently 1/3 point to make 840 Hz dominant.

12. We know the equation of a stationary wave is given by

$$y = A\sin\left[\frac{2\pi}{\lambda}(vt - x)\right] + A\sin\left[\frac{2\pi}{\lambda}(vt + x)\right]$$

$$= 2A\cos\frac{2\pi x}{\lambda}\sin\frac{2\pi vt}{\lambda} = R\sin\frac{2\pi}{\lambda}vt$$

Here, $R = 2A\cos(2\pi x/\lambda)$ is the amplitude of medium particle situated at a distance x.

The given equation can be expressed as

$$y = 10\sin\left[\frac{2\pi}{50}(5000t - x)\right] + 10\sin\left[\frac{2\pi}{50}(5000t + x)\right]$$

$$= 2 \times 10\cos\left(\frac{2\pi x}{50}\right)\sin\left(\frac{2\pi}{50}5000t\right)$$

Comparing it with standard equation of stationary wave, we get wavelength $\lambda = 50$ units

Wave velocity $v = 5000$ units

Thus amplitude $R = 2 \times 10\cos\dfrac{2\pi x}{50}$

and maximum amplitude $R_{\text{max}} = 2 \times 10 = 20$ units

Frequency $= \dfrac{v}{\lambda} = \dfrac{5000}{50} = 100$ units

And loop length is $\dfrac{\lambda}{2} = \dfrac{50}{2} = 25$ units

13. Let n_1 be frequency of vibration of left wire and n_2 that of second wire. Then given $n_2 = n_1$. If T_1 and T_2 are tensions in first and second wire, then

$$n_1 = \frac{1}{2l}\sqrt{\frac{T_1}{m}} \quad \text{and} \quad n_2 = \frac{2}{2l}\sqrt{\frac{T_2}{m}}$$

Therefore, $\dfrac{n_2}{n_1} = 2\sqrt{\dfrac{T_2}{T_1}}$

As $\quad n_1 = n_2 \quad$ or $\quad \sqrt{T_1} = 2\sqrt{T_2}$

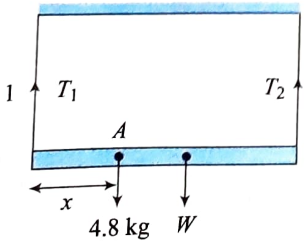
$$\therefore \quad \sqrt{\frac{T_1}{T_2}} = 2 \quad \text{or} \quad T_1 = 4T_2 \qquad \text{...(i)}$$

For vertical equilibrium of rod

$$T_1 + T_2 = 4.8 + 1.2 = 6 \text{ kg wt} \qquad \text{...(ii)}$$

From Eqs. (i) and (ii)

$T_1 = 4.8$ kg, $\quad T_2 = 1.2$ kg



Taking moments about A

$T_1 x + W(0.2 - x) = T_2(0.4 - x)$

$4.8x + 0.24 - 1.2x = 0.48 - 1.2x$

or $\quad 4.8x = 0.24$

$x = \dfrac{0.24}{4.8} = 0.05 \text{ m} = 5 \text{cm}$

Exercise 7.3

1. For open pipe: $f_1 = 300 = \dfrac{v}{2l_1} \Rightarrow l_1 = \dfrac{v}{600} = \dfrac{350}{600} = 0.58$ m

For closed pipe: $\dfrac{3v}{4l_2} = 2f_1$

$\Rightarrow \quad \dfrac{3v}{4l_2} = 600 \Rightarrow l_2 = 0.44$ m

2. The frequency of the fundamental mode of the open pipe filled with air

$$= \frac{c}{2l} = \frac{330}{2 \times 0.11} = 1500 \text{ Hz}$$

The first overtone of the closed pipe $= 3 \times \dfrac{c_{\text{gas}}}{4 \times 0.132}$

$\therefore \quad 3 \times \dfrac{c_{\text{gas}}}{4 \times 0.132} = 1500$

or $\quad c_{\text{gas}} = 264$ m/s

3. (a) $n_0 = \dfrac{c}{4l}$ where n_0 = frequency of the fundamental

$$n_0 = \frac{330}{4 \times 0.15} = 550 \text{ Hz}$$

The first four overtones are $3n_0$, $5n_0$, $7n_0$ and $9n_0$.

So, the required frequencies are 550, 1650, 2750, 3850 and 4950 Hz.

(b) $n_0 = \dfrac{c}{2l} = \dfrac{330}{2 \times 0.15} = 1100$ Hz

The first four overtones are $2n_0$, $3n_0$, $4n_0$ and $5n_0$.

So, the required frequencies are 1100, 2200, 3300, 4400 and 5500 Hz.

(c) The frequency of the nth overtone is $(2n + 1)n_0$ or $(n + 1)n_0$.

$\therefore \quad (2n + 1)\, n_0 = 20{,}000$

or $\quad (n + 1)1100 = 20{,}000$

$\Rightarrow \quad n = 17.18$ or $n = 17.18$

The acceptable value is 17.

4. Wavelength of sound

$$= \frac{v}{f} = \frac{340 \text{ m/s}}{606 \text{ s}^{-1}} = 56.1 \text{ cm}$$

Since, closed pipe allows only odd harmonics, so

$$f = (2n + 1)\frac{v}{4l} \quad \text{or,} \quad l = (2n + 1)\frac{v}{4f}; \quad n \in I$$

or, $\quad l = (2n + 1) \times 14$ cm

$\therefore \quad l = 14$ cm, 42 cm, 70 cm, 98 cm, 126 cm, 154 cm, etc.

Since $\quad l \le 150$ cm

$\therefore \quad$ Number of resonances = 5

5. Since the node-to-node distance is $\lambda/2$

$\lambda/2 = 0.08 \quad$ or $\quad \lambda = 0.16$ m

(a) $\quad c = n\lambda$

$\therefore \quad c = 2000 \times 0.16 = 320$ m/s

(b) $320 = 1600 \times \lambda \quad$ or $\quad \lambda = 0.2$ m

$\therefore$ Distance between nodes $= 0.2/2 = 0.1$ m $= 10$ cm

(c) Since there are nodes at the ends, the distance between the closed end and the membranes must be exact integrals of $\lambda/2$.

$\therefore \quad l = n \times 0.16/2$ and $l = n' \times 0.2/2$

$$\Rightarrow \quad \frac{n}{n'} = \frac{5}{4}$$

When $n = 5, \quad n' = 4$

$$l = 5 \times 0.16/2 = 0.4 \text{ m} = 40 \text{ cm}$$

(d) For the next lower frequency $n = 3$

$\therefore \quad 0.4 = 3\lambda/2$ or $\lambda = 0.8/3$

Since $c = n\lambda, \quad n = \frac{320}{0.8/3} = 1200 \text{ Hz}$

6. For closed pipe, $f = f_0(2n+1)$

Or $\quad 1000 = \frac{v}{4l}(3)$ ($\because$ For Ist overtone $n = 1$)

Or $\quad \frac{v}{l} = \frac{900 \times 4}{3} = 1200$

Then, the fundamental frequency for open pipe is

$$f = \frac{v}{2l} = \frac{1}{2}(1200) = 600 \text{ Hz}$$

7. Let f and f' be the fundamental frequencies of the open and the closed pipes respectively of lengths L and L'.

Clearly, $f = \frac{v}{2L}$ and $f' = \frac{v}{4L'}$ ...(i)

where v is the speed of sound.

Second overtone of the closed pipe $= 5f'$

Fourth overtone of the open pipe $= 5f$

Since the two notes are to be in unison,

$5f' = f$ or $f' = f$ ...(ii)

Thus, from Eqs. (i) and (ii), $\frac{v}{4L'} = \frac{v}{2L}$

or $\quad 4L' = 2L$ or $\frac{L'}{L} = \frac{2}{4} = \frac{1}{2}$

or $\quad L' : L :: 1 : 2$

8. The ratio of successive harmonics $= \frac{1372}{1764} = \frac{7}{9} = \frac{7f_0}{9f_0}$

It means the organ pipe is close organ pipe and required harmonics are 7 and 9.

The fundamental frequency $f_0 = \frac{1372}{7} = 196 \text{ Hz}$

The fundamental frequency for closed organ pipe

$$f_0 = \frac{v}{4l} \Rightarrow 196 = \frac{330}{4l}$$

Or $\quad l = \frac{3 \times 330}{4 \times 1372} = 0.42 \text{ m}$

9. The tube acts as a pipe open at one end and closed at the other. Resonance frequencies are odd harmonics. The length corresponding to the fundamental satisfies

$$f_1 = \frac{v}{4L};\ L_1 = \frac{v}{4f_1} = \frac{320 \text{ m/s}}{4\left(500 \text{ s}^{-1}\right)} = 0.16 \text{ m}$$

Since $L > 20.0$ cm, the next two nodes will be observed, corresponding to $f_2 = \frac{3v}{4L_2}$ and $f_3 = \frac{5v}{4L_3}$

or $\quad L_2 = \frac{3v}{4f_2} = 3L_1 = 0.48 \text{ m}$

and $\quad L_3 = \frac{5v}{4f_3} = 5L_1 = 0.80 \text{ m}$

10. The wavelength of the sound from the tuning fork is $\lambda = \frac{v}{f}$. The cylinder is a pipe open at the top and closed at the water surface; its resonance patterns are AN, ANAN, ANANAN, etc. Resonance occurs each time the height of the air column changes by half a wavelength: $\Delta h = \frac{v}{2f}$. The volume of the pipe between these two water levels is $\pi r^2 \Delta h$, which is also equal to the amount of water that has entered the pipe at rate R in a time interval Δt and has filled this volume.

Therefore, $R\Delta t = \pi r^2 \Delta h = \frac{\pi r^2 h}{2f} \Rightarrow \Delta t = \frac{\pi r^2 v}{2Rf}$

11. For an air column of length 0.70 m, the column may be open ended or closed at one end. For a column open at both ends:

$$f_n = n\frac{v}{2L} \text{ where } n = 1, 2, 3, \ldots..$$

$$f_n = n\frac{v}{2L} = n\frac{350 \text{ m/s}}{2(0.70 \text{ m})} = n(250 \text{ Hz}) \text{ where } n = 1, 2, 3, \ldots..$$

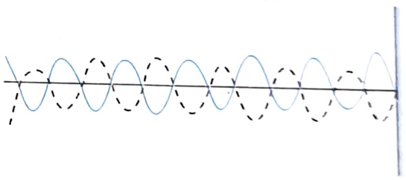
And thus 250 Hz belongs to the harmonic series of an open column (with $n = 1$), but 600 Hz does not match this harmonic series. Similarly, for a column open only at one end:

$$f_n = n\frac{v}{4L}, \text{ where } n = 1, 3, 5\ldots\ldots \text{ (only odd harmonics)}$$

$$f_n = n\frac{v}{4L} = n\frac{350 \text{ m/s}}{4(0.70 \text{ m})} = n(125 \text{ Hz}) \text{ where n} = 1, 3, 5, \ldots.$$

and 250 Hz belongs to the harmonic series of a column open at only one end (for $n = 2$), but 600 Hz does not match this harmonic series. Therefore, it is impossible because a single column could not produce both frequencies.

12. The speed of sound in the rod is, $v = \sqrt{\frac{Y}{\rho}}$



The rod is free at the vibrator end and hence has maximum displacement (displacement antinode). Since the rod is fixed at P, there displacement node occurs. Thus we have odd harmonics. if the fundamental is f_0, the harmonics will be $f_0, 3f_0, 5f_0, 5f_0, 7f_0 (2n-1)$ to etc.

$$f = (2n-1)f_0; n = 1, 2\ldots..$$

The 3rd overtone is $7f_0$

Then, $f = 7f_0$, where $f_0 = \frac{v}{\lambda} = \frac{v}{4\ell}$

or $\quad f = 7\frac{v}{4l} = \frac{7}{4l}\sqrt{\frac{Y}{\rho}} \quad \left(\because v = \sqrt{\frac{Y}{\rho}}\right)$

$$= \frac{7}{4 \times \frac{20}{100}}\sqrt{\frac{6 \times 10^8}{24 \times 10^{-3}}} = \frac{35}{8}\sqrt{10^5} = \frac{875}{2}\sqrt{10} \text{ Hz}$$

Exercise 7.4

1. Frequency of A

$n_A = 400 \text{ s}^{-1}$

Number of beats/s $= 3$.

$\therefore$ Possible frequencies of B,

$n_B = 400 \pm 3 = 403$ or 397

Verification of correct value. A small piece of wax is loaded on B, which decrease the frequency of B. Now the two forks A and B are again sounded, if the number of beats is less than 3, the possible frequency of B is 403, but if number of beats is greater than 3, the possible frequency of B is 397.

2. (a) Number of beats per second $= n_1 = n_2 = 302 - 300 = 2$.

(b) Maximum amplitude $A_{\max} = a_1 + a_2 = 0.2 + 0.3 = 0.5$ mm

Minimum amplitude, $A_{\min} = a_1 - a_2 = 0.3 - 0.2 = 0.1$ mm

(c) $$\frac{I_{\max}}{I_{\min}} = \frac{(a_1 + a_2)^2}{(a_1 - a_2)^2} = \frac{(0.5)^2}{(0.1)^2} = \frac{25}{1}$$

$$\therefore \quad I_{\max} : I_{\min} = 25 : 1$$

3. n_1, frequency of the first wave $= c/0.500$

n_2, frequency of the second wave $= c/0.505$

Obviously $n_1 > n_2 \quad \therefore n_1 - n_2 = 6$

or $$\frac{c}{0.500} - \frac{c}{0.505} = 6 \quad \text{or} \quad c = 303 \text{ m/s}$$

4. Let f_1 and f_2 be the frequencies of the tuning forks A and B respectively.

Since, the beat frequency between A and B is 4

$$\therefore \quad f_1 - f_2 = 4$$

or $$f_2 - f_1 = 4$$

From, $n = \frac{1}{2l}\sqrt{T/\mu}$, we have, $n \propto 1/l$

$$\therefore \quad f_1 \propto 1/32.4 \text{ cm} \quad \text{and} \quad f_2 \propto 1/32 \text{ cm}$$

Evidently, $f_2 > f_1$

$$\therefore \quad f_2 - f_1 = 4 \text{ s}^{-1} \qquad \text{(i)}$$

Also, $$\frac{f_2}{f_1} = \frac{324}{320} \qquad \text{(ii)}$$

Solving Eqs. (i) and (ii), we get

$$f_1 = 320 \text{ Hz} \quad \text{and} \quad f_2 = 324 \text{ Hz}$$

5. $$n_A = \frac{c}{4 \times 0.32} \quad \text{and} \quad n_B = \frac{c}{4 \times 0.33}$$

Obviously $n_A > n_B$

$$\therefore \quad n_A - n_B = \frac{40}{5}$$

$$\therefore \quad \frac{c}{4 \times 0.32} - \frac{c}{4 \times 0.33} = 8 \quad \text{or} \quad c = 337.92 \text{ m/s}$$

$$\therefore \quad n_A = \frac{337.92}{4 \times 0.32} = 264 \text{ Hz}$$

and $$n_B = 264 - 8 = 256 \text{ Hz}$$

6. $$n_1 = \frac{1}{2 \times 0.5}\sqrt{\frac{4.5 \times 10}{m}}$$

$$n_2 = \frac{1}{2 \times 0.6}\sqrt{\frac{5.12 \times 10}{m}}$$

Dividing $\frac{n_1}{n_2} = \frac{9}{8}$ (Obviously $n_1 > n_2$)

$$\therefore \quad n_1 - n_2 = 2 \Rightarrow \frac{9}{8}n_2 - n_2 = 2$$

$$\Rightarrow \quad n_2 = 16, \ n_1 = 18$$

$$\therefore \quad 16 = \frac{1}{2 \times 0.6}\sqrt{\frac{5.12 \times 10}{m}} \Rightarrow m = 0.14 \text{ kg/m}$$

7. $$n = \frac{1}{2l}\sqrt{\frac{W}{m}}$$

where W = weight of load in air

$$n' = \frac{1}{2l}\sqrt{\frac{W'}{m}}$$

where W' = weight of load in water

Obviously, $n' < n$ because $W' < W$

$$\therefore \quad n - n' = 5 \quad \text{or} \quad n' = n - 5$$

or $$\left(\frac{n}{n-5}\right)^2 = \frac{W}{W'}$$

$$W = V(9000)g$$

$$W' = W - B = V(9000)g - V(1000)g = V(8000)g$$

So $$\frac{W}{W'} = \frac{9}{8}$$

$$\therefore \quad \frac{9}{8} = \left(\frac{n}{n-5}\right)^2 \quad \text{or} \quad n = 87.4 \text{ Hz}$$

Exercises

Single Correct Answer Type

1. (1) The direction of wave must be opposite and frequencies will be same then by superposition, standing wave formation takes place.

2. (2) After 2 s, the two pulses will nullify each other. As string now becomes straight, there will be no deformation in the string. In such a situation, the string will not have potential energy at any point. The whole energy will be kinetic.

3. (3) Node means a point at which medium particles do not displace from its mean position and antinode mean a point at which particles oscillate with maximum possible amplitude. Nodes and antinodes are obtained for both types of stationary waves, transverse and longitudinal. Hence, options (1) and (2) both are wrong. To obtain a stationary wave, two waves travelling in opposite directions, having same amplitude, same frequency are required. They must have same nature, means either both of the waves should be longitudinal or both of them should be transverse. Hence, option (3) is correct.

4. (2) When a stationary wave is established in a medium then maximum deformation of the medium is produced at nodes. Hence, maximum pressure change takes place at nodes and at antinodes, no pressure change takes place. Therefore, option (1) is wrong.

$$v = \sqrt{\frac{\text{Elasticity}}{\text{Density}}}$$

Since, elasticity and density both are the characteristic property of the medium, therefore, velocity of a longitudinal wave in a medium is its physical characteristic. So, option (2) is correct.

Due to propagation of longitudinal wave in a medium pressure change

$$\Delta P = \frac{\gamma P u}{v}$$

where u is the velocity of medium particles.

Pressure change will be maximum possible when medium particles have maximum possible velocity, which is equal to $a\omega = 2\pi n a$

Hence, $\Delta P = \gamma P \dfrac{2\pi n a}{v}$

But $\gamma P = \rho v^2$

$\therefore \quad \Delta P = 2\pi n a \rho v$

So, option (3) and therefore option (4) is also wrong.

5. (2) Standing waves form when two waves of equal amplitude, same frequency, same wavelength travelling in opposite directions superimpose, as a result, the net transfer of energy through any cross-section is zero in standing waves.

6. (4) Probable frequency of A is 390 Hz and 378 Hz and after loading the beats are decreasing from 6 to 4 so the original frequency of A will 390 Hz.

7. (1) Let the frequency of first tuning fork = n and that of last = $2n$

$n, n+5, n+10, n+15, \ldots 2n$ this forms AP

Formula of AP, $l = a + (N-1)\,r$ where l = Last term, a = First term, N = Number of term, r = Common difference

$$2n = n + (41-1)\,5$$

$$2n = n + 200$$

$$n = 200 \text{ and } 2n = 400$$

8. (1) The relative velocity of sound waves w.r.t. the wall is $V + v$. Hence, the apparent frequency of the waves striking the surface of the wall is $(V + v)/\lambda$. The number of positive crests striking per second is same as frequency. In 3 s, the number is $[3(V+v)]/\lambda$.

9. (1) In both cases, the 'applied frequency' is same. So, the frequency of vibration has to be same. However, the mode of vibration of the string be different.

10. (4) $$\frac{f_1}{f_2} = \frac{101}{100}$$

$$f_1 - f_2 = 5$$

$$\frac{101}{100} f_2 - f_2 = 5 \text{ or } f_2 = 500 \text{ Hz}$$

and $f_1 = f_2 + 5 = 505$ Hz

11. (2) $\lambda = \dfrac{330}{500} = 0.66\,\text{m}$

The resonance occurs at

$$\frac{\lambda}{4}, \frac{3\lambda}{4}, \frac{5\lambda}{4}, \frac{7\lambda}{4}, \ldots$$

i.e., at 0.165 m, 0.495 m, 0.825 m, 1.115 m. As the length of the tube is only 1.0 m, hence 3 resonances will be observed.

12. (2) $\dfrac{\lambda}{2} = 46 - 16 \Rightarrow \dfrac{\lambda}{2} = 30\,\text{cm}$

or $\lambda = 60$ cm

$\therefore \quad v = \lambda f = \dfrac{60}{100} \times 500 = 300$ m/s

13. (4) $n = \dfrac{1}{2l}\sqrt{\dfrac{T}{m}}$

$$\frac{n_2}{n_1} = \frac{l_1}{l_2}\sqrt{\frac{T_2}{T_1}}$$

$$= \frac{l_1}{\left[l_1 - \dfrac{40}{100} l_1\right]} \sqrt{\left(\frac{T_1 + \dfrac{44}{100} T_1}{T_1}\right)} = \frac{100}{60} \times \frac{12}{10} = 2:1$$

14. (4) According to equation

$$2n_1 = 3n_2$$

or $\dfrac{2}{2l_1}\sqrt{\dfrac{T}{m_1}} = \dfrac{3}{2l_2}\sqrt{\dfrac{T}{m_2}}$

or $\dfrac{l_1}{l_2} = \dfrac{2}{3}\sqrt{\dfrac{m_2}{m_1}} = \dfrac{2}{3}\sqrt{\dfrac{a_2\rho}{a_1\rho}}$

or $\dfrac{l_1}{l_2} = \dfrac{2}{3}\sqrt{\dfrac{r_2^2}{r_1^2}} = \dfrac{2}{3}\sqrt{\left(\dfrac{1}{2}\right)^2}$

or $\dfrac{l_1}{l_2} = \dfrac{1}{3}$

15. (2) Length of the path for direct sound = 120 m

Length of the path for reflected sound

$$= 2\sqrt{(60)^2 + (25)^2]} = 130 \text{ m}$$

Geometrical path difference = 130 – 120 = 10 m

Two waves interfere constructively when $10 = n\lambda$

Putting, $n = 1, 2, 3, \ldots, \lambda = 10, 5, 2.5, \ldots$

16. (1) Let ϕ_1 and ϕ_2 represent angles of the first and second waves, then

$$\phi_2 = \frac{2\pi}{\lambda}[(vt - x) + x_0]$$

and $\phi_1 = \dfrac{2\pi}{\lambda}(vt - x)$

But $x_0 = \dfrac{\lambda}{2}$,

$$\phi_2 - \phi_1 = \pi$$

Hence, phase difference, $\phi = \pi$. So, amplitude of resultant wave

$$R = \sqrt{a^2 + b^2 + 2ab\cos\phi}$$

$$= \sqrt{a^2 + b^2 + 2ab\cos\pi} = \sqrt{(a-b)^2} = a - b$$

or $R = |a - b|$

17. (1) For $x = 5$, $y = 4\sin\left(\dfrac{5\pi}{15}\right)\cos(96\pi t) = 2\sqrt{3}\cos(96\pi t)$

So, y will be maximum when $\cos(96\pi t) = \max = 1$

$$y_{\max} = 2\sqrt{3} \text{ cm at } x = 5$$

18. (2) $v \propto \sqrt{T}, \quad v' \propto \sqrt{T + \dfrac{1}{100}T}$

$$\frac{v'}{v} = \left(1 + \frac{1}{100}\right)^{1/2} = 1 + \frac{1}{200}$$

or $\dfrac{v' - v}{v} = \dfrac{1}{200}$ or $\dfrac{3}{2v} = \dfrac{1}{200}$ or $v = 300$ Hz

19. (3) At 25 cm, there will be antinode. So wire will vibrate in two loops.

$$v = \frac{2}{2l}\sqrt{\frac{T \times l}{M}} \text{ or } v = \sqrt{\frac{T}{Ml}} = \sqrt{\frac{20}{5 \times 10^{-4} \times 1}}$$

$$= \sqrt{4 \times 10^4} \text{ Hz} = 200 \text{ Hz}$$

20. (4) For the given problem, $\dfrac{\sqrt{T}}{l}$ = constant

or $T \propto l^2$

If l is to be doubled, T would be quadrupled.

21. (3) $v = \frac{1}{Dl}\sqrt{\frac{T}{\pi d}}$ or $v \propto \frac{1}{D}$

Now, $\left(\frac{v'}{v} - 1\right) \times 100 = \left(\frac{30}{31} - 1\right) \times 100 = -\frac{100}{31} = -3.2.$

22. (2) $$n_1 = \frac{\omega_1}{2\pi} = \frac{400\pi}{2\pi} = 200 \text{ Hz}$$

$$n_2 = \frac{\omega_2}{2\pi} = \frac{404\pi}{2\pi} = 202 \text{ Hz}$$

Therefore, the number of beats $n = n_2 - n_1 = 2$ Hz

Again $A_1 = 4$ and $A_2 = 3$

$$\frac{I_{\max}}{I_{\min}} = \frac{(A_1 + A_2)^2}{(A_1 - A_2)^2} = \left(\frac{4+3}{4-3}\right)^2 = \frac{49}{1}$$

This is alternative (2) is correct.

23. (4) In ten forks, there are nine intervals

$$n_2 = n_{11} + 9 \times 4 \qquad \text{(Also given } n_2 = 2n_1\text{)}$$

$$2n_1 = n_1 + 36$$

$$n_1 = 36 \text{ Hz}$$

so $n_2 = 2n_1 = 72$ Hz

24. (3) When piston moves a distance x_1, path difference changes by $2x$.

Therefore, the path difference between maxima and consecutive minima $= \lambda/2$

$$2x = \lambda/2$$

or $\lambda = 4x = 4 \times 9 \text{ cm} = 36 \text{ cm} = 0.36 \text{ m}$

$$n = \frac{v}{\lambda} = \frac{360}{0.36} = 1000 \text{ Hz}$$

25. (2) Path difference $= (\pi r - 2r) = (2n-1)\lambda/2$ for minima

Given $\lambda = 0.40$ m, for smallest radius $n = 1$

$$(3.14 - 2)r = \lambda/2$$

$$r = \frac{\lambda}{2 \times 1.14} = \frac{0.40}{2 \times 1.14} = 0.175 \text{ m}$$

26. (4) For maximum path difference $\Delta = n\lambda$

$$2 \times 0.6\, l = \lambda$$

$$l = \frac{\lambda}{1.2} = \frac{6}{1.2} = 5 \text{m}$$

27. (1) Length of open organ pipe $l = 2$ m.

When it is dipped in water, it becomes closed at one end. Let l_1 be the length of air column of pipe immersed, then frequency of first overtone of pipe $= \frac{3v}{4l_1}$

Given

$$\frac{3v}{4l_1} = 170$$

$$l_1 = \frac{3v}{4 \times 170} = \frac{3 \times 340}{4 \times 170} = 1.5 \text{ m}$$

Length immersed $x = l - l_1 = 2 - 1.5 = 0.5$ m

28. (2) When the stone is suspended in air: $n = \frac{1}{2L}\sqrt{\frac{W_a}{m}}$

When the stone is suspended in water: $n = \frac{1}{2L'}\sqrt{\frac{W_w}{m}}$

Hence, $\frac{\sqrt{W_a}}{L} = \frac{\sqrt{W_w}}{L'}$

or $\frac{W_a}{W_w} = \frac{L^2}{L'^2}$

Now, specific gravity of material of the stone

$$= \frac{W_a}{W_a - W_w} = \frac{1}{1 - \frac{W_w}{W_a}} = \frac{1}{1 - \frac{L'^2}{L^2}}$$

$$= \frac{L^2}{L^2 - L'^2} = \frac{(40)^2}{(40)^2 - (22)^2}$$

29. (2) $$n_1 = \frac{1}{2l_1}\sqrt{\frac{T_1}{m}},\ n_2 = \frac{1}{2l_2}\sqrt{\frac{T_2}{m}}$$

$$\therefore \quad \frac{n_2}{n_1} = \frac{l_1}{l_2}\sqrt{\frac{T_2}{T_1}}$$

Let $l_1 = 100l,\ l_2 = 55l$

$T_1 = 100T,\ T_2 = 121\ T$

$$\therefore \quad \frac{n_2}{n_1} = \frac{100l}{55l}\sqrt{\frac{121T}{100T}} = \frac{100}{55} \times \frac{11}{10} = 2$$

$$\Rightarrow \quad n_2 = 2n_1$$

30. (2) Let l be the length of the pipes and v the speed of sound. Then frequency of open organ pipe of nth overtone is

$$f_1 = (n+1)\frac{v}{2l}$$

and frequency of closed organ pipe of nth overtone

$$f_2 = (2n+1)\frac{v}{4l}$$

Therefore, the desired ratio is $\frac{f_1}{f_2} = \frac{2(n+1)}{(2n+1)}$

31. (3) Velocity of longitudinal waves $v_1 = \sqrt{\frac{Y}{\rho}}$

and velocity of transverse waves

$$v_2 = \sqrt{\frac{T}{m}} = \sqrt{\frac{T}{\rho s}}$$

$$\therefore \quad \frac{v_1}{v_2} = \sqrt{\frac{y}{T/s}} = \sqrt{\frac{Y}{Y\left(\frac{\Delta l}{l}\right)}} = \sqrt{n} \qquad \left[\because\ \Delta l = \frac{l}{n}\right]$$

Now $f \propto v$

$$\therefore \quad \frac{f_1}{f_2} = \frac{v_1}{v_2} = \sqrt{n}$$

In the above expression, ρ = density of string, s = area of cross-section of string, Y = Young's modulus.

32. (2) Frequency $f \propto \sqrt{mg}$

or $f \propto \sqrt{g}$

In water, $f_w = 0.8 f_{\text{air}}$

$$\therefore \quad \frac{g'}{g} = (0.8)^2 = 0.64$$

or $$1 - \frac{\rho_w}{\rho_m} = 0.64$$

or $\dfrac{\rho_w}{\rho_m} = 0.36$ ...(i)

or $1 - \dfrac{\rho_L}{\rho_m} = 0.36$

or $\dfrac{\rho_L}{\rho_m} = 0.64$ ...(ii)

From Eqs. (i) and (ii),

$$\frac{\rho_w}{\rho_m} = \frac{0.64}{0.36} = 1.77$$

33. (3) Beat frequency $= f_1 - f_2 = \dfrac{v}{2l} - \dfrac{v}{2(1+x)} = \dfrac{v}{2l}\left[1 - \left(1 + \dfrac{x}{l}\right)^{-1}\right]$

$$= \frac{v}{2l}\left[1 - 1 + \frac{x}{l}\right] = \frac{vx}{2l^2}$$

34. (4) $v_B = v_A + \dfrac{3}{100}v_A$

$$v_C = v_A - \frac{2}{100}v_A$$

$$v_B - v_C = 8$$

$$\frac{3}{100}v_A + \frac{2}{100}v_A = 8$$

or $v_A \times \dfrac{5}{100} = 8$ or $v_A = 160\,\text{Hz}$

35. (3) $f = \dfrac{1}{2l}\sqrt{\dfrac{T}{\mu}}$

If radius is doubled, mass per unit length will become four times. Hence

$$f' = \frac{1}{2 \times 2l}\sqrt{\frac{2T}{4\mu}} = \frac{f}{2\sqrt{2}}$$

36. (1) $\lambda = 2l = 3\text{m}$

Equation of standing wave

(As $x = 0$ is taken as a node)

$y = 2A \sin kx \cos \omega t$,

given $2A = 4$ mm

To find value of x for which amplitude is 2 mm, we have

$$2\text{ mm} = (4\text{ mm}) \sin kx$$

$$\frac{2\pi}{\lambda}x = \frac{\pi}{6} \quad \Rightarrow x_1 = \frac{1}{4}m$$

$$\frac{2\pi}{\lambda}x = \frac{\pi}{2} + \frac{\pi}{3} \Rightarrow x_2 = 1.25\text{ m}$$

$$x_2 - x_1 = 1\text{ m}$$

37. (1) $\Delta v = 384 - 288 = 96$

Thus 288 and 384 (96 × 3; 96 × 4) are third and fourth harmonics.

For fundamental mode: $\dfrac{\lambda}{2} = 0.75$

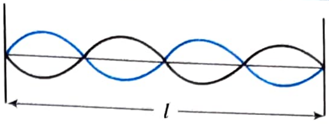
$$\lambda = 1.5$$

$$\eta = 96$$

$$v = 96 \times 1.5 = 144\text{ m/s}$$

38. (3) For a string vibrating in its nth overtone $(n + 1)$th harmonic)

$$(n+1)\frac{\lambda}{2} = L \Rightarrow \lambda = \frac{2L}{n+1}$$



$$kx = \frac{2\pi x}{\lambda} = \frac{\pi(n+1)x}{L}$$

$$y = 2A\sin\left(\frac{(n+1)\pi x}{L}\right)\cos\omega t$$

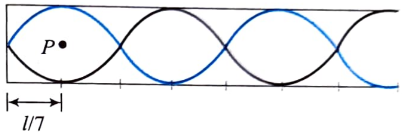
here $2A = a$ and $n = 3$.

for $x = \dfrac{L}{3}$, $y = \left[a\sin\left(\dfrac{4\pi}{L} \times \dfrac{L}{3}\right)\right]\cos\omega t$

$$= a\sin\frac{4\pi}{3}\cos\omega t = -a\left(\frac{\sqrt{3}}{2}\right)\cos\omega t$$

i.e., at $x = L/3$; the amplitude is $\sqrt{3}a/2$.

39. (1) Figure shows variation of displacement of particle in a closed organ pipe for 3rd overtone.



For third overtone

$$l = \frac{7\lambda}{4} \quad \text{or} \quad \lambda = \frac{4l}{7} \quad \text{or} \quad \frac{\lambda}{4} = \frac{l}{7}$$

Hence the amplitude at P at a distance $l/7$ from closed end is 'a' because there is an antinode at that point.

Alternatively:

Because there is node at $x = 0$ the displacement amplitude as function of x can be written as

$$A = a\sin kx = a\sin\frac{2\pi}{\lambda}x$$

For third overtone

$$l = \frac{7\lambda}{4} \quad \text{or} \quad \lambda = \frac{4l}{7}$$

$$A = a\sin\frac{2\pi 7x}{4l}$$

At $x = \dfrac{l}{7} \Rightarrow A = a$

40. (1) For minimum, $\Delta x = (2n-1)\dfrac{\lambda}{2}$

The maximum possible path difference = difference between the sources = 3m.

For no minimum

$$\frac{\lambda}{2} > 3 \Rightarrow \lambda > 6$$

$$f = \frac{V}{\lambda} < \frac{330}{6} = 55$$

If $f < 55$ Hz, no minimum will occur.

41. (2) As $y = A_b \sin(2\pi n_{av} t)$

where $A_b = 2A\cos(2\pi n_A t)$

where $n_A = \dfrac{n_1 - n_2}{2}$

42. (2)

$$y_1 = 2A\sin\omega t$$

$$y_2 = \frac{A}{2}\sin\left(\omega t + \frac{\pi}{6}\right)$$

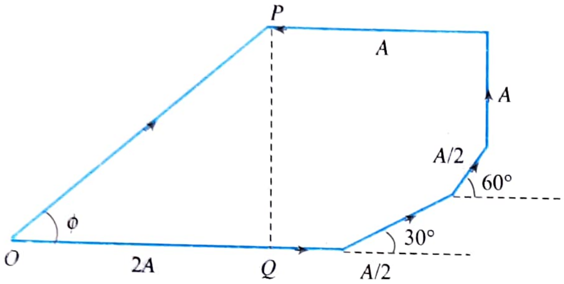
$$y_3 = \frac{A}{2}\sin\left(\omega t + \frac{\pi}{3}\right)$$

$$y_4 = A\sin\left(\omega t + \frac{\pi}{2}\right)$$

$$y_5 = A\sin(\omega t + \pi)$$

By phaser diagram, $\tan\phi = \frac{PQ}{OQ} = 1$

$$\phi = 45^\circ$$



Alternatively:

$$y = 2A\sin\omega t + \frac{A}{2}(\sin\omega t\cos 30^\circ + \cos\omega t\sin 30^\circ)$$

$$+\frac{A}{2}(\sin\omega t\cos 60^\circ + \cos\omega t\sin 60^\circ) + A\cos\omega t - A\sin\omega t$$

$$= A'\cos\phi\sin\omega t + A'\sin\phi\cos\omega t$$

where $A'\cos\phi = \left[A + \frac{A}{4}(\sqrt{3}+1)\right]$

$$A'\sin\phi = \left[A + \frac{A}{4}(\sqrt{3}+1)\right]$$

$$\tan\phi = 1$$

$$\phi = 45^\circ$$

43. (2) Maximum frequency

$$v = \frac{1}{2l}\sqrt{\frac{T}{\mu}} = \frac{1}{2l}\sqrt{\frac{T}{A\rho}} = \frac{1}{2}\sqrt{\frac{7.85\times10^8}{7.7\times10^3}} = 158\text{ Hz}$$

44. (4) As number of beats = Δv

For option (1), the frequencies are

$$v_1 = 550\text{Hz}, v_2 = 552\text{Hz}, v_3 = 553\text{Hz}, v_4 = 560\text{Hz}$$

The beats produced will be

$$\Delta v_1 = v_2 - v_1 = 2$$

$$\Delta v_2 = v_3 - v_1 = 3$$

$$\Delta v_3 = v_4 - v_1 = 10$$

$$\Delta v_4 = v_3 - v_2 = 1$$

$$\Delta v_5 = v_4 - v_2 = 8$$

$$\Delta v_6 = v_4 - v_3 = 7$$

which does not match with the given set of beat frequencies. Hence option (1) is not possible.

Similarly options (2) and (3) are also not possible.

For option (4), frequencies are $v_1 = 550$, $v_2 = 551$, $v_3 = 553$, $v_4 = 558$

$$\Delta v_1 = v_2 - v_1 = 1$$

$$\Delta v_2 = v_3 - v_1 = 3$$

$$\Delta v_3 = v_4 - v_1 = 8$$

$$\Delta v_4 = v_3 - v_2 = 2$$

$$\Delta v_5 = v_4 - v_2 = 7$$

$$\Delta v_6 = v_4 - v_3 = 5$$

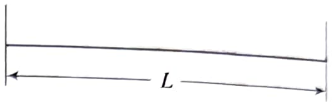
which matches with the given set of beat frequencies. Hence option (4) is correct.

45. (2)

$$V_S = \sqrt{\frac{Y}{\rho}} = \sqrt{\frac{10^{11}}{10.0\times10^4}} = 10^3\text{ m/s}$$

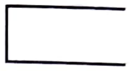
$$t = \frac{2l}{V} = \frac{2\times100}{1000} = 0.2\text{ s}$$

46. (2)



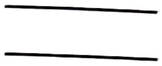
Fundamental frequency of wire $(f_{\text{wire}}) = v/2L$

(1)



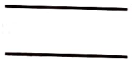
$$f = \frac{v}{4L}, \frac{3v}{4L}, \frac{5v}{4L}$$ cannot match with f_{wire}

(2)



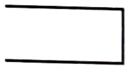
$$f = \frac{v}{2(2L)}, \frac{2v}{2(2L)}, \frac{3v}{2(2L)}$$ its second harmonic $\frac{2v}{2(2L)}$ matches with f_{wire}.

(3)



$$f = \frac{v}{2(L/2)}, \frac{2v}{2(L/2)}$$ cannot match with f_{wire}

(4)



$$f = \frac{v}{4(L/2)}, \frac{3v}{4(L/2)}, \ldots$$ cannot match with f_{wire}

47. (4) x_1 and x_2 are in successive loops of stationary waves.

so, $\phi_1 = \pi$

and $\phi_2 = K(\Delta x) = K\left(\frac{3\pi}{2K} - \frac{\pi}{3K}\right) = \frac{7\pi}{6} = \frac{\phi_1}{\phi_2} = \frac{6}{7}$

48. (1)

$$l_1 + \varepsilon = \frac{v}{4f_0}$$

$$l_2 + \varepsilon = \frac{3v}{4f_0}$$

$$l_3 + \varepsilon = \frac{5v}{4f_0}$$

On solving, we get $l_3 = 2l_2 - l_1$

49. (1) If detector moves x distance, distance from direct sound increases by x and distance from reflected sound decreases by x so path difference created = $2x$

$$2(0.14) = 14\lambda = 14\,c/f$$

$$f = \frac{14\times3\times10^8}{0.14\times2} = 1.5\times10^{10}\text{ Hz}$$

50. (2) Initially the standing wave equation is $y = 2A\sin kx\cos\omega t$

If phase difference ϕ is added to one of waves, then resulting standing wave equation is

$$y = 2A\sin\left(kx + \frac{\phi}{2}\right)\cos\left(\omega t - \frac{\phi}{2}\right)$$

Here, frequency does not change and also spacing between two successive nodes does not change as its value for both is π/k. But for a particle, in standing wave, amplitude changes.

1. (2) Velocity of the string section can be given as

$$v = \frac{\partial y}{\partial t} = 4\cos\pi x \times (50\pi)\cos(50\pi t)$$

$$v = 4\cos\left[\pi \times \frac{1}{3}\right] \times 50\pi\cos\left[50\pi \times \frac{1}{5}\right]$$

$$= 200\pi \times \frac{1}{2} \times 1 = 100\pi \text{ cm/s} = \pi \text{ m/s}$$

2. (4) For closed organ pipe, $f = \frac{v}{4L} \times (2n - 1)$

For minimum and maximum length of pipe the fundamental frequency of pipe must be 20 kHz and 20 Hz, respectively.

$$20 = \frac{320}{4L_{\text{max}}}$$

$$L_{\text{max}} = 4 \text{ m}$$

$$20 \times 10^3 = \frac{320}{4L_{\text{min}}}$$

$$L_{\text{min}} = 4 \text{ mm}$$

53. (4) In a sonometer, $f \propto \sqrt{T}$

$$\text{Thus, } \frac{f_1}{f_2} = 2 = \sqrt{\frac{T_1}{T_2}}$$

$$T_2 = \frac{T_1}{4}$$

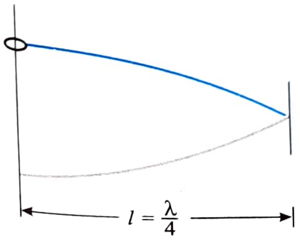
So percentage change will be

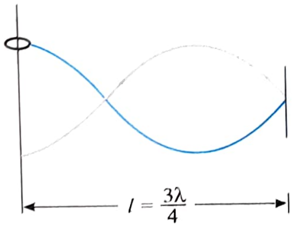
$$\frac{T_1 - T_2}{T_1} \times 100 = \frac{T_1 - \frac{T_1}{4}}{T_1} \times 100 = 75\%$$

54. (2) When the end of the string is free to move, the string being attached to weightless ring that can slide freely along the rod, the phase of reflected pulse is unchanged antinode is formed at the ring.

$$l = \frac{\lambda}{4} \Rightarrow \lambda_1 = 4l = 9.6 \text{ m}$$

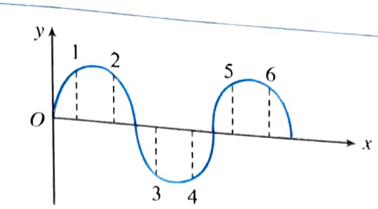
$$\lambda_2 = \frac{4l}{3} = 3.2 \text{ m}$$





$$\lambda_3 = \frac{4l}{5} = \frac{9.6}{5} = 1.92 \text{ m}$$

55. (4) Consider the wave as shown in the figure. The six particles (1–6) have been shown which all have displacements equal to ± $A/2$ form their equilibrium positions.



To get the separation between two particles having displacement of amplitude $A/2$, we have

$$\frac{A}{2} = A\sin(kx - \omega t), \text{ at } t = 0$$

$$\Rightarrow \quad kx = \frac{\pi}{6}, \frac{5\pi}{6}, \ldots \quad \text{and} \quad x_2 - x_1 = \frac{\lambda}{3}$$

separation between particles 1 and 2 comes out to be $\lambda/3$, where λ is the wavelength. Between particles 1 and 3, it is $\lambda/2$. From given information, separation between 1–2 , 3–4 or 5–6 is 8 cm.

$$\lambda/3 = 8\text{cm} \Rightarrow \lambda = 24 \text{ cm}$$

The separation between 2–3 which is equal to separation between 1–3 minus separation between 1–2

$$= \frac{\lambda}{2} - \frac{\lambda}{3} = \frac{\lambda}{6} = 4\text{cm}$$

56. (1) String 1 is heavy so it can easily pull up the lighter string 2, while string 2 being lighter would not be able to displace the point.

57. (2) If one of the natural frequencies of the string matches with the source frequency, then resonance condition will arise and the string will vibrate with source frequency.

58. (3) If none of the natural frequencies of the string matches with the frequency of the source, then string will finally vibrate with the frequency of tuning fork, but here resonance condition would not be found.

59. (3) Velocity of wave on string $= \sqrt{T/\mu} = 8\text{m/s}$.

The pulse gets inverted after reflection from the fixed end, so for constructive interference to take place between successive pulses, the first pulse has to undergo two reflections from the fixed end.

$$\text{So,} \quad \Delta t = \frac{2 \times 0.4 + 2 \times 0.4}{8} = 0.2 \text{ s}$$

60. (4) The equation of stationary wave for open organ pipe can be written as

$$y = 2A\cos\left(\frac{2\pi x}{\lambda}\right)\sin\left(\frac{2\pi ft}{v}\right)$$

where $x = 0$ is the open end from where the wave gets reflected. Amplitude of stationary wave is

$$A_S = 2A\cos\left(\frac{2\pi x}{\lambda}\right)$$

$$\text{For} \quad x = 0.1 \text{ m}, \; A_S = 2 \times 0.002\cos\left[\frac{2\pi \times 0.1}{0.4}\right] = 0$$

61. (2)

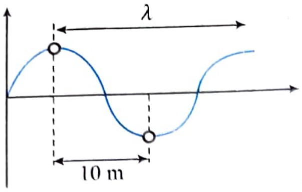
$$f_1 = f_0\left(\frac{V_0}{V_0 - V}\right) \text{ and } f_2 = f_0\left(\frac{V_0}{V_0 + V}\right)$$

$$f_1 - f_2 = f_0V_0\left(\frac{1}{V_0 - V} - \frac{1}{V_0 + V}\right)$$

$$= f_0V_0\left(\frac{V_0 + V - V_0 + V}{V_0^2 - V^2}\right) = f_0V_0 \times \frac{2V}{V_0^2} = f_0\frac{2V}{V_0}$$

Given $\frac{2Vf_0}{V_0} = 0.02 \times f_0 \Rightarrow V = 0.01\,V_0 = 3.4$ m/s

62. (2) Frequency of wave = 1/4 Hz.



Wavelength of wave, $\lambda = 2 \times 10 = 20$ m

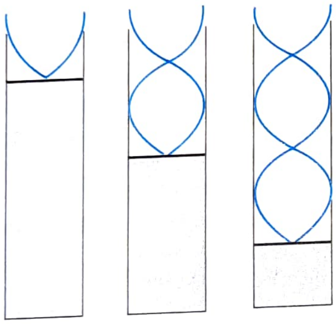
Velocity of wave $f\lambda = 5$ m/s

63. (3) Since the standing wave mode has a displacement antinode at the opening, there is a displacement node at the water–air interface. By increasing the height of the air column, to go from one harmonic to the next, an additional length equal to ½ wavelength is required. Hence

$$\frac{\lambda}{2} = (0.38 - 0.12)\text{m} \Rightarrow \lambda = 0.52 \text{ m}$$

Finally, from $v = f\lambda$, we find that $f = v/\lambda = 312/0.52 = 600$ Hz. If one checks, this problem deals with the 1st and 3rd harmonics.

64. (3) $\lambda/2 = 29.5 - 10.5 = 19$ cm



3rd resonance = 19 + 29.5 = 48.5 cm

65. (1) For minima, $\Delta x = (2n+1)\frac{\lambda}{2}$ and $\lambda = \frac{v}{f}$

$$0.5 = \frac{(2n+1)}{2}\frac{300}{f}$$

$f = (2n + 1)\ 300$

Therefore, all odd multiples of 300 are silenced.

66. (4) Wavelength of sound $= \frac{v}{f} = \frac{340 \text{ m/s}}{606 \text{ s}^{-1}} = 56.1$ cm

Since, closed pipe allows only odd harmonics, so

$$f = (2n+1)\frac{v}{4l} \quad \text{or,} \quad l = (2n+1)\frac{v}{4f};\ n \in I$$

or, $l = (2n + 1) \times 14$ cm

∴ $l = 14$ cm, 42 cm, 70 cm, 98 cm, 126 cm, 154 cm, etc.

Since $l > 150$ cm

∴ No. of resonances = 5

67. (4) For $y = x + \frac{4\pi}{\alpha}$

$$\frac{\partial r}{\partial t} = 0$$

i.e., all points lying on the $y = x + \left(\frac{4\pi}{\alpha}\right)$ are always at rest.

68. (3) Since the point $x = 0$ is a node and reflection is taking place from point $x = 0$. This means that reflection must be taking place from the fixed end and hence the reflected ray must suffer an additional phase change of π or a path change of $\lambda/2$.

So, If $y_{\text{incident}} = a\cos(kx - \omega t)$, then

$$y_{\text{reflected}} = -a\cos(\omega t + kx)$$

69. (3) For 1st reading of oscillator

$$f_A = (514 \pm 2) \text{ Hz}$$

$$f_A = 516 \text{Hz} \quad \text{or} \quad 512 \text{ Hz}$$

For 2nd reading of oscillator

$$f_A = (510 \pm 6) \text{Hz}$$

$$f_A = 516 \text{ Hz or } 504 \text{ Hz}$$

A has a frequency 516 Hz.

75.

70. (1) With clamp at the centre $L = \lambda/2$ for the fundamental.

So, $f' = \frac{v}{2L} = 4$ kHz

When clamp is moved to one end then

$$L = \frac{\lambda}{4}, \frac{3\lambda}{4}, \frac{5\lambda}{4}, \ldots, (2n-1)\frac{\lambda}{4}$$

For $n = 1, 2, \ldots$

$$f_n = (2n-1)\frac{v}{4L}$$

$f_0 = 2\,\text{kHz}$ (1st harmonic)

$f_1 = 6\,\text{kHz}$ (2nd harmonic or 1st overtone)

$f_2 = 10\,\text{kHz}$ (3rd harmonic or 2nd overtone)

71. (3) Supports for the loop are reasonable for nodes at two points.

$$\pi r = n\left(\frac{\lambda}{2}\right)$$

$$\pi\frac{D}{2} = n\frac{\lambda}{2}$$

$$f = n\left(\frac{v}{\pi D}\right)$$

Fundamental frequency = $v/\pi D$

72. (4) $$\frac{n_1}{2\left(\frac{l}{2}\right)}\sqrt{\frac{T}{\pi r^2 \rho}} = \frac{n_2}{2\left(\frac{l}{2}\right)}\sqrt{\frac{T}{\pi(4r^2)\rho}}$$

$$\Rightarrow \quad \frac{n_1}{n_2} = \frac{1}{2}$$

73. (4) 45 cm = 5(9 cm) and 99 cm = 11(9 cm)

So two other lengths between these two values are 7 (9 cm) and 9 (9 cm), i.e., 63 cm and 81 cm respectively.

So the fundamental length is 9 cm

$$9 = \frac{\lambda}{4} \text{ (for a closed organ pipe)}$$

$$\lambda = 36 \text{ cm}$$

74. (3) $$f = \frac{1}{2l}\sqrt{\frac{T}{m}}$$

$$\frac{\Delta f}{f} = \frac{1}{2}\frac{\Delta T}{T}$$

$$\frac{\Delta T}{T} = 2\left(\frac{\Delta f}{f}\right)$$

$$\frac{\Delta T}{T}\times 100 = \text{percentage change in tension}$$

$$\frac{\Delta T}{T}=2\left(\frac{5}{500}\right)$$

$$\frac{\Delta T}{T}=\frac{1}{50}$$

$$\frac{\Delta T}{T}\times 100 = 2\%$$

5. (3) $f_1=\dfrac{v}{4(24.1+0.3D)}$

$$f_3=\frac{3v}{4(74.1+0.3D)}$$

$$f_1=f_3$$

$$\Rightarrow \quad \frac{v}{4(24.1+0.3D)}=3\frac{v}{4(74.1+0.3D)}$$

$$3\ (24.1+0.3D)=74.1+0.3\ D$$

$$72.3+0.9\ D=74.1+0.3\ D$$

$$0.6\ D=74.1-72.3$$

$$0.6\ D=1.8$$

$$D=\frac{1.8}{0.6}=3\text{ cm}$$

76. (2) For interference at A: S_2 is behind of S_1 by a distance of $100\ \lambda=\lambda/4$ (equal to phase difference $\pi/2$). Further S_2 lags S_1 by $\pi/2$. Hence the waves from S_1 and S_2 interfere at A with a phase difference of $200.5\ \pi+0.5\ \pi=201\ \pi=\pi$. Hence the net amplitude at A is $2a-a=a$. For interference at B: S_2 is ahead of S_1 by a distance of $100\lambda+\lambda/4$ (equal to phase difference $\pi/2$). Further S_2 lags S_1 by $\pi/2$. Hence the waves from S_1 and S_2 interfere at B with a phase difference of $200.5\ \pi-0.5\ \pi=200\ \pi=0\ \pi$. Hence the net amplitude at A is $2a+a=3a$.

Hence, $\left(\dfrac{I_A}{I_B}\right)=\left(\dfrac{a}{3a}\right)^2=\dfrac{1}{9}$

77. (1) For 1st overtone, there will be one and half loop as shown in (1).

78. (3) $f_1:f_2:f_3=50:150:250=1:3:5$

Hence this is close pipe of fundamental frequency 50 Hz

$$50=\frac{v}{4L}\Rightarrow L=\frac{v}{200}=\frac{340}{200}=1.7\text{m}$$

79. (3) For pipe A, second resonant frequency is third harmonic thus

$$f=\frac{3V}{4L_A}$$

For pipe B, second resonant frequency is second harmonic thus

$$f=\frac{2V}{2L_B}$$

Equating, $\dfrac{3V}{4L_A}=\dfrac{2V}{2L_B}$

$$\Rightarrow \quad L_B=\frac{4}{3}L_A=\frac{4}{3}\cdot(1.5)=2\text{m}$$

Multiple Correct Answers Type

1. **(1), (2)**

In case of a stationary wave, all the particles lying between two consecutive nodes, oscillate in the same phase.

Since all the particles of given string are oscillating in the same phase, therefore, all the particles of the string lie between two consecutive nodes. Hence, the string is oscillating in the single loop. It means, it is oscillating in its fundamental tone. Hence, option (1) is correct.

Interference is a phenomenon of obtaining constant intensity at a fixed position but the intensity varies with position of the point of observation. Hence, intensity should vary from point to point. Hence, to observe interference, two sources having same frequency must be placed some distance apart. Hence option (2) is correct.

Beats is a phenomenon of obtaining an intensity which varies with time. To obtain beats, two sources having different frequencies are required. Therefore, option (3) is wrong.

2. **(1), (2), (4)**

A stationary wave is characterized by a function of type $y=f(t)\,g(x)$. Hence, choices (1) and (2) represent a stationary wave. Choice (4) is superposition of two oppositely travelling waves of the same amplitude and same frequency, which gives rise to a stationary wave. Hence choice (4) also represents a stationary wave.

3. **(2), (3)**

Comparing with the equation

$$y=2A\sin\left(\frac{n\pi x}{L}\right)\cos(\omega t)\text{, we have}$$

$$2A=2\text{mm} \quad \text{or} \quad A=1\text{ mm}$$

and $\dfrac{n\pi x}{L}=6.28x=2\pi x$

$$L=\frac{n}{2}\text{m}$$

for $n=1, L=0.5\text{ m}$

4. **(2), (3)**

On reflection from a rigid support the reflected wave suffers an additional phase change of π. When this reflected wave superimposes with incident wave stationary waves are obtained with node at the rigid support and intensity of such stationary waves vary periodically with distance.

5. **(1), (3), (4)**

$$f\propto\sqrt{T}$$

$$\frac{f_2}{f_1}=\sqrt{\frac{T_2}{T_1}}=\sqrt{\frac{1.21T_1}{T_1}}$$

$$\Rightarrow \quad \frac{f_1+15}{f_1}=1.1\Rightarrow f_1=150\text{ Hz}$$

$$\frac{v_2}{v_1}=\sqrt{\frac{T_2}{T_1}}=1.1\Rightarrow v_2=1.1\,v_1$$

So velocity increases by 10%. Fundamental wavelength is $\lambda=2L$, which remains same as the length of string remains same.

6. **(1), (2)**

Fundamental frequency: $v=\dfrac{320}{4\times 1}\text{ Hz}=80\text{ Hz}$

Now only odd harmonics are present

7. **(2), (4)**

Since $T_1>T_2, v_1>v_2$

Now, $v_1-v_2=6$

Beat frequency would remains the same even if $v_2-v_1=6$.

To decrease v_1, T_1 needs to be decreased. To increase v_2, T_2 needs to be increased.

8. (3), (4)

For a closed tube $f_n = \dfrac{nv}{4L}$

$$L = 1.1 \text{ m}, v = 330 \text{ m/s}$$

$$f_n = \frac{n \times 330}{4 \times 1.1} = 500\,\text{Hz} \Rightarrow n = 6.666$$

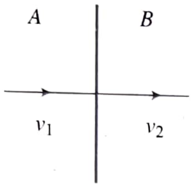
highest frequency, $f_h = \dfrac{6 \times 330}{4 \times 1.1} = 450 \text{ Hz}$

Lowest frequency, $f_1 = \dfrac{1 \times 330}{4.4} = 75 \text{ Hz}$

9. (3), (4)

Since the first wave and the third wave moving in the same direction have the phase angles ϕ and $(\phi + \pi)$, they superpose with opposite phase at every point of the vibrating medium and thus cancel out each other, in displacement, velocity, and acceleration. They in effect, destroy each other out. Hence we are left with only the second wave which progresses as a simple harmonic wave of amplitude A. The velocity of this wave is the same as if it were moving alone.

10. (1), (2), (4)



$v_2 > v_1$

Frequency does not change on changing the medium.

$$\lambda = \frac{v}{f}$$

As velocity increases, so wavelength increase.

As there is no absorption or reflection of wave, so intensity remain same.

11. (1), (2), (4)

If P divides AB in ratio 1:4, then the fundamental frequency corresponds to 5 loops, one loop in AP and 4 loops in PB which corresponds to 5th harmonic of 1 kHz. Hence fundamental = 5 kHz.

If P be taken at midpoint, the third harmonic will have three loops in each half of the wire AB. Hence total number of nodes (including A and B) will be 5 + 2 = 7.

If P divides AB in the ratio 1:2, the fundamental will have three loops, corresponding to the frequency of 3 kHz. For this string to vibrate with the fundamental of 1 kHz, the tension must be $(T/9)$.

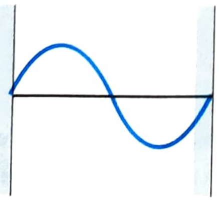
The wire AB will be symmetry, vibrate with the same fundamental frequency when P divides AB in the ratio $a:b$ or in the ratio $b:a$.

12. (1), (2), (4)

Speed of wave in wire $V = \sqrt{\dfrac{T}{\rho A}} = \sqrt{\dfrac{Y\Delta l}{l} A \times \dfrac{1}{\rho A}} = \sqrt{\dfrac{Y\Delta l}{l\rho}}$

Minimum frequency; that means fundamental mode.

$$f = \frac{V}{\lambda} = \frac{V}{2l} = \frac{1}{2l}\sqrt{\frac{Y\Delta l}{l\rho}} = 35 \text{ Hz}$$



$$\text{Stress} = Y\frac{\Delta l}{l} = 9 \times 10^{10} \times \frac{4.9 \times 10^{-4}}{1} = 4.41 \times 10^{7} \text{ N/m}^2$$

and frequency of first overtone = 70 Hz.

13. (1), (3), (4)

$$\frac{\lambda}{4} = 0.1 \Rightarrow \lambda = 0.4$$

From graph ⇒ $T = 0.2$ s and amplitude of standing wave is $2A = 4$ cm.

Equation of the standing wave

$$y(x,t) = -2A\cos\left(\frac{2\pi}{0.4}x\right)\sin\left(\frac{2\pi}{0.2}t\right)\text{cm}$$

$$Y\,(x = 0.05, t = 0.05) = -2\sqrt{2} \text{ cm}$$

$$Y(x = 0.04, t = 0.05) = -2\sqrt{2}\cos 36^\circ$$

$$\text{Speed} = \frac{\lambda}{T} = 2\,\text{m/s}$$

$$V_y = \frac{dy}{dt} = -2A \times \frac{2\pi}{0.2}\cos\left(\frac{2\pi x}{0.4}\right)\cos\left(\frac{2\pi t}{0.2}\right)$$

$$V_y\left(x = \frac{1}{15}\text{m}, t = 0.1\right) = 20\pi \text{ cm/s}$$

14. (2), (3)

At any point on line AB, the phase difference between two waves is zero and hence waves will interfere constructively.

Along CD, the phase difference changes and waves interfere constructively and destructively and, hence sound will be loud, faint and so on.

15. (2), (4).

$$\Delta\phi = \frac{2\pi x_1}{\lambda} - \omega t + \frac{\pi}{4} - 2\pi\frac{x_2}{\lambda} + \omega t - \frac{\pi}{6}$$

$$= \frac{2\pi}{\lambda}(x_1 - x_2) + \frac{\pi}{12}$$

for constructive interference

$$\frac{2\pi}{\lambda}(x_1 - x_2) + \frac{\pi}{12} = 2n\pi \qquad n = 0, 1, 2, \ldots$$

for destructive interference

$$\frac{2\pi}{\lambda}(x_1 - x_2) + \frac{\pi}{12} = (2n - 1)\pi$$

16. (2), (3)

When two waves having same frequency superimpose under given conditions the frequency of the resultant wave is the same as that of component waves. From the theory of interference of waves it can be easily understood. The amplitude of resultant wave for the given situation is given by

$$A = \sqrt{A_1^2 + A_2^2 + 2A_1A_2\cos\delta}$$

where δ is the phase difference between the waves, so A can be anything between $A_1 - A_2$ to $A_1 + A_2$ depending upon the value of δ.

17. (3), (4)

$$y_A = A\sin\left[\omega t - k(AC)\right]$$

$$y_B = \sin\left[\omega t - \frac{\pi}{2} - k\,(BC)\right]$$

For maximum intensity at C

$$k(BC - AC) + \frac{\pi}{2} = 2n\pi$$

$$BC - AC = \left(n\lambda - \frac{\lambda}{4}\right) = 15, 35, 55, 75, \ldots$$

18. (1), (4)

(a) New wave $= y_1\hat{j} + z_1\hat{k} = (a\hat{j} + a\hat{k})\sin\omega\left(\frac{t-x}{v}\right)$

Amplitude $= |a\hat{j} + a\hat{k}| = a\sqrt{2}$

(i) and (ii) are travelling in opposite directions, so they will form stationary waves.

Similarly (iii) and (iv) will make the stationary wave.

19. (3), (4)

$$y = y_1 + y_2 = A\sin(\omega t - kx) + 3A\sin(\omega t + kx)$$

$$= A\sin\omega t\cos kx - A\cos\omega t\sin kx$$

$$+3A\sin\omega t\cos kx + 3A\cos\omega t\sin kx$$

$$= 4A\sin\omega t\cos kx + 2A\cos\omega t\sin kx$$

$$= 2A\sin\omega t\cos kx + 2A\sin(\omega t + kx)$$

It is combination of a stationary and travelling wave.

Maximum amplitude = 4 A

Minimum amplitude = 2 A

Distance between points having amplitude $4A$ and $2A$ will be $= \lambda/4 = v/4f$

20. (1), (2), (3), (4)

If a wave is incident normally on a surface then it gets reflected back to its original path. The incident wave is travelling along negative x-direction and reflected wave is travelling along positive x-direction. Hence, the wave is incident normally on the surface. Therefore, option (1) is correct.

The equation of the reflected wave will be $y' = a'\sin(ct - bx + \phi)$ only when the reflecting surface is $x = 0$ plane, i.e., $y - z$ plane.

Hence option (2) is correct.

Since, ϕ is equal to zero, it means, no phase change takes place at the reflecting surface. It is possible only when the reflecting surface is boundary of a rarer medium. It means wave is travelling in a denser medium relative to the other medium. Hence, option (3) is also correct.

If the reflecting surface is perfectly elastic then whole of the incident energy gets reflected back. In that case a' will be equal to a. But if a part of wave is refracted into the other medium, then the amplitude of oscillations for the reflected wave will be less than that for incident wave. It implies that a' can never be greater than a. Hence, option (d) is also correct.

21. (1), (3)

If a string of length l has cross-sectional area A, density of its material ρ then its oscillation energy is given by

$$E = \pi^2 A\rho a_0^2 l f^2$$

where f is frequency of transverse stationary wave formed in the string.

But $\quad f = \frac{v}{\lambda} = \frac{1}{\lambda}\sqrt{\frac{T}{m}}$

where λ is wavelength, T is tension in the string and $m = A\rho$.

Since, string has a fixed length, therefore, wavelength of a tone excited in the string is constant. Hence, energy $E \propto T$.

Therefore, option (1) is correct.

If the frequency of fundamental tone is f_0, then frequency of nth overtone will be equal to $(n+1)f_0$.

Hence, oscillation energy of the string will be equal to:

$$E_n = \pi^2 A\rho a_0^2\, l\, f_0^2 (n+1)^2$$

Since, E_n is not directly proportional to n^2, therefore, option (2) is wrong.

Since, every particle of the string performs SHM, therefore, r.m.s. speed of a particle $= 1/\sqrt{2}$ × its maximum speed

Hence, average KE is half of maximum KE. But maximum KE is equal to oscillation energy of the string. Therefore, option (3) is correct.

22. (3), (4)

$$t = 0, \frac{1}{(f_1 - f_2)}, \frac{2}{(f_1 - f_2)}, \frac{3}{(f_1 - f_2)}, \ldots$$

are times at which maxima are obtained

$$t = \frac{\frac{1}{2}}{(f_1 - f_2)}, \frac{\frac{3}{2}}{(f_1 - f_2)}, \frac{\frac{5}{2}}{(f_1 - f_2)}, \ldots$$

are times at which minima are obtained.

23. (2), (3)

This is the case of sustained interference in which position of maxima and minima remains fixed all over the screen.

$$\frac{I_{\min}}{I_{\max}} = \left(\frac{a_1 - a_2}{a_1 + a_2}\right)^2$$

And both waves must have been travelling in the same direction with a constant phase difference (condition for coherence)

Linked Comprehension Type

For Problems 1–4

1. (3), 2. (1), 3. (3), 4. (2)

$$y = y_1 + y_2 = (12\sin 5x)\cos 4t$$

Maximum value of y-positions in SHM of an element of the string that is located at an antinode = ± 12 cm ($\sin 5x = \pm 1$)

For the position nodes amplitude should be zero.

So, $\quad \sin 5x = 0 \quad \Rightarrow 5x = n\pi$

$$x = \frac{n\pi}{5}$$

where $n = 0, 1, 2, 3, \ldots$

Value of amplitude at $x = 1.8$ cm

$$A = 12\sin(5 \times 1.8) = 4.9 \text{ cm}$$

At any instant say $t = 0$, instantaneous velocity of points on the string is zero for all points as at extreme position velocities of particles are zero.

For Problems 5–7

5. (1) $y_1 = A\cos(0.5\pi x - 100\pi t)$

$y_2 = A\cos(0.46\pi x - 92\pi t)$

For the first wave angular frequency is $\omega_1 = 100\pi, f_1 = 50$ Hz

For the second wave angular frequency is $\omega_2 = 92\pi, f_2 = 46$ Hz.

Frequency at which the amplitude of resultant wave varies

$$f_A = \frac{f_1 - f_2}{2} = \frac{50 - 46}{2} = 2$$

Time interval between this is maximum.

$$\Delta t = \frac{1}{2f_A}$$

$$\Delta t = \frac{1}{4}$$

Therefore, the number of time intensity is maximum in time 1 s is 4.

6. (3) Wave velocity of louder sound

$$v = \frac{\omega}{k} = 200 \text{ m/s}$$

7. (1) At $x = 0$

$$y = y_1 + y_2 = A \cos (100\,\pi t) + A \cos (92\,\pi t)$$

$$\Rightarrow \quad y = 2A \cos (96\,\pi t) \sin (4\,\pi t)$$

for $y = 0$, we have $\sin (4\,\pi t) = 0$

$$\Rightarrow \quad 4\pi t = n\pi$$

$$\Rightarrow \quad t = \frac{n}{4}$$

$$\Rightarrow \quad t = 0, \frac{1}{4}, \frac{2}{4}, \frac{3}{4}, \frac{4}{4} \text{ in 1 s}$$

This is 5 times

also for $y = 0$, we have $\cos (96\,\pi t) = 0$

$$\Rightarrow \quad 96\pi t = \frac{(2m-1)\pi}{2}$$

$$\Rightarrow \quad t = \frac{2m-1}{192}$$

$$\Rightarrow \quad t = \frac{1}{192}, \frac{3}{192}, \ldots, \frac{191}{192}$$

This is 95 times.

So the total number of times, the value of y is zero is 100.

For Problems 8–10

8. (4) Speed, $V = \sqrt{\dfrac{T}{(m/l)}}$, so it is dependent on tension, mass of the string and its length.

9. (2) A stationary wave is shown as $y = 2A\sin kx\cos \omega t$ in general. So the amplitude of the original wave is A.

10. (3) Third overtone (fourth harmonic) frequency in string is

$$\frac{4v}{2l} = 200 \text{ Hz} = 200 \text{ Hz}$$

$$l = \frac{4v}{400} = \frac{v}{100} = \frac{30}{100} 0.30 \text{ m} = 30 \text{ cm}$$

For Problems 11–13

11. (4), 12. (3), 13. (1)

$$\mu = \frac{1.2}{2} = 0.6 \text{ kg/m}$$

$$n = 5 \text{ Hz}$$

$$\lambda = 2l = 4 \text{ m}$$

$$V = n\lambda = 5 \times 4 = 20 \text{ m/s}$$

Using $v = \sqrt{\dfrac{T}{\mu}}$

$$T = 20^2 \times 0.6 = 240 \text{ N}$$

$$\left(\frac{\partial y}{\partial t}\right)_{\max} = 3.14 \text{ m/s}$$

$$(2A)\omega = 3.14$$

$$\text{Amplitude, } 2A = \frac{3.14}{2 \times (3.14) \times 5} = 0.1 \text{m}$$

Equation of standing wave is $y = (0.1)\sin\left(\dfrac{\pi}{2}\right)x\sin(10\pi)t$

For Problems 14–16

14. (4) For largest mass, $p = 1$

$$n = \frac{p}{2L}\sqrt{\frac{T}{\mu}}$$

$$700 = \frac{p}{2L}\sqrt{\frac{T}{\mu}}$$

$$m = 2 \times 10^{-3} \text{ kg/m}, L = 1\text{m}$$

$$T = [(700)^2 \times 4 \times 1 \times 2 \times 10^{-3}] = 3920 \text{ N}$$

Largest mass to be hang, $M_{\max} = 3920/9.8 = 400$ kg

15. (3) Number of loops formed

$$P = 2nL\sqrt{\frac{\mu}{T}} = 2 \times 700 \times 1\sqrt{\frac{2 \times 10^{-3}}{16 \times 9.8}} = \frac{2 \times 700}{280} = 5$$

16. (1) $A(x) = 6 \sin \dfrac{\pi x}{10}$

Maximum displacement at $x = 5$ cm

$$A(x) = 6\sin\left(\frac{\pi}{10} \times 5\right) = 6 \text{ cm}$$

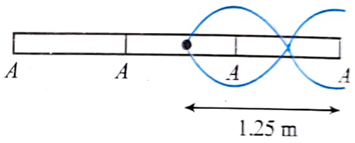
For Problems 17–19

17. (2) Velocity of the longitudinal waves in the rod

$$v = \sqrt{Y/d} = \sqrt{2 \times 10^{11}/8000} = 5000 \text{ m/s}$$

The wavelength of the wave for the mode of vibration in which 2 antinodes occur is

$$\lambda = \frac{1.25}{(3/4)} = \frac{5}{3}\text{m}$$



Hence frequency of vibration

$$n = \frac{V}{\lambda} = \frac{5000}{5/3}\text{Hz} = 3000 \text{ Hz}$$

18. (1) In its fundamental mode of vibration, the wavelength is

$$\lambda_0 = 4 \times 1.25 \text{ m} = 5\text{m}$$

And the angular frequency

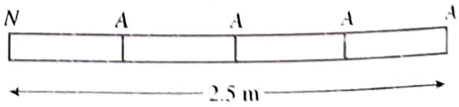
$$\omega_0 = 2\pi n_0 = \frac{2\pi \times 5000}{5} \text{rad/s}$$

Hence the maximum velocity of the wave at the antinode is

$$v_0 = A\omega_0 = 2 \times 10^{-6} \times \frac{2\pi \times 5000}{5} \text{m/s}$$

$$= 1.25 \times 10^{-2} \text{ m/s}$$

19. (3) Since the full length of the rod is set into a single mode of vibration in which four antinodes are seen.



Hence if λ be the wavelength for this mode of vibration. We have

$$\frac{3\lambda}{2} + \frac{\lambda}{4} = 2.5$$

$$\lambda = \frac{4 \times 2.5}{7} = \frac{10}{7}\text{m}$$

$$\text{Hence frequency of vibration } = \frac{5000}{10/7} = 3500 \text{ Hz}$$

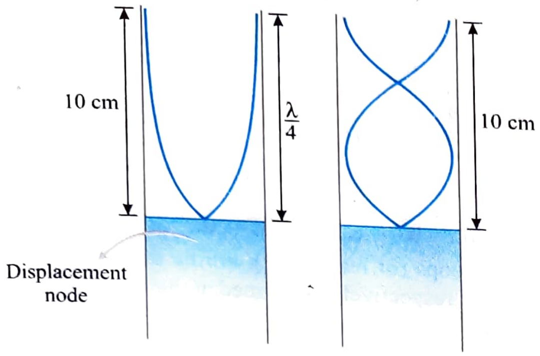
or Problems 20–24

0. (2), 21. (3), 22. (1), 23. (4), 24. (3)

Displacement node corresponds to pressure antinode.

$$L = \frac{\lambda_0}{4}$$

$\lambda_0 = 4L = 40$ cm (First harmonic)



$$v = \lambda f$$

$$f = \frac{v}{\lambda} = \frac{344}{40 \times 10^{-2}} = 860 \text{ Hz}$$

For the second resonance

$$\frac{3\lambda_0}{4} = L$$

$$L = 30 \text{ cm}$$

For the third resonance $5\lambda_0/4 = L$

$$L = 50 \text{ cm}$$

Also, $v = v/\lambda$

3rd harmonic is 2nd overtone.

Hence, frequency for 2nd overtone $= \frac{5v}{4L} = 4300$ Hz

For Problems 25–28

25. (2) Frequency of source = 680 Hz

Velocity of sound = 340 m/s

Wave length $= \lambda = v/f = 340/680\text{m} = 1/2\text{m}$

Let the person is at distance D when he observes first minimum intensity.

Hence the path difference between two source = $\lambda/2$

Path difference at that point

$$\sqrt{D^2 + d^2} - D = \frac{\lambda}{2}$$

$$\sqrt{9^2 + D^2} - D = \frac{\lambda}{2} \qquad (d = 3)$$

$$D\left(1 + \frac{9}{D^2}\right)^{1/2} - D = \frac{\lambda}{2}$$

Using Binomial theorem

$$D = 18 \text{ m}$$

Hence option (b) closest for second minimum.

26. (1) $\Delta x = \frac{3}{2}\lambda$

$$D\left(1 + \frac{9}{D^2}\right)^{1/2} - D = \frac{3}{2}\lambda$$

$D = 6$ m closest 5.6 m

27. (4) Number of maxima

$$d = n\lambda$$

$$n = \frac{d}{\lambda} = \frac{3}{1/2} = 6$$

Here minima will be also 6.

28. (3) When the person is at distance 4 m from pole then the path travelled by wave from $s_2 = 5$ m.

∴ Path difference = 5 – 4 = 1 m

As $\Delta x = n\lambda$, $\lambda = \frac{1}{2}$m

$$n = \frac{1}{\lambda} = 2$$

Therefore, path difference = 1 m = 2λ.

Path difference of 2λ corresponds to 4π

$$y = A\cos(kx - \omega t + 4\pi)$$

It can be expressed as

$$y = A\cos(kx - \omega t + 2\pi)$$

For Problems 29–32

29. (1) $\xi = (0.1 \text{ mm})\cos\frac{2\pi}{0.8}(y + 1 \text{ cm})\cos(400t)$

End correction is 1 cm, so at $y = -1$ cm

$$\xi = (0.1\text{mm})\cos\frac{2\pi}{0.8}(-1\text{cm} + 1\text{cm})$$

$$= (0.1\text{mm})\cos(0) = \text{Antinode}$$

So upper end is open.

At lower end $y = 99$ cm $= 0.99$ m

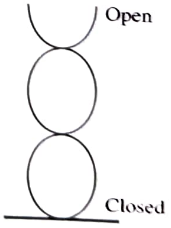
$$\xi = (0.1\text{mm})\cos\frac{2\pi}{0.8}(0.99 + 0.01)$$

$$= 0.01\cos\frac{5\pi}{2} = 0 \Rightarrow \text{Node}$$

So tube is open closed.

30. (2) $\frac{2\pi}{0.8} = \frac{2\pi}{\lambda}$ so $\lambda = 0.8$

And effective length of air column = 0.99 + 0.01 = 1m



So $\frac{l}{\lambda} = \frac{5}{4}$

$$l = 5\frac{\lambda}{4}$$

so five half loops will be formed

$$l = 5\left(\frac{\lambda}{4}\right)$$

so second overtone.

31. (1) $P_{ex} = -B\frac{d\xi}{dx}$

$$= (5\times10^5)\times(0.1\times10^{-3})\frac{2\pi}{0.8}\sin\frac{2\pi}{0.8}(y+1\text{ cm})\cos(400)t$$

$$= (125\pi\text{ N/m}^2)\sin\frac{2\pi}{0.8}(y+1\text{ cm})\cos(400t)$$

32. (1) End correction = (0.3) $d = 1$ cm

$$d = \frac{10}{3}\text{cm}$$

Volume of tube

$$= \left(\pi\frac{d^2}{4}\right)l = \frac{\pi}{4}\left(\frac{10}{3}\right)^2\times100\text{cm}^3$$

(take $l = 0.99$ m $\approx$ 1m)

$$= \frac{10\pi}{36}\text{ litres}$$

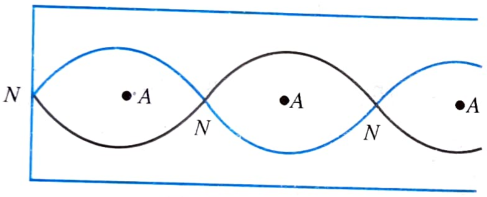
$$\text{No. of moles} = \frac{10\pi}{36\times22.4}\text{moles}$$

(22.4 l contains 1 mol and $10\pi/36l$ contains $10\pi/36\times22.4$ mol)

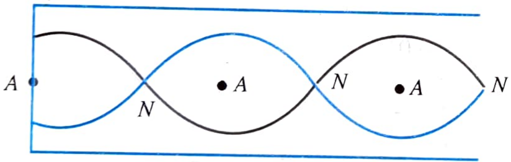
Matrix Match Type

1. i. → a., c.; ii. → b., d.; iii. → b., d.; iv. → a., c.

Second overtone is shown in the figure.



Displacement wave



Pressure wave

Distance between nearest node and antinode is 2/5

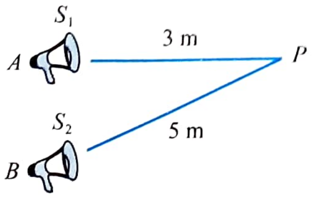
2. i. → a., d.; ii. → a., b., d.; iii. → c.; iv. → a., b., c., d.

When the string is resonating in 1st harmonic or fundamental tone, all the particles of the string are vibrating in phase. When the string is resonating in even harmonic the particles near the ends of the sting are vibrating out of phase as even number of loops are there, while reverse is the case for odd harmonics. In all modes all the particles of the string are crossing mean position or extreme position simultaneously twice in one cycle.

3. i. → c.; ii. → a.; iii. → d.; iv. → b.

Intensity at a distance r from a source of power output P is given by

$$I = \frac{P}{4\pi r^2}$$



$$I_1 = \frac{0.9\pi\text{mW}}{4\pi(3)^2} = \frac{1}{40}\text{mW} = 25\ \mu\text{W/m}^2$$

$$I_2 = \frac{9\pi\text{mW}}{40\pi(5)^2} = \frac{9}{1000}\text{mW} = 9\ \mu\text{W/m}^2$$

For incoherent source,

$$I_R = I_1 + I_2 = (25+9) = 34\ \mu\text{W/m}^2$$

For coherent source, $\Delta = 0$

$$I_R = I_1 + I_2 + 2\sqrt{I_1I_2}$$

$$\left(\sqrt{I_1}+\sqrt{I_2}\right)^2 = (5+3)^2 = 64\ \mu\text{W/m}^2$$

For $\delta = \pm\pi, I_R = \left(\sqrt{I_1}-\sqrt{I_2}\right)^2 = (5-3)^2 = 4\ \mu\text{W/m}^2$

If the speaker S_2 is switched off, $I_R = I_1 = 25\ \mu\text{W/m}^2$

4. i. → a., b., d., ii. → c., d., iii. → d., iv. → c. ,d.

Number of loops (of length $\lambda/2$) will be even or odd and node or antinode will respectively be formed at the middle.

Phase difference between two particles in same loop will be zero and that between two particles in adjacent loops will be π.

Number of loops will not be integral. Hence neither a node nor an antinode will be formed in the middle.

Phase difference between two particle in same loop will be zero and that between two particles in adjacent loops will be π.

5. i. → c.; ii. → a.; iii. → d.; iv. → b.

$v_1 = 1310$ Hz, $v_2 = 1834$ Hz, $v_3 = 2358$ Hz

$$\frac{v_2}{v_1} = \frac{7}{5};\quad \frac{v_3}{v_1} = \frac{9}{5}$$

$$v_1 : v_2 : v_3 = 5:7:9$$

(This corresponds to a pipe closed at one end)

Fundamental frequency $n_0 = \dfrac{2358}{9} = 262$ Hz

Frequency of the first overtone $n_1 = 3n_0 = 786$ Hz

Frequency of the fifth overtone $= 5n_0 = 1310$ Hz

For fundamental frequency

$$\lambda_0/4 = l$$

$$\lambda_0 = 4l$$

$$n_0 = \frac{v}{4l}$$

$$\Rightarrow\quad l = \frac{v}{4n_0} = \frac{340}{4\times262} = 32.4\times10^{-2}\text{ m}$$

Length of the pipe = 32.4 cm

6. i. → b.; ii. → a.; iii. → c.; iv. → d.

For (i): $2\left(\dfrac{\lambda}{2}\right) = L \Rightarrow \lambda = L$

$$K = \frac{2\pi}{\lambda} = \frac{2\pi}{L}$$

at $x = 0$ and at $x = L$, we have $y = 0$ for all t.

Option (b) is suitable for above all.

For (ii): $\dfrac{\lambda}{2} = L \Rightarrow \lambda = 2L$

$$K = \frac{2\pi}{\lambda} = \frac{2\pi}{2L} = \frac{\pi}{L}$$

at $x = 0$ and at $x = L$, we have $y = 0$ for all t.

Option (a) is suitable for above all.

For (iii): $\frac{\lambda}{4} = L \Rightarrow \lambda = 4L$

$$K = \frac{2\pi}{\lambda} = \frac{2\pi}{4L} = \frac{\pi}{2L}$$

Option (c) is suitable for this.

For (iv): $\frac{\lambda}{4} + \frac{\lambda}{4} = L \Rightarrow \lambda = 2L$

$$K = \frac{2\pi}{\lambda} = \frac{2\pi}{2L} = \frac{\pi}{L}$$

Option (d) is suitable for this.

7. **i. → a., c.; ii. → b., d.; iii. → a., c.; iv. → b., d.**

In string $V = \sqrt{T/\mu} = 320$ m/s

Open pipe and string fixed at both ends.

$$f = \frac{nv}{2L} = 320, 640, 960, \ldots$$

Closed pipe and string free end

$$f = (2n-1)\frac{v}{4L} = 160, 480, 800, \ldots$$

8. **i. → a., b.; ii. → d.; iii. → a., c; iv. → d.**

(i) $y = 4\sin(5x - 4t) + 3\cos(4t - 5x + \pi/6)$
is super position of two coherent waves, so their equivalent will be an another travelling wave.

(ii) $y = 10\cos\left(t - \frac{x}{330}\right)\sin(100)\left(t - \frac{x}{330}\right)$

Let us check at any point, say at $x = 0$.

$y = (10\cos t)\sin(100\,t)$

at any point amplitude is changing sinusoidally, so this is equation of beats.

(iii) $y = 10\sin(2\pi x - 120\,t) + 10\cos(120\,t + 2\pi x)$ = superposition of two coherent waves travelling in opposite direction.

(iv) $y = 10\sin(2\pi x - 120\,t) + 8\cos(118 - 59/30\,\pi x)$ = superposition of two waves whose frequencies are slightly different ($\omega_1 = 120$; $\omega_2 = 118$) ⇒ equation of beats.

9. **i. → a., c.; ii. → b., d.; iii. → c.; iv. → d.**

Bass strings have low fundamental frequency and larger wavelengths. For having low frequency the string has to be long according to expression, $f \propto v/L$. For low f, v should be low, i.e., string should be thick. For treble strings also, the same explanation holds true.

10. **i. → b., d.; ii. → b., d.; iii. → a.; iv. → b.; c.**

Velocity of wave on a string is given by $v = \sqrt{T/\mu}$. Frequency is the property of source. Wavelength = v/f

Numerical Value Type

1. **(2)** $l = 15.0$ m, $v = 12$ ms^{-1}

Since there are 6 nodes, with the ends as nodes there will be five half wavelength in the string.

So, $\frac{5\lambda}{2} = l = 15 \Rightarrow \lambda = 6.0$ m

Using $f = \frac{v}{\lambda} = \frac{12}{6} = 2.0$ Hz

2. **(1)** Since frequencies are in odd number ratio, the pipe has to be a closed pipe.

Ratio of 3 frequencies = 425 : 595 : 765 = 5 : 7 : 9

So fundamental frequency = $f = \frac{425}{5} = 85$ Hz

For fundamental frequency $l = \frac{v}{4f} = \frac{340}{4 \times 85} = 1$ m

3. **(3)** The pattern corresponds to $l = \frac{5\lambda}{4} = 2.0$ m $\quad \lambda = \frac{8}{5}$ m

With speed $v = 5.0$ ms^{-1}

$$f = \frac{v}{\lambda} = \frac{5 \times 5}{8} = 3.1 \text{ Hz}$$

4. **(3)** $f \propto \sqrt{T}$ for strings.

On increasing the tension by 1%

$$f' = \sqrt{101T}$$

$$\frac{f'}{f} = \frac{\sqrt{1.01T}}{\sqrt{T}} = (1 + 0.01)^{\frac{1}{2}} = 1 + \frac{1}{200}$$

Beat frequency, $f' - f = f\left(\frac{f'}{f} - 1\right) = 1$

Number of beats in 3 s = 1 × 30 = 30

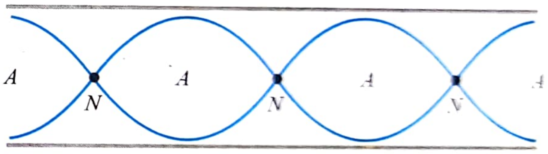
5. **(7)** $f_o - f_c = 2$

$$V\left[\frac{1}{2L} - \frac{1}{4L}\right] = 2 \quad \text{or} \quad V/L = 8$$

In the second case,

$$f_0' - f_c' = \frac{V}{L} - \frac{V}{8L} = \frac{7V}{8L} = \frac{7}{8}(8) = 7$$

6. **(3)** We have to find the number of pressure antinodes (displacement nodes), which is 3 (from the diagram).



7. **(9)** $n\left(\frac{V}{4L_c}\right) = m\left(\frac{V}{2L_0}\right)$ ...(i)

Also $3\left(\frac{V}{4L_c}\right) = 2\left(\frac{V}{2L_0}\right)$ ...(ii)

From Eq. (ii) $\frac{L_c}{L_0} = \frac{3}{4}$

From Eq. (i) $\frac{n}{m} = 2\left(\frac{L_c}{L_0}\right) = \frac{6}{4} = \frac{3}{2} = \frac{9}{6}$

$n = 9$ if $m = 6$

8. **(4)** $f \propto \frac{(T/\mu)^{1/2}}{L}$

Where μ = mass per unit length = $\rho a = \rho(\pi r^2)$

So, $f \propto \frac{(T/\rho)^{1/2}}{rL}$

$$\frac{f_2}{f_1} = \left(\frac{T_2}{T_1}\right)^{1/2}\left(\frac{\rho_1}{\rho_2}\right)^{1/2}\left(\frac{r_1 L_1}{r_2 L_2}\right) = \left(\frac{1}{\sqrt{2}}\right)(\sqrt{2})(4) = 4$$

9. **(2)** $f = 500$ Hz, $v = 300$ ms^{-1}

$$\lambda = \frac{v}{f} = \frac{300}{500} = \frac{3}{5} \text{ m}$$

Resonating length $l = \frac{(2n-1)v}{4f}$

$$l = \frac{(2n-1)\times 300}{4\times 5} \le 1 \text{ m}$$

$$n \le 23/6 = 3.83$$

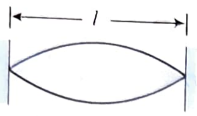
Since only odd harmonics are possible there will be only two resonant lengths.

10. (1) For fundamental frequency

$$\mu = \frac{3.2\text{ g}}{40\text{ cm}} = \frac{3.2\times 10^{-3}}{40\times 10^{-2}} = \frac{3.2}{400} = \frac{32}{4000} \text{ kg/m}$$

$$l = \lambda/2$$

$$\lambda = 2l \qquad \text{...(i)}$$



$$f = \frac{v}{\lambda} = \frac{1}{2l}\sqrt{\frac{T}{\mu}}$$

$$\frac{1000}{64} = \frac{1}{2\times 40\times 10^{-2}}\sqrt{\frac{T}{32/4000}}$$

$$\left[\frac{1000}{64}\times 2\times 40\times 10^{-2}\right]^2 \frac{32}{4000} = T$$

$$\Rightarrow \quad T = \frac{10}{8}\text{ N}$$

$$Y = \frac{\text{stress}}{\text{strain}} = \frac{\dfrac{10/8}{10^{-6}}}{\dfrac{0.05\times 10^{-2}}{40\times 10^{-2}}} = 10^9 \text{ N/m}^2$$

11. (1.17)

$$\lambda_{\text{air}} = \frac{v_{\text{air}}}{f} = \frac{330}{1000} = 0.33\text{ m}$$

$$v_{\text{water}} = \sqrt{\frac{\beta}{\rho}} = \sqrt{\frac{2.25\times 10^9}{1000}} = 15\times 10^3 = 1500$$

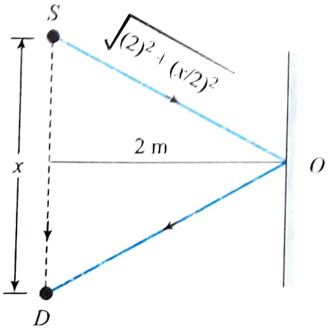
$$\lambda_{\text{water}} = \frac{1500}{1000} = 1.5\text{ m}$$

$$\lambda_{\text{water}} - \lambda_{\text{air}} = 1.5 - 0.33 = 1.17\text{ m}$$

12. (3) The situation is shown in the figure.

Suppose the detector is placed at a distance of x metre from the source. The wave received from the source travels a distance x metre. The wave reaching the detector after reflection from the wall has travelled a distance of $2[\sqrt{(2)^2 + x^2/4}]$ metre. Thus the path difference between the two waves at detector is

$$\Delta = \left\{2\sqrt{(2)^2 + \frac{x^2}{4}} - x\right\} \qquad \text{...(i)}$$



Constructive interference will take place when $\Delta = \lambda, 2\lambda, \dots$. Thus the minimum distance x for which a maximum occurs at detector, the path difference will be

$$\Delta = \lambda \qquad \text{...(ii)}$$

The wavelength is

$$\lambda = \frac{v}{n} = \frac{360}{180} = 2\text{ m}$$

$$\Delta = 2\sqrt{2^2 + \frac{x^2}{4}} - x = 2$$

$$\sqrt{4 + \frac{x^2}{4}} = 1 + \frac{x}{2}$$

$$4 + \frac{x^2}{4} = 1 + \frac{x^2}{4} + x$$

$$x = 3\text{ m}$$

Thus, the detector should be placed at a distance of 3 m from the source to detect a maximum sound.

13. (220)

Frequency of first fork = n. Frequency of 56th fork = $2n$

$$\therefore \quad n + (56-1)4 = 2n$$

$$\therefore \quad n + 220 = 2n$$

$$n = 220\text{ Hz}$$

14. (256)

$$n_A - n_B = 8$$

Also $n_A = \dfrac{v}{4(0.32)}$, $\quad n_B = \dfrac{v}{4(0.33)}$

$$\frac{v}{4(0.32)} - \frac{v}{4(0.33)} = 8$$

$$v = 338\text{ m/s}$$

$$\therefore \quad n_A = \frac{v}{4\times 0.32} = \frac{338}{4\times 0.32} = 264\text{ Hz}$$

$$n_B = n_A - 8 = 256\text{ Hz.}$$

15. (532)

$$n/\sqrt{T} = \text{constant} \qquad \text{...(i)}$$

If f is frequency of fork, the frequency of string = $f \pm 4$ ...(ii)

From Eqs. (i) and (ii), we get

$$\frac{f+4}{\sqrt{10.2}} = \frac{f-4}{\sqrt{9.9}}$$

Solving for f, we get f = 532 Hz.

16. (540)

$$n_A = \frac{v}{4\times 0.15}, \quad n_B = \frac{v}{2\times 0.305}$$

Clearly $n_A > n_B$

$$n_A - n_B = 9$$

$$\frac{v}{0.60} - \frac{v}{0.61} = 9$$

$$v = 329.4\text{ m/s}$$

$$n_A = \frac{329.4}{0.60} = 549\text{ Hz}$$

$$n_B = n_A - 9 = 549 - 9 = 540\text{ Hz.}$$

17. (39.2)

Kundt's tube is a closed organ pipe. Nodes are formed at ends. The separation between six antinodes is $5\lambda/2$.

Length of Kundt's tube $= \dfrac{\lambda}{4} + \dfrac{5\lambda}{2} + \dfrac{\lambda}{4} = 3\lambda$

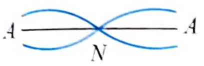
If steel rod is clamped in the middle, node is formed in the middle and antinodes at ends.

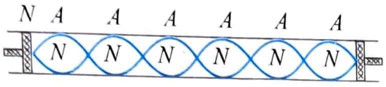
$$\therefore \quad l_{\text{steel}} = \frac{\lambda_{\text{steel}}}{2}$$

or $\quad \lambda_{\text{steel}} = 2l_{\text{steel}} = 2\text{ m}$

Also $n_{\text{air}} = n_{\text{steel}}$

$$\therefore \quad \frac{v_{\text{air}}}{\lambda_{\text{air}}} = \frac{v_{\text{steel}}}{\lambda_{\text{steel}}}$$





$$\lambda_{\text{air}} = \frac{v_{\text{air}}}{v_{\text{steel}}} \times \lambda_{\text{steel}} = \frac{343}{5250} \times 2 = 0.1306\text{ m}$$

$$\therefore \quad l_{\text{air}} = 3\lambda_{\text{air}} = 3 \times 0.1306 = 39.2\text{ cm}$$

18. (50) $\quad \dfrac{v_{51}}{\lambda} - n = 4$

$$\frac{v_{51}}{\lambda} = n + 4 \qquad \text{...(i)}$$

and $\quad \dfrac{v_{16}}{\lambda} - n = 1 \qquad$ ...(ii)

$$\frac{v_{16}}{\lambda} = n + 1$$

Dividing Eq. (ii) by Eq. (i)

$$\frac{v_{51}}{v_{16}} = \frac{n+4}{n+1}$$

As $\quad v \propto \sqrt{T}$

$$\Rightarrow \quad \sqrt{\frac{273+51}{273+16}} = \frac{n+4}{n+1}$$

or $\quad \sqrt{\dfrac{324}{289}} = \dfrac{n+4}{n+1}$ or $\dfrac{18}{17} = \dfrac{n+4}{n+1}$

$n = 50$ Hz.

19. (500)

Wave speed $\quad v = \sqrt{\dfrac{T}{\mu}} = \sqrt{\dfrac{Mg}{\mu}}$

If the string oscillates with n loops when $M = 16$ kg then there will be $(n - 1)$ loops for $M = 25$ kg

$$\therefore \quad f = \frac{n}{2L}\sqrt{\frac{T}{\mu}}$$

But frequency is same in both cases,

$$\therefore \quad \frac{n}{2L}\sqrt{\frac{16g}{\mu}} = \frac{(n-1)}{2L}\sqrt{\frac{25\,g}{\mu}}$$

$$\therefore \quad 4n = 5(n-1) \Rightarrow n = 5$$

$$\therefore \quad f = \frac{5}{2\times 2}\sqrt{\frac{16\times 10}{1\times 10^{-3}}} = \frac{5}{4}\times 4\times 10^2 = 500\text{ Hz}$$

20. (3.5) $\quad f_0 = \dfrac{1}{2L}\sqrt{\dfrac{T}{\mu}}$

$$f = \frac{1}{2(L+\Delta L)}\sqrt{\frac{T+\Delta T}{\mu}} = \eta f_0$$

$$\therefore \quad \eta = \frac{1}{L+\Delta L}\sqrt{\frac{T+\Delta T}{T}} = \frac{\sqrt{\dfrac{T+\Delta T}{T}}}{1+\dfrac{\Delta T}{T}}$$

$$\therefore \quad \eta_1 = \frac{\sqrt{1-0.96}}{1-0.35} = \frac{1.4}{0.65} \text{ and } \eta_2 = \frac{\sqrt{1-0.36}}{1+0.3} = \frac{0.8}{1.3}$$

$$\therefore \quad \frac{\eta_1}{\eta_2} = \frac{1.4}{0.65} \times \frac{1.3}{0.8} = 3.5$$

Archives

JEE Main

Single Correct Answer Type

1. (3) Let the given sources of sound produce frequencies, $(f - 1)$, f and $(f + 1)$.

For two sources of frequencies f_1 and f_2.

$$y_1 = A\cos 2\pi f_1 t$$

$$y_2 = A\cos 2\pi f_2 t$$

If these two waves superpose then

$$y = 2A\cos 2\pi\left(\frac{f_1 - f_2}{2}\right)t\cos 2\pi\left(\frac{f_1+f_2}{2}\right)t.$$

The resultant frequency obtained is $\dfrac{f_1+f_2}{2}$ and this wave is modulated by a wave of frequency $\dfrac{f_1 - f_2}{2}$

Hence the intensity waxes and wanes. For a cosine curve (or sine curve), the number of beats $= f_1 \sim f_2$

Frequencies	Mean	Beats
$f + 1$ and f	$(f + 0.5)$ Hz	1
f and $f - 1$	$f - 0.5$	1
$(f + 1)$ and $f - 1$	f	2

Total number of beats = 2.

2. (1) $\quad f = \dfrac{v}{2l}$

Now, it will act like one end open and other closed.

$$f = \frac{v}{4l'} = \frac{v}{4\dfrac{l}{2}} = \frac{v}{2l} = f$$

3. (1) Fundamental frequency of vibration of sonometer wire is

$$f_0 = \frac{1}{2L}\sqrt{\frac{T}{\mu}}$$

where L is the length of the wire, T is the tension in the wire and μ is the mass per length of the wire

As $\quad \mu = \rho A$

where ρ is the density of the material of the wire and A is the area of cross-section of the wire.

$$\therefore \quad f_0 = \frac{1}{2L}\sqrt{\frac{T}{\rho A}}$$

But we have $Y = \dfrac{\text{Stress}}{\text{Strain}} = \dfrac{TL}{A\Delta L}$

The tension in the wire is due to elasticity of wire

$$\therefore \quad T = YA\left[\frac{\Delta L}{L}\right]$$

Hence, $f_0 = \frac{1}{2L}\sqrt{\frac{Y\Delta L}{\rho L}}$

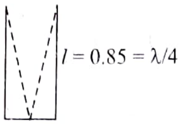
Here, $Y = 2.2\times10^{11}\ \text{N/m}^2, \rho = 7.7\times10^3\ \text{kg/m}^3$

$$\frac{\Delta L}{L} = 0.01, \quad L = 1.5\,\text{m}$$

Substituting the given values, we get

$$f_0 = \frac{1}{2\times1.5}\sqrt{\frac{2.2\times10^{11}\times0.01}{7.7\times10^3}} = \frac{10^3}{3}\sqrt{\frac{2}{7}}\ \text{Hz} = 178.2\,\text{Hz}$$

4. **(1)** In fundamental mode



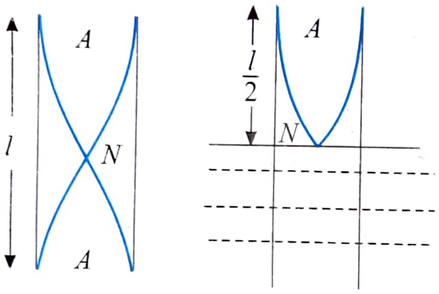
$$\frac{\lambda}{4} = 0.85$$

$$\lambda = 4\times0.85$$

$$f = \frac{v}{\lambda} = \frac{340}{4\times0.85} = 100\,\text{Hz}$$

∴ Possible frequencies = 100 Hz, 300 Hz, 500 Hz, 700 Hz, 900 Hz and 1100 Hz below 1250 Hz.

5. **(4)**



For open organ pipe $l = \frac{\lambda}{4} + \frac{\lambda}{4} = \frac{\lambda}{2} \Rightarrow \lambda = 2l$

Hence frequency, $f = \frac{v}{\lambda} = \frac{v}{2l}$

The pipe is dipped vertically in water, it acts as closed pipe

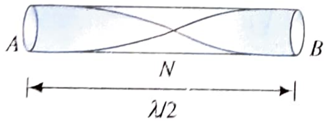
$$\frac{l}{2} = \frac{\lambda}{4} \Rightarrow \lambda = 2l$$

$$\therefore \quad f' = \frac{v}{\lambda} = \frac{v}{2l} = f$$

6. **(2)** Velocity of wave $= \sqrt{\frac{Y}{\rho}} = \sqrt{\frac{9.27\times10^{10}}{2.7\times10^3}} = \sqrt{3.433\times10^7}$

$$\Rightarrow v = 5.85\times10^3\ \text{m/sec}$$

The rod is clamped at middle fundamental wave shape is as follow



$$\frac{\lambda}{2} = L \Rightarrow \lambda = 2L = 2\times0.60 = 1.20\,\text{m}$$

Fundamental frequency

$$f = \frac{v}{\lambda} = \frac{5.85\times10^3}{1.2} = 4.88\times10^3\,\text{Hz} = 5\,\text{kHz}$$

JEE Advanced

Single Correct Answer Type

1. **(2)** $\frac{v_S}{4L_P} = \frac{2}{2l_S}\sqrt{\frac{T}{\mu}} \Rightarrow \frac{320}{4\times0.8} = \frac{1}{0.5}\sqrt{\frac{50}{\mu}}$

$$\Rightarrow \quad \mu = 1/50$$

Mass of string, $m = \mu l_S = \frac{1}{50}\times0.5 = 0.01\ \text{kg} = 10\ \text{g}$

2. **(2)** $\frac{V}{4(l+e)} = f$

$$\Rightarrow \quad l + e = \frac{V}{4f}$$

$$\therefore \quad l = \frac{V}{4f} - e$$

Here $e = (0.6)\,r = (0.6)\,(2) = 1.2$ cm

So $\quad l = \frac{336\times10^2}{4\times512} - 1.2 = 15.2$ cm

3. **(4)** For minimum height, $\frac{\lambda}{4} = e$

$$\frac{\lambda}{4} = 0.356\pm0.005 \text{ or } \lambda = 1.400\pm0.020$$

$$v = f\lambda = 244(1.4000\pm0.020)$$

$$\Rightarrow \quad v = (341.6\pm4.88)\frac{\text{m}}{\text{s}}$$

$$v = \sqrt{\frac{\gamma RT}{M\times10^{-3}}} = \sqrt{\frac{100\gamma RT\times10}{M}}$$

For monatomic, $\gamma = 1.67$

$$v = \sqrt{167RT}\cdot\sqrt{\frac{10}{M}}$$

For neon (monoatomic): $640\times\frac{7}{10} = 448\ \text{ms}^{-1}$

For argon (monoatomic): $640\times\frac{17}{32} = 340\ \text{ms}^{-1}$

For diatomic $\gamma = 1.4$

$$v = \sqrt{140RT}\cdot\sqrt{\frac{10}{M}}$$

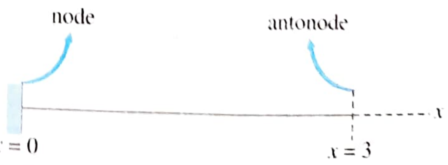
For nitrogen (diatomic): $590\times\frac{3}{5} = 384\ \text{ms}^{-1}$

For oxygen (diatomic): $590\times\frac{9}{16} = 351.875\ \text{ms}^{-1}$

Hence possible answer is (4).

Multiple Correct Answers Type

1. **(1), (3), (4)**



The fixed end is a node while the free end is an antinode. Therefore, at $x = 0$ is a node and at $x = 3$ m is an antinode. Possible modes of variation are

$$L = (2n+1)\frac{\lambda}{4} \text{ where } n = 0, 1, 2, 3, \ldots$$

or
$$\lambda = \frac{4L}{2n+1} = \frac{12}{2n+1} \qquad (\because L = 3 \text{ m (given)})$$

$$k = \frac{2\pi}{\lambda} = \frac{2\pi}{12/(2n+1)} = \frac{(2n+1)\pi}{6}$$

$$\omega = vk = 100(2n+1)\frac{\pi}{6} = \frac{(2n+1)50\pi}{3}$$

We have stationary wave equation $y = A \sin kx \cdot \cos \omega t$

For $n = 0$

$$y(t) = A \sin\frac{\pi x}{6} \cos\frac{50xt}{3}$$

For $n = 2$

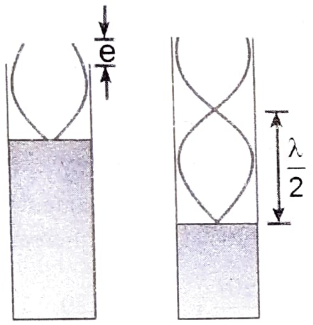
$$y(t) = A \sin\frac{5\pi x}{6} \cos\frac{250xt}{3}$$

For $n = 7$

$$y(t) = A \sin\frac{5\pi x}{2} \cos 250xt$$

2. (1), (2), (3)

Let n_1 harmonic is corresponding to 50.7 cm and n_2 harmonic is corresponding to 83.9 cm since both are consecutive harmonics. The difference in the lengths of air columns of two resonances should be $\frac{\lambda}{2}$



$$\therefore \quad \frac{\lambda}{2} = (83.9 - 50.7)\text{cm}$$

or
$$\frac{\lambda}{2} = 33.2\text{cm}$$

Hence $\lambda = 66.4$ cm

Hence option (3) is correct.

Hence the length corresponding to fundamental mode must be close to $\frac{\lambda}{4}$.

$$\therefore \quad \frac{\lambda}{4} = 16.6\text{cm}$$

It means the length 50.7 cm must be an odd multiple of this length 16.6 × 3 = 49.8 cm. Therefore 50.7 is 3rd harmonic.

If end correction is e, then $e + 50.7 = \frac{3\lambda}{4}$

$$e = 49.8 - 50.7 = -0.9\text{cm}$$

Hence option (2) is correct.

Speed of sound, $v = f\lambda$

$$\therefore \quad v = 500 \times 66.4\text{cm/sec} = 332 \text{ m/s}$$

Hence option (1) is correct.

Matrix Match Type

1. i. → a., c.; ii. → a., d.; iii. → b., d.; iv. → b., c.

(i) Sound waves are longitudinal waves

$$\frac{\lambda_f}{4} = L \Rightarrow \lambda_f = 4L$$

(ii) Sound waves are longitudinal waves

$$\frac{\lambda_f}{2} = L \Rightarrow \lambda_f = 2L$$

(iii) String waves are transverse waves

$$\frac{\lambda_f}{2} = L \Rightarrow \lambda_f = 2L$$

(iv) String waves are transverse waves

$$\frac{2\lambda_f}{2} = L \Rightarrow \lambda_f = L$$

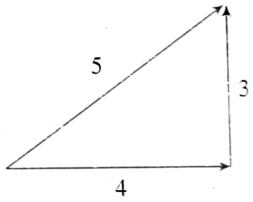
Numerical Value Type

1. (5)
$$v = \sqrt{\frac{T}{\mu}} = 10 \text{ m/s}$$

$$\lambda = \frac{v}{f} = \frac{10}{100} = 10 \text{ cm}$$

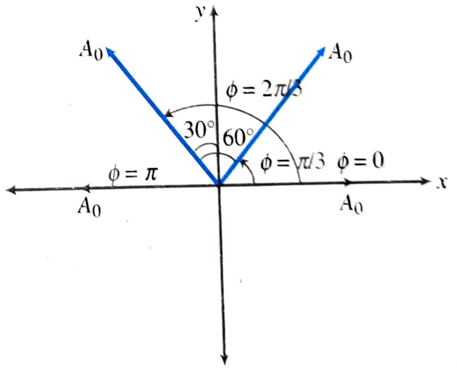
Distance between the successive nodes = $\lambda/2$ = 5 cm

2. (5)



Two waves have phase difference $\pi/2$.

3. (3) Using phasor method of superposition,



Resultant amplitude $A = 2A_0 \cos 30° = A_0\sqrt{3}$

As $I \propto A^2$

$\therefore$ Resultant intensity = $3I_0$

Chapter 8

Concept Application Exercises

Exercise 8.1

1. Compare the given equation with $\Delta p = p_0 \sin\left[\frac{2\pi}{\lambda}(x-vt)\right]$

   (a) The velocity of the wave $v = 330$ m/s

   (b) The frequency of the wave $f=\frac{v}{\lambda}=\frac{330}{2}=165\,\text{Hz}$

   (c) The maximum pressure (pressure amplitude) $\Delta p_{\max}=1.5\,\text{Pa}$

   (d) The pressure changes at $x=(1/6)$ m and $t=0$

$$\Delta p = 1.5\sin\left[\frac{2\pi}{2}\left(\frac{1}{6}-0\right)\right]=1.5\sin\frac{\pi}{2}=0.75\,\text{Pa}$$

The pressure at this point,

$$p=1.01\times10^5+0.75=1.76\times10^5\,\text{Pa}$$

2. Comparing the given equation with $\Delta p=\Delta p_{\max}\sin(\omega t-kx)$, we get

$$p_0=0.01\ \text{N/m}^2,\ \omega=600\pi\ \text{s}^{-1}\ \text{and}\ k=2.0\pi\ \text{m}^{-1}$$

   (a) The frequency $f=\frac{\omega}{2\pi}=\frac{600\pi\ \text{s}^{-1}}{2\pi}=300\,\text{Hz}$

   The speed of sound $v=\frac{\omega}{k}=\frac{600\pi\ \text{s}^{-1}}{2.0\pi\ \text{m}^{-1}}=300$ m/s

   The wavelength of sound wave $\lambda=\frac{v}{f}=\frac{300}{300}=1.0$ m

   (b) Maximum pressure

$$p_{\max}=p_{\text{atm}}+\Delta p_{\max}=(1.01\times10^5+0.01)\,\text{N/m}^2$$

   Minimum pressure

$$p_{\min}=p_{\text{atm}}-\Delta p_{\max}=(1.01\times10^5-0.01)\,\text{N/m}^2$$

3. $\frac{v_t}{v_0}=\sqrt{\frac{T}{T_0}}=\sqrt{\frac{273+t}{273}}\approx\left(1+\frac{t}{546}\right)$

As $\quad \frac{v_t-v_0}{v_0}=1\%=\frac{1}{100},\ \frac{v_t}{v_0}-1=\frac{1}{100}$

or $\quad \left(1+\frac{t}{546}\right)-1=\frac{1}{100}$

or $\quad \left(\frac{t}{546}\right)100=1$ or $t=5.5°\text{C}$

4. $\frac{v_2}{v_1}=\sqrt{\frac{T_2}{T_1}}=\sqrt{\frac{293}{283}}=\left(1+\frac{10}{283}\right)^{1/2}$

or $\quad \frac{v_2}{v_1}=1+\frac{5}{283}$

percentage change in velocity $=\left(\frac{v_2-v_1}{v_1}\right)\times100$

$$=\left(\frac{v_2}{v_1}-1\right)\times100=\frac{5}{283}\times100=1.77\%$$

5. Density of mixture, $\rho_M=\frac{V_O\rho_O+V_H\rho_H}{V_O+V_H}$

Thus, $\quad \frac{\rho_H}{\rho_M}=\frac{\rho_H(V_O+V_H)}{V_O\rho_O+V_H\rho_H}=\frac{V_O+V_H}{V_O\left(\frac{\rho_O}{\rho_H}\right)+V_H}$

or $\quad \frac{\rho_H}{\rho_M}=\frac{V_O+V_H}{16V_O+V_H}\quad\left(\text{as }\frac{\rho_O}{\rho_H}=16\right)\quad$ ...(i)

As $\quad \frac{v_M}{v_H}=\sqrt{\frac{\rho_H}{\rho_M}},\ \sqrt{\frac{\rho_H}{\rho_M}}=\frac{500}{1200}=\frac{5}{12}$

or $\quad \frac{\rho_H}{\rho_M}=\frac{25}{144}\quad$ ...(ii)

From eqns. (i) and (ii), $\frac{V_O+V_H}{16V_O+V_H}=\frac{25}{144}$

or $\quad \frac{V_H}{V_O}=\frac{256}{119}$

6. The speed of sound in the gas, $v=\sqrt{\frac{\gamma RT}{M}}\quad$ ...(i)

The column of air (closed at one end and open at the end) is in resonance with the tunning fork at fundamental mode. Then,

$$f=\frac{v}{4l}\quad ...(ii)$$

Eliminating v form eqns. (i) and (ii),

$$\sqrt{\frac{\gamma RT}{M}}=4lf$$

or $\quad M=\frac{\gamma RT}{16l^2f^2}$

Exercise 8.2

1. Let the initial intensity be I and the sound level be L_1. When the intensity is increased to 20 I, the level increases to L_2.

   Thus, $L_2-L_1=10\log\left(\frac{20I}{I}\right)=10\log 20=13$ dB

2. In terms of their intensities, the difference in the decibel level of two sounds is

$$L_2-L_1=(10dB)\log\left(\frac{I_2}{I_0}\right)-(10dB)\left(\frac{I_1}{I_0}\right)$$

$$=(10dB)\log\left(\frac{I_2}{I_0}\cdot\frac{I_0}{I_1}\right)=(10dB)\log\left(\frac{I_2}{I_1}\right)$$

Thus, $\quad \frac{I_2}{I_1}=10^{(L_2-L_1)/10}$

or $\quad I_2=I_1\times10^{(\beta_2-\beta_1)/10}$

if $L_2-L_1=30\,dB$ and $I_1=3.0\times10^{-11}\,\text{W/m}^2$,

then $I_2=(3.0\times10^{-11}\,\text{W/m}^2)\times10^3=3.0\times10^{-8}\,\text{W/m}^2$

3. $L_1=10\log_{10}\left(\frac{I}{I_0}\right)$

$$100=10\log_{10}\left(\frac{I}{10^{-12}}\right)$$

Solving we get, $I=10^{-2}\ \text{W/m}^2$

Now, $I=\frac{P}{4\pi r^2}$

$\therefore\quad P=I(4\pi r^2)=(10^{-2})(4\pi)(40)^2=64\pi$ W

$$I = \frac{v(\Delta p)^2_{\max}}{2B} \Rightarrow v = \sqrt{\frac{B}{\rho}}$$

$\therefore$
$$B = \rho v^2$$

$\therefore$
$$I = \frac{(\Delta p)^2_{\max}}{2\rho v} = \frac{(\sqrt{33}\times 10^{-5})^2}{2\times 1.25\times 330} = 4.0\times 10^{-12}\ \text{W/m}^2$$

$$L = 10\log_{10}\left(\frac{I}{I_0}\right) = 10\log_{10}\left(\frac{4.0\times 10^{-12}}{10^{-12}}\right) = 6.0\,\text{dB}$$

(a) At a distance r from a point source of power P, the intensity of the sound is

$$I = \frac{P}{4\pi r^2} = \frac{0.9}{(4\pi)(1.5)^2} = \frac{1}{10\pi}\,\text{W/m}^2 \qquad \ldots\text{(i)}$$

Further, the intensity of sound in terms of $(\Delta p)_m, \rho$ and v is given by $I = \dfrac{(\Delta p)^2_m}{2\rho v}$ ...(ii)

From equations (i) and (ii),

$$(\Delta p)_m = \sqrt{2\times\frac{1}{10\pi}\times 10^{-2}\times\frac{\pi}{2}\times 360} = 6.0\ \text{N/m}^2$$

(b) Pressure oscillation amplitude $(\Delta p)_m$ and displacement oscillation amplitude A are related by the equation

$$(\Delta p)_m = BAk$$

Substituting $B = \rho v^2, k = \dfrac{\omega}{v}$ and $\omega = 2\pi f$

We get, $(\Delta p)_m = 2\pi A\rho v f$

$\therefore$
$$A = \frac{(\Delta p)_m}{2\pi\rho v f} = \frac{6.0}{(2\pi)(\pi/2)(360)(500)} = 3.3\times 10^{-6}\,\text{m}$$

6. If $I_{\max}$ and $I_{\min}$ are the maximum and minimum intensities of the resultant waves, then the intensity level between them will be

$$L = 10\log\left(\frac{I_{\max}}{I_{\min}}\right)\text{dB}$$

or, $$20\ \text{dB} = 10\log\left(\frac{I_{\max}}{I_{\min}}\right)\text{dB}$$

or, $$\frac{I_{\max}}{I_{\min}} = \frac{100}{1} \qquad [\log 100 = 2]$$

If A_1 and A_2 are the amplitudes of the individual waves, interferring, then

$$\frac{I_{\max}}{I_{\min}} = \frac{(A_1 + A_2)^2}{(A_1 - A_2)^2} \qquad [\text{Assuming } A_1 > A_2]$$

$\therefore$
$$\left(\frac{A_1 + A_2}{A_1 - A_2}\right)^2 = \frac{100}{1}$$

$\therefore$
$$\frac{A_1 + A_2}{A_1 - A_2} = \frac{10}{1}$$

or, $$\frac{A_1}{A_2} = \frac{11}{9}$$

Therefore, the intensity ratio between the individual waves will be

$$\frac{I_1}{I_2} = \left(\frac{A_1}{A_2}\right)^2 = \frac{121}{81}$$

7. Intensity level $= \log_{10}\dfrac{I_2}{I_1}$ bels $= 10\log\dfrac{I_2}{I_1}$ dB

Here $I_1 = 10\times 10^{-9}\times 10^4 = 10^{-4}\ \text{W/m}^2$

$\therefore$
$$6 = 10\log\frac{I_2}{10^{-4}}$$

or $$I_2 = \frac{10^{0.6}}{10^4} = \frac{3.98}{10^4} = 4.0\times 10^{-4}\ \text{W/m}^2$$

8. Increase in number of dB, $N_2 - N_1 = 60 - 50 = 10$ dB

$\therefore$ Ratio of increase in intensity

$$10 = 10\log_{10}\frac{I_2}{I_1} \Rightarrow \frac{I_2}{I_1} = (10)^1 = 10$$

The relation between pressure amplitude and intensity is $I = \Pi_0^2/2\rho v$.

i.e., $$I \propto p_0^2 \Rightarrow \frac{p_2}{p_1} = \sqrt{\frac{I_2}{I_1}} = \sqrt{10} = 3.16$$

9. Loudness level is given as $L = 10\log\left(\dfrac{I}{I_0}\right)$

Thus,

$$10\log\frac{I}{I_0} = 60$$

$$\Rightarrow \frac{I}{I_0} = 10^6$$

$$\Rightarrow I = (10^{-12}\times 10^6) = 10^{-6}\ \text{W/m}^2 = 1\ \mu\text{W/m}^2$$

And the power of sound is given by

$$P = IS = 1\times 10^{-6}\times 2 = 2\,\mu\text{W}$$

Thus energy is given by

$$E = P\times t = 2\times 10^{-6}\times 5\times 60\times 60 = 36\times 10^{-3}\ \text{J}$$

10. (a) As intensity is power per unit area, for a given source $P \propto I$. If L_1 and L_2 are the initial and final loudness levels, then we have

$$L_2 - L_1 = 10\log(I_2/I_1)$$

The increase in loudness level is

$$\Delta L = 10\log\frac{P_2}{P_1} = 10\log\frac{400}{20}$$

$$\Rightarrow \Delta L = 10[\log 20] = 13\ \text{dB}$$

(b) Increase in loudness level of sound is

$$L_2 - L_1 = 10\log(I_2/I_1)$$

$\therefore$ $$80 - 20 = 10\log(I_2/I_1)$$

$$\Rightarrow 6 = \log(I_2/I_1)$$

$$\Rightarrow \left(\frac{I_2}{I_1}\right) = 10^6$$

11. For maximum possible sound intensity, maximum pressure amplitude of wave can be at most equal to atmospheric pressure. So we have,

$$\Delta p_0 = p = 1.01\times 10^5\ \text{N/m}^2$$

$$I = \frac{\Delta p_0^2}{2\rho v} = \frac{(1.01\times 10^5)^2}{2\times 1.3\times 332} = 1.18\times 10^7\ \text{W/m}^2$$

Thus loudness level is

$$L = 10\log\frac{I}{I_0} \approx \log_{10}\frac{10^7}{10^{-12}} = 190\ \text{dB}$$

Exercise 8.3

1. No. It is done by considering the component of the velocity of the source along the line joining it with the observer.

2. $f' = f\left(\dfrac{v}{v+v'}\right)$

where $v' = v_s$

$$\because \quad \frac{f}{f'} = \eta\ (\text{given}) = 1 + \frac{v'}{v} \qquad \text{...(i)}$$

$$f'' = f\left(\frac{v+v'}{v-v'}\right)$$

where $v_0 = v'$

$$\because \quad \frac{f}{f''} = \frac{v-v'}{v+v'} = \frac{1-(v'/v)}{1+(v'/v)}$$

$$\frac{1-(\eta-1)}{1+(\eta-1)} = \frac{2-\eta}{\eta} = \frac{2}{\eta} - 1$$

3. (a) In front of the train: $f' = f\left(\dfrac{v-V_0}{v-V_s}\right) = f\left(\dfrac{v-0}{v-30}\right)$

$$\because \quad f' = \frac{500 \times 345}{345-30} = 547.62 \text{ Hz}$$

(b) Behind the train: $f' = f\left(\dfrac{v-0}{v-(-30)}\right) = \dfrac{500 \times 345}{375} = 460 \text{ Hz}$

4. $n' = \dfrac{300-0}{300-u} \times 1000$

$$n'' = \frac{300-0}{300-(-u)} \times 1000$$

Dividing, $\dfrac{n'}{n''} = \dfrac{300+u}{300-u}$

It is given that $n'/n'' = 11/9$, therefore $\dfrac{11}{9} = \dfrac{300+u}{300-u}$

or $\quad u = 30$ m/s

5. Here, the bat is the 'observer' and the 'image' is the source.

$$n' = \frac{v-v_0}{v-v_s} \times n = \left[\frac{340+(-6)}{340-6}\right] 450 = 466 \text{ kHz}$$

6. $v_s = r\omega = 1.5 \times 20 = 30$ m/s

$$n_{\min} = \frac{v}{v+v_s} n = \frac{330}{330+30} \times 440 \text{ Hz} = 403 \text{ Hz}$$

$$\therefore \quad n_{\max} = \frac{330}{330-30} \times 440 = 484 \text{ Hz}$$

7. The 'image' of the source approaches the driver at the same speed. Here, the image or echo is the source. Therefore,

$v_s = +60$ m/s, $\quad v_0 = -60$ m/s

$$n' = \frac{v-v_0}{v-v_s} \times n$$

$$\therefore \quad n' = \frac{340-(-60)}{340-60} \times 133 \Rightarrow n' = 190 \text{ Hz}$$

8. (a) The frequency of sound heard directly, $f_1 = f_0\left(\dfrac{v}{v+v_s}\right)$

$v_s = 8$ m/s

$$\therefore \quad f_1 = \left(\frac{330}{330+8}\right) \times 2000$$

$$f_1 = \frac{330}{338} \times 2000 = 1953 \text{ Hz}$$

(b) The frequency of the reflected sound is given by

$$f_2 = f_0\left(\frac{v}{v-v_s}\right)$$

$$\therefore \quad f_2 = \left(\frac{330}{330-8}\right) \times 2000$$

$$f_2 = \frac{330}{322} \times 2000 = 2050 \text{ Hz}$$

9. Let the fork at A be source S_1 and the fork at B source S_2.
For the sound from S_1:

Apparent frequency $= f_1 = f\left(\dfrac{v-V_0}{v-V_s}\right)$

Here $v = 345 + 5 = 350$ m/s as wind and sound are in same directions.

$V_0 = +25$ m/s, $\quad V_s = 0$ m/s

$$\Rightarrow \quad f_1 = 500\left(\frac{350-25}{350-0}\right) = 464.3 \text{ Hz}$$

For the sound from source S_2: $v = 345 - 5 = 340$ m/s as wind is in opposite direction to that of sound.

$V_s = 0$ m/s, $\quad V_0 = -25$ m/s

$$\Rightarrow \quad f_2 = f\left(\frac{v-V_0}{v-V_s}\right) = 500\left(\frac{v-(-25)}{v-0}\right) = \frac{500 \times 365}{340}$$

$$= 536.80 \text{ Hz}$$

$$\Rightarrow \quad f_2 - f_1 = (536.80 - 464.30) = 72.5 \text{ Hz}$$

10. (a) Due to motion of the source, the wavelength (and hence, the frequency) is actually changed from λ and λ' such that if n = actual frequency,

$$\lambda' = \frac{v-v_s}{n} = \frac{332-32}{1000} = 0.3 \text{ m}$$

(b) The number of waves arriving at the reflecting surface is the same as the number of waves received by an observer moving towards the source. This is given by the apparent frequency.

$$n' = \frac{v-v_0}{v-v_s} \times n = \frac{332-(-64)}{332-32} \times 1000 = 1320 \text{ Hz}$$

(c) Same as that of the incident wave because the speed of wave depends only on the characteristics of the medium.
Therefore, speed of the reflected wave = 332 ms^{-1}

(d) Now reflector will act as a source of frequency 1320 Hz. So wavelength emitted by this:

$$\lambda' = \frac{v-v_{\text{ref}}}{1320} = \frac{332-64}{1320} = 0.2 \text{ m}$$

11. If there is no wind, n waves emitted by the source in each second will form a train of waves of length equal to speed c of the sound. When the wind blows at speed w, the same number of waves will form a train of length $v + w$ on the windside.

$$\therefore \quad \lambda' = \frac{v+w}{n} = \frac{v+w}{v}\lambda \qquad (\because n\lambda = v)$$

$$\Rightarrow \quad (\lambda' - \lambda)/\lambda = w/v$$

$$\therefore \quad \%\ \text{increase in wavelength} = \frac{w}{v} \times 100 = \frac{0.1v}{v} \times 100 = 10$$

Let n' be the apparent frequency. Then $n'\lambda' = v + w$ (Since speed of sound on the wind-side is $v + w$)

or $\quad n' = \dfrac{v+w}{\lambda'} = n$

so, there is no change in the frequency of the wave.

When the source moves, the wavelength is not changed but the frequency appears to change. The apparent frequency is given by

$$n' = \frac{v - v_0}{v - v_s} \times n$$

Here $v_s = 0$, $v_0 = -0.1\ v$, therefore $n' = \frac{v - (-0.1v)}{v} \times n = 1.1n$

$$\%\ \text{increase} = \frac{n' - n}{n} \times 100 = 10$$

$$n' = \frac{v - v_0}{v - v_s} \times n$$

Here $n' = \frac{340 - 0}{340 - v} \times 680 = \frac{340}{340 - v} \times 680$

$$n'' = \frac{340 - 0}{340 + v} \times 680 = \frac{340}{340 + v} \times 680$$

Now $n' - n'' = 2$

or $340 \times 680 \left(\frac{2v}{340^2 - v^2} \right) = 2$ or $v = \frac{340}{680} = 0.5$ m/s

Exercises

Single Correct Answer Type

1. (3) $B = 10 \log_{10} \left(\frac{I}{I_0} \right) = 10 \log_{10} \left(\frac{10^{-6}}{10^{-12}} \right) = 60$ dB

2. (3) $100 = 10 \log_{10} \frac{I_1}{I_0}$

$$50 = 10 \log_{10} \frac{I_2}{I_0}$$

or $\frac{I_1}{I_0} = 10^{10}$ and $\frac{I_2}{I_0} = 10^5$

Dividing, $\frac{I_1}{I_2} = 10^5$

3. (2) $L_2 - L_1 = 30$ dB

$$10\ \text{dB} \log \frac{I_2}{I_0} - 10\ \text{dB} \log \frac{I_1}{I_0} = 30\ \text{dB}$$

$$\log_{10} \frac{I_2}{I_1} = 3 \Rightarrow \frac{I_2}{I_1} = 10^3$$

Hence, the sound intensity increases by 10^3.

4. (2) Towards right wavelength gets compressed and towards left wavelength gets expanded.

5. (3) $P = -B \frac{dy}{dx}$

At R, dy/dx is most negative. So pressure is maximum.

6. (1) The frequency of direct and reflected sound is same.

7. (4) No doppler effect, because velocity is perpendicular to line joining vehicle and observer.

8. (3) Since source and both the observers are stationary, therefore no change will be observed by the two observers. It means both the observers will receive waves with natural frequency, which is equal to n.

9. (3) Let v be the speed of sound, u be the speed of train.

Then, $v_s = v_o = u$

and $f' = f \left(\frac{v + u\cos\theta}{v + u\cos\theta} \right) = f$

10. (2) $f = f_o \left(\frac{v_s + v_o}{v_s} \right) = f_0 \left[\frac{v + \frac{v}{5}}{v} \right] = \frac{6}{5} f_o$

Hence, percentage increase is $\left[\frac{\frac{6}{5} f_0 - f_0}{f_0} \right] \times 100 = 20\%$

11. (1) As the source and the observer are approaching one another, so n' would be larger.

$$f = \left(\frac{v + v/15}{v - v/10} \right) 600 = 711\ \text{Hz}$$

12. (1) $\lambda' = \left(\frac{v - v_s}{v} \right) \lambda = \left(\frac{320 - 20}{320} \right) 60$

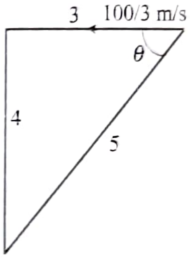
$$= 56.25\ \text{cm}$$

13. (2) $v' = \frac{v}{v - v_s} v$, $v'' = \frac{v}{v + v_s} v$

$$\frac{v'}{v''} = \frac{v + v_s}{v - v_s} \quad \text{or} \quad \frac{6}{5} = \frac{330 + v_s}{330 - v_s}$$

$$11 v_s = 330 \quad \text{or} \quad v_s = 30\ \text{m/s}$$

14. (2) Effective value of velocity of source is



$$v_s = \frac{100}{3} \cos\theta = \frac{100}{3} \times \frac{3}{5} = 20\ \text{m/s}$$

$$v' = \frac{v}{v - v_s} v$$

$$v' = \frac{340}{340 - 20} \times 640\ \text{Hz} = 680\ \text{Hz}$$

15 (3) $f = \left(\frac{v + v_m}{v + v_m - v_{\text{source}}} \right) 1000 = \left(\frac{340 + 20\cos 60°}{340 + 20\cos 60° - 30} \right) 1000$

$$= 1094\ \text{Hz}$$

16. (1) Apparent frequency due to train which is coming in is

$$n_1 = \frac{v}{v - v_s} n$$

Apparent frequency due to train which is leaving is

$$n_2 = \frac{v}{v + v_s} n$$

So the number of beats is

$$n_1 - n_2 = \left(\frac{1}{316} - \frac{1}{324} \right) 320 \times 240 \Rightarrow n_1 - n_2 = 6$$

17. (2) Intensity after passing through one slab

$$I' = \left[I - \frac{20}{100} \times I \right] = \left[I - \frac{I}{5} \right] = \frac{4I}{5}$$

So, intensity after passing through two slabs

$$I'' = \left[I' - \frac{20}{100} \times I' \right] = \frac{4I'}{5} = \frac{16I}{25}$$

$$\therefore \quad \%\ \text{decrease} = \left[\frac{\left(I - \frac{16I}{25} \right)}{I} \right] \times 100 = 36\%$$

18. (4) When the man is approaching the factory,

$$n' = \left(\frac{v + v_0}{v} \right) n = \left(\frac{320 + 2}{320} \right) 800 = \left(\frac{322}{320} \right) 800$$

When the man is going away from the factory,

$$n'' = \left(\frac{v - v_0}{v}\right)n = \left(\frac{320 - 2}{320}\right)800 = \left(\frac{318}{320}\right)800$$

$$\therefore \quad n' - n'' = \left(\frac{322 - 318}{320}\right)800 = 10\,\text{Hz}$$

19. (2) Apparent frequency due to source A is

$$n' = \frac{v - u}{v} \times n$$

Apparent frequency due to source B is

$$n'' = \frac{v + u}{v} \times n$$

$$\therefore \quad n'' - n' = \frac{2u}{v} \times n = 10$$

$$\therefore \quad u = \frac{10v}{2n} = \frac{10 \times 340}{2 \times 680} = 2.5\,\text{m/s}$$

20. (2) When the train is approaching,

$$n_1 = \frac{v}{v - v_s} \times n = \frac{320}{320 - 4} \times 243 = \frac{80}{79} \times 243$$

When the train is receding,

$$n_2 = \frac{v}{v + v_s} \times n = \frac{320}{324} \times 243 = \frac{80}{81} \times 243$$

Beat frequency is

$$n = n_1 - n_2 = 80 \times 243\left(\frac{1}{79} - \frac{1}{81}\right) = 6\text{ Hz}$$

21. (2) $f_1 = 900\left(\dfrac{300}{300 + v_1}\right)$

$$\cong 900\left(1 + \frac{v_1}{300}\right)^{-1} = 900 - 3v_1$$

Similarly, $f_2 = 900\left(\dfrac{300}{300 + v_2}\right) = 900 - 3v_2$

$$f_2 - f_1 = 6$$

$$\therefore \quad 3(v_1 - v_2) = 6$$

$$\therefore \quad 3(v_1 - v_2) = 6$$

$$\text{or} \quad v_1 - v_2 = 2\,\text{m/s}$$

22. (4) When the source approaches the observer,

$$f_1 = f\left(\frac{v}{v - v_s}\right) = f\left(1 - \frac{v_s}{v}\right)^{-1} \approx f\left(1 + \frac{v_s}{v}\right)$$

$$\text{or} \quad \left(\frac{f_1 - f}{f}\right) \times 100 = \frac{v_s}{v} \times 100 = 10 \qquad \text{...(i)}$$

In the second case, when the source recedes from the observer

$$f_2 = f\left(\frac{v}{v + v_s}\right) = f\left(1 + \frac{v_s}{v}\right)^{-1} = f\left(1 - \frac{v_s}{v}\right)$$

$$\therefore \quad \left(\frac{f_2 - f}{f}\right) \times 100 = -\frac{v_s}{v} \times 100 = -10 \qquad \text{[from Eq. (i)]}$$

In the first case, observed frequency increases by 10% while in the second case, observed frequency decreases by 10%.

(3) Apparent frequency for reflector (which will act here as an observer) would be $f_1 = \left(\dfrac{v + u}{v}\right)f$

where f is the actual frequency of source. The reflector will now behave as a source. The apparent frequency will now become

$$f_2 = \left(\frac{v}{v - u}\right)f_1$$

Substituting the value of f_1', we get $f_2 = \left(\dfrac{v + u}{v - u}\right)f$

24. (1) Given that velocity of source $v_s = 0$ (because it is stationary).

Velocity of observer $v_o = (1/5)v = 0.2v$ (where v is the velocity of sound). Actual frequency of source is f and actual wavelength of source is λ. We know from the Doppler's effect that the apparent frequency recorded, when the observer is moving towards the stationary source, is given by

$$n' = \left(\frac{v + v_o}{v - v_s}\right) \times n = \left(\frac{v + 0.2v}{v - 0}\right) \times n = \frac{1.2v}{v} \times n = 1.2\,n = 1.2\,f$$

Since the source is stationary, therefore the apparent wavelength remains unchanged, i.e., λ.

25. (1) When the source is coming to stationary observer,

$$n' = \left(\frac{v}{v - v_s}\right)n$$

$$\text{or} \quad 1000 = \left(\frac{350}{350 - 50}\right)n$$

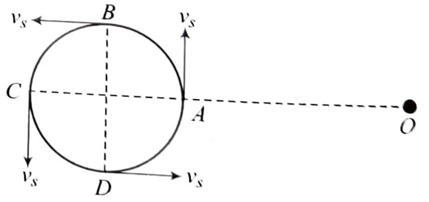
$$\text{or} \quad n = (1000 \times 300/350)\text{ Hz}$$

When the source is moving away from the stationary observer.

$$n'' = \left(\frac{v}{v + v_s}\right)n = \left(\frac{350}{350 + 50}\right)\left(\frac{1000 \times 300}{350}\right) = 750\text{ Hz}$$

26. (4) Frequency heard by the observes will be maximum when the source is in position D. In this case, source will be approaching towards the stationary observer, almost along the line of sight (as observer is stationed at a larger distance).

$$n_{\max} = \frac{v}{v - v_s}n = \frac{330 \times 440}{330 - 1.5 \times 20} = 484\text{ Hz}$$



Similarly, frequency heard by the observer will be minimum when the source reaches at position B. Now, the source will be moving away from the observer.

$$n_{\min} = \frac{v}{v + v_s} \times n = \frac{330}{330 + 1.5 \times 20} \times 440$$

$$= \frac{330 \times 440}{360} = 403.3\text{ Hz}$$

27. (1) $n' = n\left(\dfrac{v - v_o}{v - v_s}\right)$

$$\Rightarrow \quad 605 = 550\left(\frac{330 - 0}{330 - v_s}\right)$$

$\therefore \quad v_s = 30$ m/s

28. (4) Intensity,

$$I = \frac{P}{A} = \frac{200\pi}{4\pi \times (10)^2} = 0.5 \text{ W/m}^2$$

No. of decibels is given by

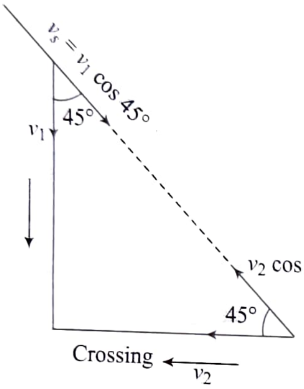
$$10 \log_{10} : p\frac{I}{I_0} = 10 \log_{10} \frac{0.5}{10^{-12}} = 10 \log_{10} (5 \times 10^{11})$$

$$= 10 \log_{10} \left(\frac{10^{12}}{2}\right) = 117 \text{ dB}$$

29. (2) The component of velocity of source along line joining the car is

$$v_S = v_1 \cos 45° = 36 \times \frac{1}{\sqrt{2}} \text{ km/h } = 5\sqrt{2} \text{ m/s}$$

Component of velocity of observer (second car) along the line joining the car is



$$v_0 = v_2 \cos 45° = 72 \times \frac{1}{\sqrt{2}} \text{ km/h } = 10\sqrt{2} \text{ m/s}$$

$$n' = \frac{v + v_0}{v - v_S} n = \frac{330 + 10\sqrt{2}}{330 - 5\sqrt{2}} \times 280$$

$$= \frac{344}{323} \times 280 \text{ Hz} = 298 \text{ Hz}$$

30. (4) Initially wall behaves as an approaching observer, so frequency of sound reaching the wall is

$$n_1 = \frac{c + v}{c} n$$

While reflecting, the wall behaves as an approaching source, so frequency received by stationary observer is

$$n_2 = \frac{c}{c - v} n_1 = \frac{c}{c - v} \times \frac{c + v}{c} n = \frac{c + v}{c - v} n$$

Direct frequency received by observer is n. the number of beats is

$$x = n_2 - n = \frac{c + v}{c - v} n - n = \frac{2nv}{c - v}$$

31. (2) The frequency of direct sound of whistle heard by observer is

$$n_1 = \frac{v}{v - v_s} n = \frac{340}{340 - 1} \times n = \frac{340}{339} n \qquad \text{...(i)}$$

Frequency of sound of whistle reflected by wall is

$$n_2 = \frac{v}{v + v_s} n = \frac{340}{341} n \qquad \text{(ii)}$$

Given, $n_1 - n_2 = 4$

Therefore, $\frac{340}{339} n - \frac{340}{341} n = 4$

$\Rightarrow \quad n = 680$ Hz

32. (2) $60 \text{ dB} = 10 \text{ dB} \log \frac{I}{I_0}$

$\Rightarrow \quad I = (10^6 \times 10^{-12}) \text{ W/m}^2 = 10^{-6} \text{ W/m}^2$
$[I_0 = 10^{-12} \text{ W/m}^2]$

$$I = \frac{(\Delta P_m)^2}{2\rho v}$$

where $\rho = 15/11 \text{kg/m}^3$, $v = 330$ m/s

$\therefore \quad (\Delta P_m)^2 = 2\rho v I = 2 \times \frac{15}{11} \times 330 \times 10^{-6}$

$\Rightarrow \quad \Delta P_m = 0.03 \text{ N/m}^2$

33. (1) The wavelength in front of the car is

$$\lambda = \frac{v - u_S}{f_S} = \frac{340 \text{ m/s} - 34 \text{ m/s}}{400 \text{ Hz}} = 0.765 \text{ m}$$

34. (3) $SL = 10 \log \frac{I}{I_0} = 10 \log \frac{k}{I_0 r^2}$

$= 10 \log K - 10 \log (I_0 r^2)$

$= 10 \log k - 10 \log I_0 - 20 \log r$

$= a - b \log r$

35. (2) Let v_m be the velocity of motorcyclist and v be the velocity of sound.

$$v' = \frac{90}{100} v_0 = \frac{v_0 (v - v_m)}{v}$$

$\Rightarrow \quad 9v = 10v - 10v_m$

$$v_m = \frac{v}{10}$$

$$v_m^2 - 0 = 2as = \frac{v^2}{100}$$

$$\therefore \quad S = \frac{v^2}{200a} = \frac{(330)^2}{200 \times 2.2} = 247.5 \text{ m}$$

36. (2) Velocity of sound is not affected by the change in pressure of air. Velocity of sound at 1°C,

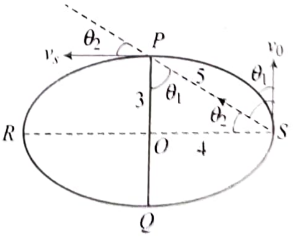
$v_t = (332 + 0.61\, t)$ m/s

At $\quad -5°\text{C}, \ v_{-5°\text{C}} = (332 - 0.61 \times 5)$ m/s

At $\quad 30°\text{C}, \ v_{30°\text{C}} = (332 + 0.61 \times 30)$ m/s

$\therefore \quad v_{30°\text{C}} - v_{-5°\text{C}} = (0.61 \times 35) \text{ m/s } = 21.35$ m/s

37. (2) Frequency received by guard is



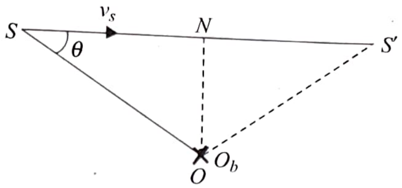
$$n_0 = n_0 \frac{v_0 (v + v_0 \cos\theta_1)}{(v + v_s \cos\theta_2)}$$

$v = 330$ m/s

Here, $v_0 = v_s = v/3$, $\cos\theta_1 = 3/5$, $\cos\theta_2 = 4/5$.

$$\therefore \quad n = n_0 \frac{\left(v+\frac{v}{3}\times\frac{3}{5}\right)}{\left(v+\frac{v}{3}\times\frac{4}{5}\right)} = \left(\frac{6}{5}\times\frac{15}{19}\right)n_0 = \frac{18n_0}{19} = 1800 \text{ Hz}$$

38. (1) This frequency–time curve corresponds to a source moving at an angle to a stationary observer.



In the region *SN*, the source is moving towards the observer, i.e., the apparent frequency

$$n' = n_0\left(\frac{v}{v - v_s\cos\theta}\right)$$

$$n' = n_0\left(\frac{300}{300 - 30\cos\theta}\right)$$

When $\theta = \pi/2$, i.e., at *N*,

$n' = n_0 = 1000$ Hz, i.e., natural frequency of source.

In the region *NS′* the source is moving away from the observer, i.e., apparent frequency

$$n' = n_0\left(\frac{300}{300 - 30\cos\theta}\right)$$

When $\theta = 0$, i.e., $\cos\theta = 1$,

$$n_{\max} = n_0\frac{v}{v - v_s} = \frac{(1000 \text{ Hz})(300 \text{ m/s})}{(300 \text{ m/s} - 30 \text{ m/s})}$$

$$= \frac{10}{9}\times 1000 \text{ Hz} = 1111 \text{ Hz}$$

$$n_{\min} = n_0\frac{v}{v + v_s} = \frac{1000\times 300}{330} = 909 \text{ Hz}$$

39. (4) Time taken is given by

$$T = t_1 + t_2 = \frac{d_1}{v_1} + \frac{d_2}{v_2}$$

$$v_1 = v_{0^\circ C} = 330 \text{ m/s}$$

$$v_2 = (330 + 0.6t) = 342 \text{ m/s}$$

$$d = 1662 \text{ m}$$

$$\therefore \quad T = \frac{d_1}{330} + \frac{(d - d_1)}{342} = 5 \text{ s}$$

$$\frac{d_1(342 - 330)}{330\times 342} + \frac{d}{342} = 5 \text{ s}$$

$$12d_1 = 5(342\times 330) - 330\times 1662$$

$$d_1 = 1320 \text{ m}$$

$$d_2 = 342 \text{ m}$$

40. (3) The frequency that the observer receives directly from the source has frequency $n_1 = 500$ Hz. As the observer and source both move towards the fixed wall with velocity u, the apparent frequency of the reflected wave coming from the wall to the observer will have frequency

$$n_2 = \left(\frac{V}{V - u}\right) 500 \text{ Hz}$$

where V is the velocity of sound wave in air. The apparent frequency of this reflected wave as heard by the observer will then be

$$n_3 = \left(\frac{V+u}{V}\right)n_2 = \left(\frac{V+u}{V}\right)\left(\frac{V}{V-u}\right)500 = \left(\frac{V+u}{V-u}\right)500$$

It is given, that the number of beat per second is $n_3 - n_1 = 10$

$$\therefore \quad (n_3 - n_1) = 10 = \left(\frac{V+u}{V-u}\right)500 - 500$$

$$\Rightarrow \quad 10 = 500\left[\frac{V+u}{V-u} - 1\right]$$

$$\Rightarrow \quad 10 = \frac{2\times u\times 500}{V - u}$$

Hence, $10V = 1000u + 10u = 1010u$

Putting $u = 4$ m/s,

we have $V = \frac{1}{10}[4040] = 404$ m/s

41. (3) Apparent frequency is given by

$$n' = n\frac{(u + v_w)}{(u + v_w - v_s\cos 60^\circ)} = \frac{510(330 + 20)}{330 + 20 - 20\cos 60^\circ}$$

$$= 510\times\frac{350}{340} = 525 \text{ Hz}$$

42. (1) Wavelength of the incident sound is

$$\lambda_i = \frac{10u - \frac{u}{2}}{f} = \frac{19u}{2f}$$

Frequency of the incident sound is

$$F_i = \frac{10u - u}{10u - \frac{u}{2}}f = \frac{18}{19}f = f_r$$

when f_r is the frequency of the reflected sound.

Wavelength of the reflected sound is

$$\lambda_r = \frac{10u + u}{f_r} = \frac{11u}{18f}\times 19 = \frac{11\times 19}{18}\frac{u}{f}$$

$$\therefore \quad \frac{\lambda_i}{\lambda_r} = \frac{19u}{2f}\times\frac{18f}{11\times 19u} = \frac{9}{11}$$

43. (4) $\xi = A\sin(kx - \omega t)$

$$P_{ex} = -B\frac{d\xi}{dx} = -BAk\cos(kx - \omega t)$$

Amplitude of P_{ex} is

$$BAk = (5\times 10^5)(10^{-4})\left(\frac{2\pi}{0.2}\right) = 5\pi\times 10^2 \text{ Pa}$$

44. (3) Let relative velocity be v and the speed of sound be v_0. Then,

$$f_1 = \frac{v_0 - (-v)}{v_0}\times f_0 = \frac{v_0 + v}{v_0}f_0$$

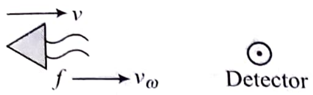
$$f_2 = \frac{v_0}{v_0 - v}\times f_0$$

$$f_3 = \frac{v_0 + v/2}{v_0 - v/2}\times f_0$$

It is clear from above that $f_1 > f_2 > f_3$, $f_3 > f_0$ and we can prove that $f_2 > f_3 > f_1$.

(3) $$\lambda' = \frac{\text{Wave speed relative to listener}}{f}$$

$$\Rightarrow \quad \lambda' = \frac{v + v_\omega}{f} = \frac{v + v_\omega}{v}\lambda$$



Wind blowing

(2) $$450 = 400\left(\frac{340 + v_s}{340 - v_s}\right)$$

$$\Rightarrow \quad \frac{9}{8} = \frac{340 + v_s}{340 - v_s}$$

$$\Rightarrow \quad 9(340) - 9v_s = 8(340) + 8v_s$$

$$\Rightarrow \quad 17v_s = 340$$

$$\Rightarrow \quad v_s = 20 \text{ m/s}$$

7. (2) $$\frac{f_{\text{approach}} - f_{\text{recede}}}{f} = \frac{\Delta f}{f} = \frac{v}{v - v_s} - \frac{v}{v + v_s}$$

$$\therefore \quad \frac{\Delta f}{f} = \frac{v(v + v_s - v + v_s)}{v^2 - v_s^2} = \frac{2vv_s}{v^2 - v_s^2}$$

But $\frac{\Delta f}{f} \times 100 = 2\%$

$$\Rightarrow \quad 0.02 = \frac{2(300)v_s}{(300)^2 - v_s^2}$$

$$\Rightarrow \quad 0.02 = \frac{2(300)v_s}{(300)^2} = \frac{2}{300}v_s$$

$$\therefore \quad v_s = (0.01)\,300 = 3 \text{ m/s}$$

48. (3) $$f' = \left(\frac{c + v_a}{c - v_a}\right)f$$

where c is the velocity of the radio wave, an electro-magnetic wave, i.e., $c = 3 \times 10^8$ m/s and v_s is velocity of aeroplane.

$$f' - f = \left[\frac{c + v_a}{c - v_a} - 1\right]f$$

$$\Rightarrow \quad \Delta f = \frac{2v_a f}{c - v_a}$$

Since approaching aeroplane cannot have a speed comparable to the speed of electromagnetic wave, so $v_s << c$.

$$\therefore \quad \Delta f = \frac{2v_a f}{c}$$

$$\Rightarrow \quad 2.6 \times 10^3 = \frac{2v_A(780 \times 10^6)}{3 \times 10^8}$$

$$\Rightarrow \quad v_A = 0.5 \times 10^3 \text{ m/s} = 0.5 \text{ km/s}$$

49. (4) $$f_1 = \left(\frac{340}{340 - 34}\right)f = \frac{10}{9}f$$

$$f_2 = \left(\frac{340}{340 - 17}\right)f = \frac{20}{19}f$$

$$\therefore \quad \frac{f_1}{f_2} = \frac{\frac{10}{9}}{\frac{20}{19}} = \frac{19}{18}$$

50. (2) $$(f_{\text{approach}})_A = 5.5 \text{ kHz} = \left(\frac{v + v_A}{v}\right)5 \qquad \text{...(i)}$$

$$(f_{\text{approach}})_B = 6 \text{ kHz} = \left(\frac{v + v_B}{v}\right)5 \qquad \text{... (ii)}$$

where v is the velocity of sound. Now,

$$5.5 = \left(1 + \frac{v_A}{v}\right)5$$

$$\Rightarrow \quad \frac{v_A}{v} = 0.1 \qquad \text{... (iii)}$$

Similarly, $6 = \left(1 + \frac{v_B}{v}\right)5$

$$\Rightarrow \quad \frac{v_B}{v} = 0.2 \qquad \text{... (iv)}$$

$$\Rightarrow \quad \frac{v_B}{v_A} = 2$$

51. (3) $$40 = 10\log_{10}\left(\frac{I_1}{I_0}\right)$$

$$\therefore \quad \frac{I_1}{I_0} = 10^4 \qquad \text{... (i)}$$

Also, $20 = 10\log_{10}\left(\frac{I_2}{I_0}\right)$

$$\Rightarrow \quad \frac{I_2}{I_0} = 10^2 \qquad \text{(ii)}$$

$$\therefore \quad \frac{I_2}{I_1} = 10^{-2} = \frac{r_1^2}{r_2^2}$$

$$\therefore \quad r_2 = 100r_1^2 \quad \Rightarrow r_2 = 10 \text{ m} \quad (\because \; r_1 = 1 \text{ m})$$

52. (2) $n_1 = n_2 \quad \Rightarrow \quad \frac{V - V_m}{V - V_C}n' = \frac{V + V_m}{V}n$

$$\Rightarrow \quad \frac{V - V_m}{V - 22} \times 176 = \frac{V + V_m}{V} \times 165$$

$$\Rightarrow \quad v_m = 22 \text{ m/s}$$

53. (2) Time recorded in summer is more accurate. The velocity of sound is directly proportional to the square root of absolute temperature. Hence, the sound of the gun fired at the starting point will reach the finishing point quicker in summer than in winter. The lapse of time due to the time taken by the sound in reaching the finish point will be less in summer and hence the time recorded will be more accurate in summer than in winter.

54. (2) $$v' = \frac{v}{v + v_s}v$$

or $$\frac{6}{7}v = \frac{330}{330 + v}v$$

or $$6 \times 330 + 6v = 7 \times 330$$

or $6v = 330$ or $v = 55$ m/s

55. (2) $v_1 = \sqrt{\frac{Y}{\rho}}$ = velocity of longitudinal waves

$$v_2 = \sqrt{\frac{T}{\rho S}} = \text{velocity of transverse waves}$$

$$v_1 v_2 = \frac{1}{\rho}\sqrt{Y.Y\frac{\Delta l}{l}} = \frac{Y}{\rho}\sqrt{\frac{\Delta l}{l}} \qquad \left(\text{As } \frac{T}{S} = Y\frac{\Delta l}{l}\right)$$

But $\frac{\Delta l}{l} = \frac{l/n}{l} = \frac{1}{n}$

$$v_1 v_2 = \frac{Y}{\rho n^{1/2}}$$

$$v^2 = \frac{Y}{\rho n^{1/2}}$$

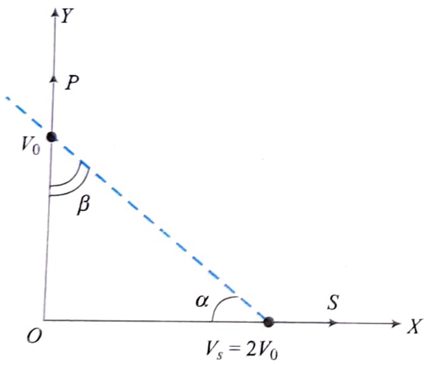
$$v = \sqrt{\frac{Y}{\rho n^{1/2}}}$$

56. (3) OS = 2OP

$$\cos\alpha = \frac{2}{\sqrt5}, \cos\beta = \frac{1}{\sqrt5}$$

$$f' = f\left[\frac{V - V_0\cos\beta}{V + V_s\cos\alpha}\right] = f\left[\frac{V - V_0/\sqrt5}{V + (2V_0)_s \times (2/\sqrt5)}\right]$$

$$= f\left[\frac{V - V_0/\sqrt5}{V + 4V_0/\sqrt5}\right]$$

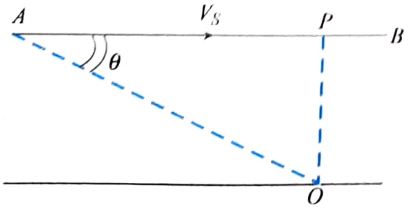


Let $\frac{V_0}{\sqrt5} = x$. Given $\frac{f'}{f} = \frac{3}{4} = \frac{V - x}{V + 4x}$

or $1 - \frac{3}{4} = \frac{5x}{V + 4x}$ or $V = 16x = \frac{16}{\sqrt5}V_0$

57. (2) For AP, $f = f_0\left(\frac{V}{V - V_S\cos\theta}\right)$

For PB, $f = f_0\left(\frac{V}{V + V_s\cos\theta}\right)$



$$f_{\min} = 1800 = f_0\left(\frac{V}{V + V_s}\right) = 2000\left(\frac{V}{V + V_s}\right)$$

$$f_{\max} \doteq 2250 = f_0\left(\frac{V}{V - V_s}\right) = 2000\left(\frac{V}{V - V_s}\right)$$

On solving = 33.3 m/s

58. (2) $f = \left(\frac{V + V_0}{V}\right)f_1 = f_1 + f_1\frac{V_0}{V}$

$$V_0 = gt$$

So, $f = f_1 + \left(\frac{f_1 g}{V}\right)t$

Slope of graph $= \frac{f_1 g}{V}$

$$\frac{(2)(10^{-3}) - f_1}{30} = \frac{(f_1)(10)}{300}$$

or $f_1 = 10^3$ Hz

59. (4) Number of waves encountered by the moving plane per unit time $= \frac{\text{distance travelled}}{\text{wavelength}} = \frac{c + v}{\lambda} = \frac{c}{\lambda}\left(1 + \frac{v}{c}\right) = f\left(1 + \frac{v}{c}\right)$

f' is the frequency of the incident wave and f'' is the frequency of the reflected wave that a stationary observer meets.

$$f'' = \frac{f'}{(1 - v/c)} = \frac{f(1 + v/c)}{(1 - v/c)} = \frac{f(c + v)}{(c - v)}$$

$$\lambda'' = \frac{c}{f''} = \frac{c}{f}\left(\frac{c - v}{c + v}\right)$$

Beat frequency $= f'' = f = \frac{f(1 + v/c)}{(1 - v/c)} - f = \frac{2vf}{(c - v)}$

Multiple Correct Answers Type

1. (2), (3)

$$v = \sqrt{\frac{\gamma RT}{M}}$$

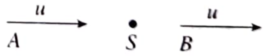
Change in temperature affects the velocity of sound in air but as long as temperature remains same change in pressure has no effect.

$$v = \sqrt{\frac{B}{p}}$$

Bulk modulus of water is very high, so velocity of sound in water is higher than that in air.

2. (1), (3)

Let velocity of each observer be u as shown in the figure.



Then frequency received by A will be $n_1 = n_0\left(\frac{v + u}{v}\right)$

where n_0 is natural frequency of the source and v is sound propagation velocity. The frequency received by B will be

$$n_2 = n_0\left(\frac{v - u}{v}\right)$$

Since $(n_1 + n_2)/2 = n_0$, therefore, option (1) is correct.

3. (2), (3), (4)

When observer P approaches the stationary source at speed v,

$$n_1 = \frac{V + n}{V} \times n_0 \qquad \text{...(i)}$$

(V is speed of sound)

When source S approaches the stationary observer P at speed v,

$$n_2 = \frac{V}{V-v} \times n_0 \qquad \text{...(ii)}$$

Thus, $n_2 > n_1$, i.e., choice (b) is correct when both S and P approach each other with speed $v/2$

$$n_3 = \frac{V+(v/2)}{V-(v/2)} n_0 \qquad \text{...(iii)}$$

Hence, $n_3 > n_0$ and n_3 lies between n_1 and n_2.

4. (2), (3), (4)

$$v \propto \sqrt{T}$$

$$60 \text{ dB} = 10 \log \frac{I_1}{I_0}$$

$$30 \text{ dB} = 10 \log \frac{I_2}{I_0}$$

$$30 = 10 \log \frac{I_1}{I_2} \Rightarrow \frac{I_1}{I_2} \neq 2$$

5. (1), (4)

In both cases (1) and (4) the source and observer are relatively at rest, thus neither of them is approaching or separating from each other. Effectively, it is the medium that moves in each of these cases. The received (apparent) frequency differs from the emitted frequency if and only if the time required for the wave to travel from the source to observer is different for different wavefronts. With a uniform steady motion of the medium, past the observer and source, the transit time from source to observer is the same for all wavefronts. Hence it follows that apparent frequency is equal to the true emitted frequency. Thus there is no Doppler effect. In cases (2) and (3), Doppler effect will be observed as the source and observer have a relative speed and so they will approach or recede from each other.

6. (2), (3), (4)

When the vibrating tuning fork is brought in contact with the table, the vibrations of the tuning fork are being transmitted to the surface of table whose surface area is very large as compared to the surface area of tuning fork and hence sound becomes louder and due to the energy transmitted over the table, the sound dies sooner.

7. (2), (3)

As time increases, the source and detector are relatively approaching each other up to $t = t_0$, where t_0 is the instant when the source and detector are located perpendicular to direction of motion.

$$v_0 \times t_0 = \frac{d \cot \theta_0}{2}$$

$$t_0 = \frac{d \cot \theta_0}{2v_0}$$

For $t < t_0$, $f_{ap} > f_0$

For $t > t_0$, $f_{ap} < f_0$

8. (1), (2), (3)

Pressure amplitude $P = \dfrac{2\pi a E}{\lambda}$

where E is the coefficient of elasticity. We have,

$$E = v^2 \rho \left\{ \because \; v = \sqrt{\frac{E}{\rho}} \right\}$$

$$\therefore \quad P_0 = \frac{2\pi a v^2 \rho}{\lambda} = \frac{2\pi a v^2 \rho}{\left(\dfrac{v}{f}\right)}$$

$$\Rightarrow \quad P_0 = 2\pi a v f \rho = 13.8 \text{ N/m}^2$$

Energy density is $2\pi^2 a^2 f^2 \rho = 6.4 \times 10^{-4}$ J/m^3

Energy flux is $2\pi^2 a^2 f^2 \rho v = 0.22$ J/(m^2/s)

9. (1), (3), (4)

If intensity at a point is I, then energy density at that point is $E = I/v$, where v is wave propagation velocity.

It means that $E \propto I$, Hence, the graph between E and I will be a straight line passing through the origin. Therefore, (1) is correct and (2) is wrong. Intensity is given by

$$I = 2\pi^2 n^2 a^2 \rho v$$

Hence, $E = 2\pi^2 n^2 a^2 \rho$

It means that $E \propto n^2$

Hence, the graph between E and n will be a parabola passing through origin, having increasing slope and symmetric about E-axis. Hence, option (4) is correct.

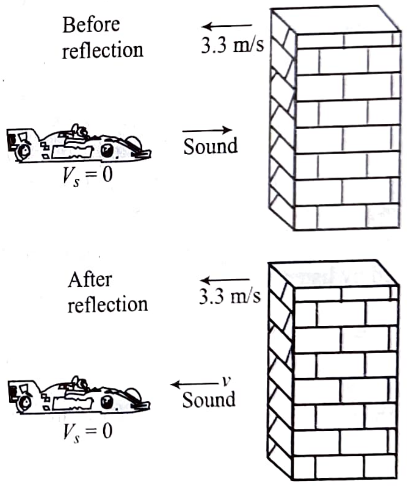
Particle maximum velocity is $u_0 = a\omega = 2\pi n a$

$$\Rightarrow \quad \pi n a = \frac{u_0}{2}$$

Hence, $E = \dfrac{1}{2} \rho u_0^2$

It means that graph between E and u_0 will be a parabola, have increasing slope and will be symmetric about E-axis. Hence, option (3) is also correct.

10. (1), (3)



$$f' = f\left(\frac{v + v_{\text{wall}}}{v - v_{\text{wall}}}\right)$$

$$f' = 1000\left(\frac{340+3.3}{340-3.3}\right) = 1020 \text{ Hz}$$

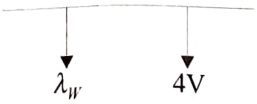
$$\frac{f'-f}{f} \times 100 = \frac{\Delta f}{f} \times 100 = 2\%$$

11. (1), (3), (4)

For observer O_1, $\lambda_4 = \dfrac{V - V_s}{f} = \dfrac{V - V/5}{f} = \dfrac{4V}{5f}$

For O_2, there is change of medium. Hence, at the surface of water, keeping frequency unchanged,

$$\frac{V}{\lambda_a} = \frac{4V}{\lambda_w}$$



$$\Rightarrow \quad \lambda_w = 4\lambda_a = \frac{16V}{5f}$$

$$f'' = \frac{\text{Velocity of wave relative to observer}}{\lambda_w} = \frac{4V + \frac{V}{5}}{\lambda_w}$$

$$= \frac{21V}{5}\frac{5f}{16V} = \frac{21f}{16}$$

Linked Comprehension Type

For Problems 1–3

1. (3) $I = \frac{P}{A}$

so, $I_1 = \frac{P}{A_1}$ and $I_1 = I_2 = \frac{P}{A_1}$

2. (3) The intensity of sound received by listener B is less, so he hears a quieter sound.

3. (3) Power received by A's ear is $I_1 \times A_A = 12 \times 10^{-15}$ W

Power received by B's ear is $= I_1 \times A_A = 12 \times 10^{-15}$ W. So, both hear the same sound.

For Problems 4–7

4. (3) The frequency heard directly from source is given by

$$f_1 = \left(\frac{v}{v - v_S}\right) f$$

Here, $v = 340$ m/s, $v_S = 2$ m/s, $f = 512$.

$$f_1 = \frac{340}{338} \times 512 = 515 \text{ Hz}$$

The frequency of the wave reflected from wall will be same (no relative motion between wall and listener, so no change in frequency). Hence no beats are observed.

5. (2) As the source is moving away from the listener, hence frequency observed by listener is

$$f_1 = \frac{v}{v + v_S} f = \frac{340}{340 + 2} \times 512 = \frac{340}{342} \times 512 = 509 \text{ Hz}$$

The frequency reflected from wall (we can assume an observer at rest) is

$$f_2 = \frac{v}{v - v_S} \times f = \frac{340}{338} \times 512 = 515 \text{ Hz}$$

Therefore, beats heard by observer (L) is 515 – 509 = 6.

6. (4) As no relative motion is there between observer and listener, hence frequency heard by observer is 512 Hz. He will observe frequency reflected from wall, $f_1 = 515$ Hz.

Hence, the wave reflected from wall will act as another source of sound of frequency 515 Hz.

Therefore, the frequency received by the observer from wall is

$$f_2 = \frac{v + v_O}{v} f_1 = \frac{342}{340} \times 515 = 518 \text{ Hz}$$

Hence, beats observed is $f_2 - f = 518 - 512 = 6$.

7. (2) Frequency received from source directly by observer will remain same. Hence frequency received by observer is 512 Hz. Let f_1 be the frequency reflected by wall. Then,

$$f_1 = \frac{v}{v - v_S} \times f = \frac{340}{338} \times 512 = 515 \text{ Hz}$$

The frequency received by observer (reflected from wall) is

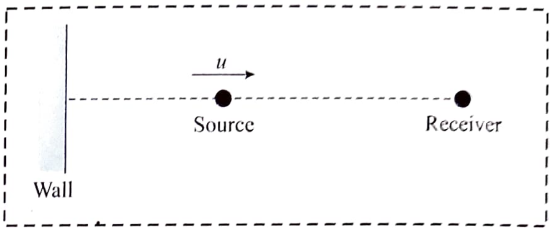
$$f_2 = \left(\frac{v + v_O}{v}\right) f = \frac{342}{340} \times 515 = 518 \text{ Hz}$$

Hence, beats heard is $f_2 - f_1 = 518 - 512 = 6$.

For Problems 8–10

8. (1), 9. (1), 10.(4)

Since source moves away from the wall, it means that its velocity is towards the receiver as shown in the figure. Hence, frequency of direct sound received by it is greater than natural frequency of the source.



Frequency of direct waves, $n_d = n_0\left(\frac{v}{v - u}\right) = 660$ Hz

Frequency of reflected sound is equal to frequency received by the wall. Since source is moving away from the wall, therefore, frequency received by the wall is less than natural frequency of the source which is equal to

$$n_0\left(\frac{v}{v + u}\right)$$

Therefore, the frequency of reflected sound is

$$n_r = n_0\left(\frac{v}{v + u}\right) = 550 \text{ Hz}$$

Hence, the beat frequency recorded by the receiver is

$n_r - n_d = 110$ Hz

Since the receiver is stationary, therefore, velocity of both direct and reflected sound relative to the receiver is equal to u. Hence, wavelength of direct waves is

$$\frac{v}{n_d} = 0.5 \text{ m} = 50 \text{ cm}$$

and wavelength of reflected waves is

$$\frac{v}{n_r} = 0.6 \text{ m} = 60 \text{ cm}$$

For Problems 11–13

11. (3) Since the source is moving on a small circle in a plane perpendicular to the direction of the wave moving towards the observer located on the axis of the circle, there would be no change in the observed frequency which will be the same as the real frequency, i.e., 500 Hz.

12. (2) Since the observer is in the same plane as the circle, at one instant of time, the source will move directly towards the observer when the apparent frequency will be maximum given by

$$f_{\max} = \left(\frac{V}{V - u}\right) 500 = \left(\frac{330}{330 - u}\right) 500$$

Now, $u = r\omega = \left(\frac{100}{\pi}\right) 5 \times 2\pi \times 10^{-2} \text{ m/s} = 10 \text{ m/s}$

$$\therefore \quad f_{\max} = \left(\frac{330}{330 - 10}\right) 500 = \frac{330}{320} \times 500 = 515 \text{ Hz}$$

Similarly, at another instant, the source will move directly away from the observer with velocity $u = 10$ m/s. At this instant, the apparent frequency will be minimum given by

$$f_{\min} = \left(\frac{330}{330+10}\right)500 = \frac{33}{34} \times 500 = 485 \text{ Hz}$$

13. (1) The frequency of the source considered stationary at the centre is $n_0 = 500$ Hz when the observer moves towards the centre with constant speed u, the apparent frequency is

$$n_a = \left(\frac{V+u}{V}\right)500 = \frac{330+20}{330} \times 500 = \frac{35}{33} \times 500$$

Change in frequency is given by

$$n_a - n_0 = \left(\frac{35}{33} \times 500 - 500\right) = \frac{2 \times 500}{33} \text{ Hz}$$

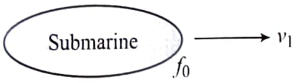
Fractional change of frequency is

$$\left(\frac{n_a - n_0}{n_0}\right) = \left(\frac{2 \times 500}{33}\right) \times \frac{1}{500} = \frac{2}{33} = 0.06$$

Hence, percentage change is 6%.

For Problems 14–17

14. (2)

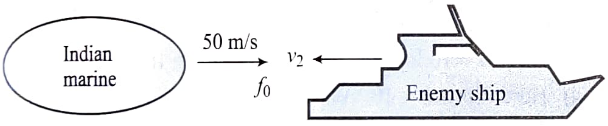


$$f' = f_0\left(\frac{v+v_1}{v-v_1}\right),\ v = 1050$$

$$f' = \frac{110 f_0}{100}$$

Solve to get: $v_1 = 50$ m/s

15. (1)



$$f' = f_0\left(\frac{v+v_2}{v-50}\right),\ f'' = f'\left(\frac{v+50}{v-v_2}\right)$$

$$f'' = f_0\left(\frac{(v+v_2)(v+50)}{(v-v_2)(v-50)}\right) = 1.21 f_0$$

[21% greater than sent waves]

We get $v_2 = 50$ m/s towards Indian submarine.

16. (2) $$\lambda' = \frac{v \text{ w.r.t. observer}}{f'} = \frac{v+v_2}{f_0\dfrac{(v+v_2)}{(v-50)}} = \frac{v-50}{f_0}$$

$$\lambda'' = \frac{v+50}{f_0\dfrac{(v+v_2)(v+50)}{(v-v_2)(v-50)}} = \frac{(v-v_2)(v-50)}{f_0(v+v_2)}$$

$$\therefore \quad \frac{\lambda'}{\lambda''} = \frac{v+v_2}{v-v_2} = \frac{1050+50}{1050-50} = 1.1$$

17. (2) $$v = \sqrt{\frac{B}{\rho}} \Rightarrow 1050 = \sqrt{\frac{B}{1000}}$$

$$\therefore \quad B \approx 10^9 \text{ N/m}^2$$

For Problems 18–19

18. (3) In the propagation of sound waves, let pressure amplitude be Δp_0 and displacement amplitude be A. Then,

$$\Delta p_0 = BAK$$

where symbols have their usual meanings. We have,

$$SL = 10 \log \frac{I}{I_0}$$

$$\Rightarrow \quad 40 = 10 \log \frac{I}{10^{-12}}$$

$$\Rightarrow \quad I = 10^{-8} \text{ W/m}^2$$

$$I = \frac{\Delta p_0^2}{2\rho v}$$

$$\Rightarrow \quad \Delta p_0 = \sqrt{I \times 2\rho v} = \sqrt{10^{-8} \times 2 \times \frac{15}{11} \times 330} \text{ N/m}^2$$

$$= 3 \times 10^{-3} \text{ N/m}^2$$

19. (3) We have, $\Delta p_0 = BAK$

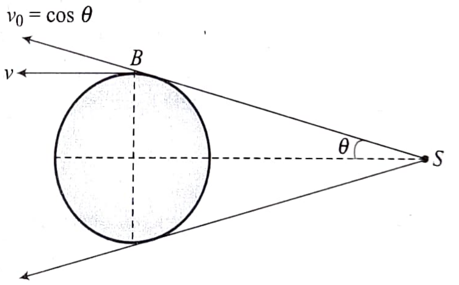
$$\Rightarrow \quad \Delta p_0 = \rho v^2 A \times k \quad (\because \ B = Pv^2)$$

$$\Rightarrow \quad \Delta p_0 = \frac{\rho v^2 A \times 2\pi}{\lambda}$$

$$\therefore \quad \frac{A}{\lambda} = \frac{\Delta p_0}{\rho v^2 \times 2\pi} = 3.22 \times 10^{-9}$$

For Problems 20–22

20. (4) The location of detector at required instant is shown in figure.



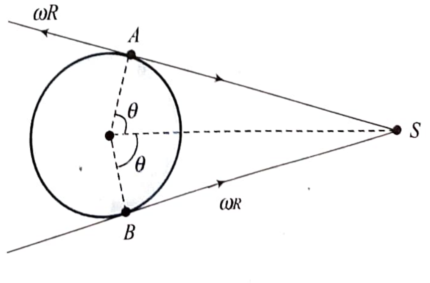
We have, $v_0 = R\omega$ [speed of detector]

$$f_{ap} = \frac{v - v_0 \cos\theta}{v} \times f$$

$$\cos\theta = \frac{2R}{\sqrt{5}R}$$

$$f_{ap} = \frac{v - \omega R \times \dfrac{2}{\sqrt{5}}}{v} \times f$$

21. (2) The instants at which the detector hears maximum and minimum frequencies are shown in Fig. S8.27. A corresponds to minimum frequency while B corresponds to maximum frequency.



Now, $\cos\theta = \frac{R}{2R} = \frac{1}{2}$

$$\Rightarrow \quad \theta = \frac{\pi}{3}$$

Let the detector reaches A in time t_0 (measured from the starting instant). Then

$$\theta = \omega t_0$$

$$\Rightarrow \quad \frac{\pi}{3} = \omega t_0$$

$$\Rightarrow \quad t_0 = \frac{\pi}{3\omega}$$

Let the detector reaches B in time t_{01} (measured from A). Then,

$$2\pi - 2\theta = \omega \times t_{01}$$

$$\Rightarrow \quad t_{01} = \frac{4\pi}{3\omega}$$

The time taken after which the detector hears the maximum frequency for the first time is

$$t_0 + t_{01} = \frac{5\pi}{3\omega}$$

22. (3) The interval between maximum and minimum frequencies as received by the detector for the first time is

$$t_{01} = \frac{4\pi}{3\omega}$$

For Problems 23–25

23. (1) Actual frequency emitted by source does not depend upon the velocity of source but frequency heard may change due to relative motion between the observer and the source.

24. (2) As the source is initially projected with velocity v_0 upwards, the velocity of the source after a time $v_0/2g$ is

$$v = v_0 - \frac{v_0}{2g} \times g = \frac{v_0}{2}$$

And the source is moving away from observer, so, frequency

$$\text{heard} = \left(\frac{v}{v + \frac{v_0}{2}}\right) f_0$$

$$\text{So wavelength of sound} = \frac{v + \frac{v_0}{2}}{f_0}$$

25. (3) As we known, velocity at $t = v_0/2g$ is $v_0/2$.

Number of waves reaching the reflector per second.

$$f_1 = \frac{v + v_0}{v - \frac{v_0}{2}} f_0$$

Now, the wall acts as the source with above frequency and the detector is at rest. So, frequency heard by the detector is

$$f = \frac{v}{v - v_0} f_1 = \frac{v}{v - v_0} \times \frac{v + v_0}{v - \frac{v_0}{2}} f_0 = \frac{v}{v - v_0} \times \frac{v + v_0}{2v - v_0} 2f_0$$

Matrix Match Type

1. i.→a., b.; ii.→a., b.; iii.→c., d.; iv.→c., d.

Power transferred in a string wave is given by

$$P = \mu v A^2\omega^2 \cos^2(\omega t - kx)$$

and $\quad P_{\text{av}} \frac{\mu v A^2\omega^2}{2}$

2. i. →c.; ii.→d.; iii.→a.; iv.→b.

Pressure amplitude is given by

$P_0 = \rho\omega vA_0 = 1.29 \times 2\pi \times 10^3 \times 340 \times (0.01 \times 10^{-3}) = 27.55$ N/m^2

Intensity is given by

$$I = \frac{1}{2}\rho\omega^2A^2v = \frac{1}{2} \times 1.29 \times (2\pi \times 10^3)^2 \times (10^{-5})^2(340) = 0.865 \text{ W/m}^2$$

Power, $P = IA = (0.865)\,\pi\,(0.1)^2 = 0.027$ W $= 2.7 \times 10^{-2}$ W

Intensity at $r = 10$ m is $I = \frac{P_{av}}{A} = \frac{2.7 \times 10^{-2}}{4\pi \times 10^2} = 2.15 \times 10^{-5}$ W/m^2

3. i. → c.; ii. → d.; iii. → a.; iv. → b.

(i) The wavelength of the waves coming towards the observer from source $= (V - V_W + V_S)/f$

(ii)The wavelength of the waves incident on the wall $= (V + V_W - V_S)/f$

(iii) The wavelength of the waves coming towards observer from the wall $= (V - V_W - V_D)/f_r$

(iv) Frequency of the waves (as detected by O) coming from wall after reflection $= (V - V_W - V_{O/V} - V_W - V_D)\,fr$

4. i.→a., b.; ii.→b., c., d.; iii.→a., b.; iv.→c., d.

To solve this question the only concept required is that sound is a mechanical wave and requires some medium for its propagation while light can travel through vacuum also.

Numerical Value Type

1. (1) Intensity is given by $I = \frac{p_0^2}{2\rho v}$.

Here v and ρ are same for both. And also given that I is same for both. So pressure amplitude is also same for both.

2. (6) Loudness due to $S_1 = I_1 = ka^2$ where a is the amplitude and loudness due to S_1 and S_2 both $= I_2 = k(2a)^2 = 4I_1$

$$n = 10\log_{10}(4I_1/I_1) = 10\log_{10}(4) = 10(0.6) = 6$$

3. (4) Here $I_1 = 1.0 \times 10^{-8}$ W/m^2

$r_1 = 5.0$ m, $I_2 = ?$, $r_2 = 25$ m

We know that $I \propto \left(\frac{1}{r^2}\right)$

$$I_1r_1^2 = I_2r_2^2$$

$$I_2 = \frac{I_1r_1^2}{r_2^2} = \frac{1.0 \times 10^{-8} \times 25}{625} = 4.0 \times 10^{-10} \text{ W/m}^2$$

4. (3) We know that $\beta = 10\log_{10}\frac{I}{I_0}$

According to the problems $\beta_A = 10\log_{10}\frac{I}{I_0}$

$$\beta_B = 10\log_{10}\left(\frac{2I}{I_0}\right)$$

$$\beta_B - \beta_A = 10\log\left(\frac{2I}{I}\right) = 10 \times 0.3010 = 3 \text{ dB}$$

5. (3) $f_1 = 900\left(\frac{300}{300 + V_1}\right)$

Or $\quad f_1 = 900\left[1 + \frac{V_1}{300}\right]^{-1} = 900 - 3V_1$

Likewise, $f_2 = 900 - 3V_2$

Given $f_2 - f_1 = 9$

$$3(V_1 - V_2) = 9 \quad \Rightarrow V_1 - V_2 = 3 \text{ m/s}$$

6. (7) Intensity from a point source varies with distance as $I \propto \frac{1}{r^2}$

Let at distance $r_1 = 0.7$ m, intensity is I_1,

Then given $\quad 20 = 10 \log \frac{I_1}{I_0} \qquad \ldots\text{(i)}$

Let for $r = r_2$, sound level be zero. Then intensity at that point should be $I_2 = I_0$.

And $\quad \frac{I_1}{I_2} = \left(\frac{r_2}{r_1}\right)^2 \Rightarrow \frac{I_1}{I_0} = \left(\frac{r_2}{r_1}\right)^2 \qquad \ldots\text{(ii)}$

From Eqs. (i) and (ii), we get

$$20 = 10 \log\left(\frac{r_2}{r_1}\right)^2 \Rightarrow 20 = 20 \log\left(\frac{r_2}{r_1}\right)$$

$$\Rightarrow \quad \frac{r_2}{r_1} = 10 \Rightarrow r_2 = 10\, r_1 = 7 \text{ m}$$

7. (8) The observer will hear a sound of the source moving away from him and another sound after reflection from the wall. The apparent frequencies of these sounds are

$$f_1 = \left(\frac{v}{v+u}\right) f, \; f_2 = \left(\frac{v}{v-u}\right) f$$

Number of beats $= f_2 - f_1$

$$\left(\frac{v}{v-u} - \frac{v}{v+u}\right) f = \frac{2uvf}{v^2 - u^2} \approx \frac{2uf}{v} = 8$$

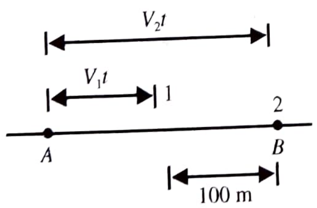
8. (4) Let the velocities of car 1 and car 2 be V_1 and V_2, respectively. Apparent frequencies of sound emitted by car 1 and car 2 as detected at end point are

$$f_1 = f_0 \frac{V}{V - V_1}, \; f_2 = f_0 \frac{V}{V - V_2}$$

$$330 = 300 \frac{330}{330 - V_1}, \; 360 = 300 \frac{330}{330 - V_2}$$

$\therefore \quad V_1 = 30$ m/s and $V_2 = 55$ m/s

The distance between both the cars just when the car 2 reaches end point B (as shown in figure.) is

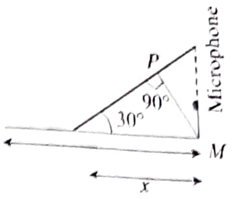


$$100 m = V_2 t - V_1 t \;\Rightarrow\; t = 4 \text{ s}$$

9. (1230)

Since aeroplane is producing intensity level of 100 dB at a distance of 200 m from it, and this is also the maximum allowable sound level so as not to violate the regulations, the minimum distance of the plane from microphone has to be 200 m.

The diagram that follows shows the situation exactly. Let the aeroplane leave the runway, before x metres of the location of microphone. Then, PM is the shortest distance between the microphone and the aeroplane.



Now, $PM = x \sin 30°$

$$\Rightarrow \quad PM = 200 = \frac{x}{2}$$

$$\Rightarrow \quad x = 400 \text{ m}$$

So, the required distance is $1630 - 400 = 1230$ m.

10. (32) The sound level in dB is $L = 10 \log_{10}\left(\frac{I}{I_0}\right)$

If L_1 and L_2 are the sound levels and I_1 and I_2 are the corresponding intensities in the two cases, then

$$L_2 - L_1 = 10\left[\log_{10}\left(\frac{I_2}{I_0}\right) - \log_{10}\left(\frac{I_1}{I_0}\right)\right]$$

$$\Rightarrow \quad 30 = 10 \log_{10}\left(\frac{I_2}{I_1}\right)$$

$$\Rightarrow \quad \frac{I_2}{I_1} = 10^3$$

As the intensity is proportional to the square of the pressure amplitude, thus we have

$$\frac{\Delta p_2}{\Delta p_1} = \sqrt{\frac{I_2}{I_1}} = \sqrt{1000} \approx 32$$

11. (8) Tuning fork A is approaching the listener. Therefore apparent frequency of sound heard by listener is

$$n'_A = \frac{v}{v - v_s} n_A = \frac{330}{330 - 5} \times 256 = 260 \text{ Hz}$$

Tuning fork B is receding away from the listener. Therefore apparent frequency of sound of B heard by listener is

$$n'_B = \frac{v}{v + v_s} n_B = \frac{330}{330 + 5} \times 256 = 252 \text{ Hz}$$

Therefore the number of beats heard by listener per second is

$$n'_A = n'_B = 260 - 252 = 8$$

12. (2) Let S be the stationary car and W the reflecting wall approaching with velocity u. S' is the image formed of S by the reflecting wall. If reflecting surface moves with velocity u, the relative velocity of source and image is $2u$. Therefore, it may be assumed that image S' approaches the stationary driver S with velocity $2u$. Hence the apparent frequency is

$$n' = \frac{v}{v - 2u} n = \frac{v}{v\left(1 - \frac{2u}{v}\right)} n = \left(1 - \frac{2u}{v}\right)^{-1} n$$

If $u << v$, then $n' = \left(1 + \frac{2u}{v}\right) n$

Here $u = 3.3$ m/s, $v = 330$ m/s, $n = 1000$.

$$\therefore \quad n' = \left(1 + \frac{2 \times 3.3}{330}\right) \times 1000 = 1020 \text{ Hz}$$

percentage change in frequency is

$$\frac{n'-n}{n}\times 100\% = \frac{1020-1000}{1000}\times 100\% = 2\%$$

13. (7.7)

Direct sound: $V_s = -5$ m/s because positive direction is from S to O.

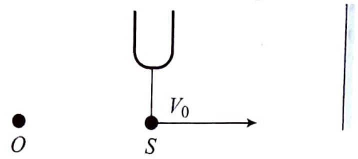
$$f = f_0\left(\frac{c-V_0}{c-V_s}\right) = 256\left(\frac{330-0}{330+5}\right) = 252.2\text{ Hz}$$

(i) Frequency received at wall (f_1) is given by

$(V_s = +5\text{ m/s})$

$$f = \frac{f_0\left(c-V_{\text{wall}}\right)}{c-V_s} = \frac{f_0(c-0)}{c-V_s} = \frac{256\times 330}{330-5}\text{Hz}$$

(ii) Frequency received by observer (f_2) is given by



$$f_2 = \frac{f_1\left(c-V_0\right)}{c-V_{\text{wall}}} = \frac{f_1(c-0)}{c-0} = f_1$$

$$\Rightarrow \quad f_2 = \frac{256\times 330}{325} = 259.9\text{ Hz}$$

Beats per second is $f_2 - f$ = (259.9 – 252.2) Hz = 7.7 Hz.

14. (1.154)

The number of revolutions per second is 2. Radius of the circle is 1.988 m.

The linear velocity of the tuning fork is

$$v = 2\times 2\pi r = 4\times\frac{22}{7}\times 1.988 = 25\text{ m/s}$$

(i) Apparent frequency when the tuning fork is approaching the listener is

$$n_1 = \frac{vn}{v-v_s} = \frac{350\,n}{350-25} = \frac{14}{13}n \text{ [highest note]}$$

(ii) Apparent frequency when the tuning fork is moving away from the listener is

$$n_2 = \frac{vn}{v+v_s} = \frac{350n}{(350+25)} = \frac{14}{15}n \text{ [lowest note]}$$

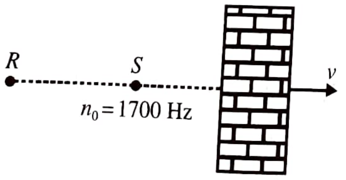
The ratio of highest note to the lowest note is given by

$$\frac{n_1}{n_2} = \frac{14n}{13}\times\frac{15}{14n} = \frac{15}{13} = 1.154$$

15. (0.6)

As both source and receiver are at rest, the frequency of sound waves which directly reach receiver will be 1700 Hz and the frequency of sound which the wall will receive as a moving observer is

$$n_1 = n_0\left[\frac{v}{v+u}\right]$$



The wall now behaves as a moving source of frequency n_1 which when received by receiver, the frequency observed is

$$n_2 = n_1\left[\frac{v-u}{v}\right] = n_0\left(\frac{v-u}{v+u}\right) = 1700\left[\frac{340-0.06}{340+0.06}\right]$$

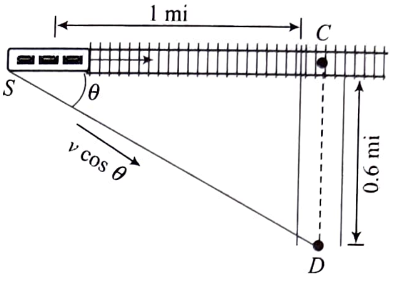
$$= 1700\times\frac{339.94}{340.06} = 1699.4\text{ Hz}$$

Thus beat frequency received by detector is

$$\Delta n = 1700 - 1699.4 = 0.6\text{ Hz.}$$

16. (442)

The situation is shown in figure.



$$SD = \sqrt{1^2+0.6^2} = 1.166\text{ mi}$$

$$\cos\theta = \frac{SC}{SD} = \frac{1}{1.166} = 0.857$$

Thus speed of source along the line of sight is

$$v_S = v\cos\theta = \frac{80}{60\times 60}\times 0.857 = 0.019\text{ mi/s}$$

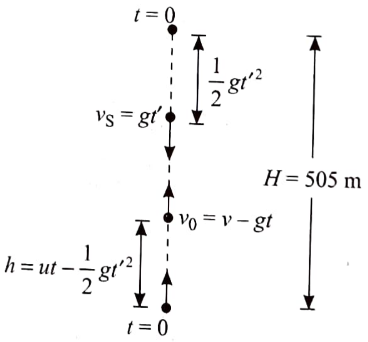
Thus the apparent frequency observed is

$$n = n_0\left[\frac{v_{\text{sound}}}{v_{\text{sound}}-v_S}\right] = 400\left[\frac{0.2}{0.2-0.019}\right] = 442\text{ Hz}$$

17. (1500)

Sound waves takes finite time to travel, it means the sound wave received at $t = 5$ s must be emitted earlier. Let t' be the time at which the source emits a sound wave which is detected at $t = 5$ s. Then, the time for which the sound move is $t - t'$. Distance travelled by the wave is $v\ (t - t')$. Let the distance moved by the detector be h, then

$$h = ut - \frac{1}{2}gt^2$$



and $$H - \frac{1}{2}gt'^2 - h = v(t-t')$$

or $$505 = \frac{1}{2}gt'^2 - \left(ut - \frac{1}{2}gt^2\right) = v(t-t')$$

Substituting numerical values,

$t = 5\text{s}, g = 10\text{ms}^{-2}$ and $v = 300\text{ms}^{-1}$, we have $505 - 5t'^2 - 125 = 300(5 - t')$

or $\quad t'^2 - 60t' + 224 = 0 \Rightarrow t' = 4\text{s}$

Now applying Doppler's formula, $f_{\text{app}} = f_0 \dfrac{v + v_0}{v - v_S}$

where $v_S = gt' = (10)(4) = 40\ \text{ms}^{-1}$

$$v_0 = u - gt = 50 - (10)(5) = 0$$

$$f_{\text{app}} = 1300\left(\frac{300}{300 - 40}\right) = 1500\ \text{Hz}$$

Archives

JEE Main

Single Correct Answer Type

1. (2) For the motor cycle, $u = 0$ and $a = 2\ \text{m/s}^2$

$$\therefore \quad n' = n\frac{v - v_0}{v + v_S}$$

$$\Rightarrow \quad \frac{94}{100}n = n\frac{330 - v_0}{330}$$

$$\Rightarrow \quad 330 - v_0 = \frac{330 \times 94}{100}$$

$$\Rightarrow \quad v_0 = 330 - \frac{94 \times 33}{10} = \frac{33 \times 6}{10}\ \text{m/s}$$

$$\therefore \quad S = \frac{v^2 - u^2}{2a} = \frac{9 \times 33 \times 33}{100} = \frac{9 \times 1098}{100} \approx 98\ \text{m}$$

2. (2) $f_1 = f\left[\dfrac{v}{v - v_s}\right] = f\left[\dfrac{320}{320 - 20}\right] = f \times \dfrac{320}{300}\text{Hz}$

$$f_2 = f\left[\frac{v}{v + v_s}\right] = f \times \frac{320}{340}\ \text{Hz}$$

$$100 \times \left(\frac{f_2}{f_1} - 1\right) = \left(\frac{f_2 - f_1}{f_1}\right) \times 100 = 100\left[\frac{300}{340} - 1\right] = 12\%$$

JEE Advanced

Single Correct Answer Type

1. (2) We know that $C = \sqrt{\left(\dfrac{\gamma RT}{M}\right)}$

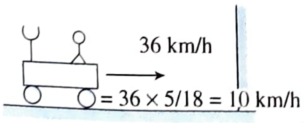
Hence $C \propto \sqrt{\dfrac{1}{m}}$

$$\therefore \quad \frac{C_1}{C_2} = \sqrt{\left(\frac{m_2}{m_1}\right)}$$

2. (1) $f_{\text{incident}} = f_{\text{reflected}} = \dfrac{320}{320 - 10} \times 8\ \text{kHz}$

$$f_{\text{observed}} = \frac{320 + 10}{320} f_{\text{reflected}} = 8 \times \frac{330}{310}$$

$$= 8.51\ \text{kHz} \approx 8.5\ \text{kHz}$$



Multiple Correct Answers Type

1. (1), (2)

If wind blows from source to observer $f_2 = f_1\left(\dfrac{V + w + u}{V + w - u}\right)$

When wind blows from observer towards source

$$f_2 = f_1\left(\frac{V - w + u}{V - w - u}\right)$$

In both cases, $f_2 > f_1$.

2. (1), (3), (4)

$$f_M = \frac{C + V\cos\theta}{C} f_1$$

$$f_N = \frac{C + V\cos\theta}{C} f_2$$

$$\Delta f = f_N - f_M = \frac{C + V\cos\theta}{C}(f_2 - f_1)$$

$$\frac{d(\Delta f)}{dt} = -\frac{V}{C}(f_2 - f_1)\sin\theta\frac{d\theta}{dt}$$

$\therefore \quad \dfrac{d(\Delta f)}{dt}$ is maximum when $\theta = 90°$ [∴ (3) is correct]

$$v_P = \left(1 + \frac{V}{C}\cos\theta\right)\Delta f$$

$$v_Q = \Delta f$$

$$v_R = \left(1 - \frac{V}{C}\cos\theta\right)\Delta f$$

$$\therefore \quad v_P + v_r = 2v_Q$$

Numerical Value Type

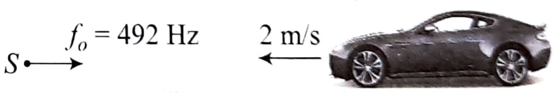
1. (7) $f_{\text{app}} = f_0\dfrac{c + v}{c - v}$, where c is speed of sound

$$\Rightarrow \quad df = \frac{2f_0c}{(c - v)^2}dv \approx \frac{2f_0c}{(c - v)^2}dv = \frac{2f_0}{c}dv \qquad \text{...(i)}$$

Given $df = \dfrac{1.2}{100}f_0$

From Eqs. (i) and (ii) $dv \approx 2$ m/s = 7 km/h.

2. (6)



Frequency received by car $f_c = f_n\left[\dfrac{v - v_0}{v - v_s}\right]$

$$= 492\left[\frac{330 - (-2)}{330}\right] = 492 \times \frac{332}{330} = 495\ \text{Hz}$$

Frequency received by source after reflection with car

$$f_2 = f_c\left[\frac{v - v_0}{v - v_s}\right] = 495\left[\frac{330}{330 - 2}\right] = 498\ \text{Hz}$$

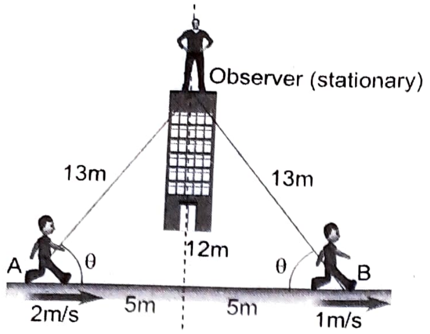
Original frequency f_1 = 492 Hz

Final frequency f_2 = 498 Hz

Beat frequency = $|f_1 - f_2|$ = 6 Hz

3. (5) The frequency of the whistle heard by observer

$$f_A = f\left[\frac{v}{v - v_A\cos\theta}\right]$$



$$\cos\theta = \frac{5}{13}$$

$$f_A = 1430\left[\frac{330}{330 - 2\cos\theta}\right] = 1430\left[\frac{1}{1 - \frac{2\cos\theta}{330}}\right]$$

$$= 1430\left[1 + \frac{2\cos\theta}{330}\right]$$

(By binomial expansion)

Similarly the frequency of the whistle heard by observer

$$f_B = f\left[\frac{v}{v + v_B\cos\theta}\right]$$

$$f_A = 1430\left[\frac{330}{330 + 1\cos\theta}\right] = 1430\left[1 - \frac{\cos\theta}{330}\right]$$

$$\Delta f = f_A - f_B = 1430\left[\frac{3\cos\theta}{330}\right] = 13\cos\theta$$

$$= 13\left(\frac{5}{13}\right) = 5.00 \text{ Hz}$$

1

APPENDIX

Chapterwise Solved January 2019 JEE Main Questions (All Sets)

Chapter 1
Thermometry, Thermal Expansion and Calorimetry

1. Two rods A and B of identical dimensions are at temperature 30°C. If A is heated upto 180°C and B upto T°C, then the new lengths are the same. If the ratio of the coefficients of linear expansion of A and B is 4 : 3, then the value of T is

(1) 270°C (2) 230°C

(3) 250°C (4) 200°C

2. A thermometer graduated according to a linear scale reads a value x_0 when in contact with boiling water, and $x_0/3$ when in contact with ice. What is the temperature of an object in °C, if this thermometer in the contact with the object reads $x_0/2$?

(1) 35 (2) 25

(3) 60 (4) 40

3. A metal ball of mass 0.1 kg is heated upto 500°C and dropped into a vessel of heat capacity 800 JK^{-1} and containing 0.5 kg water. The initial temperature of water and vessel is 30°C. What is the approximate percentage increment in the temperature of the water ? [Specific Heat Capacities of water and metal are, respectively, 4200 $Jkg^{-1}K^{-1}$ and 400 $JKg^{-1}K^{-1}$]

(1) 30% (2) 20%

(3) 25% (4) 15%

4. Ice at –20°C is added to 50 g of water at 40°C. When the temperature of the mixture reaches 0°C, it is found that 20 g of ice is still unmelted. The amount of ice added to the water was close to (Specific heat of water = 4.2 J/g/°C) Specific heat of Ice = 2.1 J/g/°C

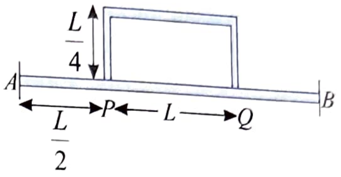
Heat of fusion of water at 0°C = 334 J/g)

(1) 50 g (2) 40 g

(3) 60 g (4) 100 g

Chapter 2
Transmission of Heat

5. Temperature difference of 120°C is maintained between two ends of a uniform rod AB of length $2L$. Another bent rod PQ, of same cross-$3L$ section as AB and length $\frac{3L}{2}$, is connected across AB (see figure). In steady state, temperature difference between P and Q will be close to



(1) 60°C (2) 75°C

(3) 35°C (4) 45°C

6. A heat source at $T = 10^3$ K is connected to another heat reservoir at $T = 10^2$ K by a copper slab which is 1 m thick. Given that the thermal conductivity of copper is 0.1 WK^{-1} m^{-1}, the energy flux through it in the steady state is:

(1) 90 Wm^{-2} (2) 200 Wm^{-2}

(3) 65 Wm^{-2} (4) 120 Wm^{-2}

7. A cylinder of radius R is surrounded by a cylindrical shell of inner radius R and outer radius $2R$. The thermal conductivity of the material of the inner cylinder is K_1 and that of the outer cylinder is K_2. Assuming no loss of heat, the effective thermal conductivity of the system for heat flowing along the length of the cylinder is:

(1) $K_1 + K_2$ (2) $\frac{K_1 + K_2}{2}$

(3) $\frac{2K_1 + 3K_2}{5}$ (4) $\frac{K_1 + 3K_2}{4}$

Chapter 3
Kinetic Theory of Gases

8. A 15 g mass of nitrogen gas is enclosed in a vessel at a temperature 27°C. Amount of heat transferred to the gas, so that rms velocity of molecules is doubled, is about [Take R = 8.3 J/ K mole]

(1) 10 kJ (2) 0.9 kJ

(3) 6 kJ (4) 14 kJ

9. A mixture of 2 moles of helium gas (atomic mass = 4 u), and 1 mole of argon gas (atomic mass = 40 u) is kept at 300 K in a container. The ratio of their rms speeds $\left[\frac{V_{rms}(\text{helium})}{V_{rms}(\text{argon})}\right]$, is close to

(1) 2.24 (2) 0.45

(3) 0.32 (4) 3.16

10. The gas mixture consists of 3 moles of oxygen and 5 moles of argon at temperature T. Considering only translational and rotational modes, the total internal energy of the system is

(1) 12 RT (2) 20 RT

(3) 15 RT (4) 4 RT

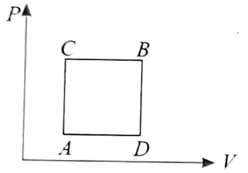
Chapter 4
Thermodynamics

11. Two Carnot engines A and B are operated in series. The first one, A, receives heat at T_1(= 600 K) and rejects to a reservoir at temperature T_2. The second engine B receives heat rejected by the first engine and, in turn, rejects to a heat reservoir at T_3(= 400 K). Calculate the temperature T_2 if the work outputs of the two engines are equal

(1) 400 K (2) 600 K

(3) 500 K (4) 300 K

12. A gas can be taken from A to B via two different processes ACB and ADB.

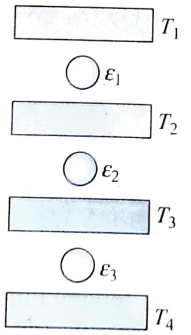


When path ACB is used 60 J of heat flows into the system and 30 J of work is done by the system. If path ADB is used work done by the system is 10 J. The heat Flow into the system in path ADB is

(1) 80 J (2) 20 J

(3) 100 J (4) 40 J

13. Three Carnot engines operate in series between a heat source at a temperature T_1 and a heat sink at temperature T_4 (see figure). There are two other reservoirs at temperature T_2, and T_3, as shown, with $T_1 > T_2 > T_3 > T_4$. The three engines are equally efficient if



(1) $T_2 = (T_1^2 T_4)^{1/3}; T_3 = (T_1 T_4^2)^{1/3}$

(2) $T_2 = (T_1 T_1^2)^{1/3}; T_3 = (T_1^2 T_4)^{1/3}$

(3) $T_2 = (T_1^3 T_4)^{1/4}; T_3 = (T_1 T_4^3)^{1/4}$

(4) $T_2 = (T_1 T_4)^{1/2}; T_3 = (T_1^2 T_4)^{1/3}$

14. In a process, temperature and volume of one mole of an ideal monoatomic gas are varied according to the relation $VT = K$, where K is a constant. In this process the temperature of the gas is increased by ΔT. The amount of heat absorbed by gas is (R is gas constant)

(1) $\frac{R}{2}\Delta T$ (2) $\frac{2K}{3}\Delta T$

(3) $\frac{K}{2}R\Delta T$ (4) $\frac{3}{2}R\Delta T$

5. A rigid diatomic ideal gas undergoes an adiabatic process at room temperature. The relation between temperature and volume of this process is TV^x = constant, then x is

(1) $\frac{5}{3}$ (2) $\frac{2}{5}$

(3) $\frac{2}{3}$ (4) $\frac{3}{5}$

16. A vertical closed cylinder is separated into two parts by a frictionless piston of mass m and of negligible thickness. The piston is free to move along the length of the cylinder. The length of the cylinder above the piston is l_1, and that below the piston is l_2, such that $l_1 > l_2$. Each part of the cylinder contains n moles of an ideal gas at equal temperature T. If the piston is stationary, its mass, m, will be given by (R is universal gas constant and g is the acceleration due to gravity)

(1) $\frac{nRT}{g}\left[\frac{1}{l_1}+\frac{1}{l_2}\right]$ (2) $\frac{nRT}{g}\left[\frac{l_1-l_2}{l_1 l_2}\right]$

(3) $\frac{RT}{g}\left[\frac{2l_1+l_2}{l_1 l_2}\right]$ (4) $\frac{RT}{g}\left[\frac{l_1-3l_2}{l_1 l_2}\right]$

17. An ideal gas is enclosed in a cylinder at pressure of 2 atm and temperature, 300 K. The mean time between two successive collisions is 6×10^{-8} s. If the pressure is doubled and temperature is increased to 500 K, the mean time between two successive collisions will be close to

(1) 4×10^{-8} s (2) 3×10^{-6} s

(3) 2×10^{-7} s (4) 0.5×10^{-8} s

18. An ideal gas occupies a volume of 2m^3 at a pressure of 3×10^6 Pa. The energy of the gas is

(1) 3×10^2 J (2) 10^8 J

(3) 6×10^4 J (4) 9×10^6 J

Chapter 5
Linear and Angular Simple Harmonic Motion

19. A rod of mass 'M' and length '$2L$' is suspended at its middle by a wire. It exhibits torsional oscillations; If two masses each of 'm' are attached at distance '$L/2$' from its centre on both sides, it reduces the oscillation frequency by 20%. The value of ratio m/M is close to

(1) 0.17 (2) 0.37

(3) 0.57 (4) 0.77

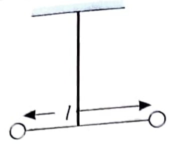
20. A particle is executing simple harmonic motion (SHM) of amplitude A, along the x-axis, about $x = 0$. When its potential Energy (PE) equals kinetic energy (KE), the position of the particle will be

(1) $\frac{A}{2}$ (2) $\frac{A}{2\sqrt{2}}$

(3) $\frac{A}{\sqrt{2}}$ (4) A

21. Two masses m and $\frac{m}{2}$ are connected at the two ends of a massless rigid rod of length l. The rod is suspended by a thin wire of torsional constant k at the centre of mass of the rod-mass system (see figure). Because of torsional constant k, the restoring torque is $\tau = k\theta$ for angular displacement θ.

If the rod is rotated by θ_0 and released, the tension in it when it passes through its mean position will be



(1) $\frac{3k\theta_0^2}{l}$ (2) $\frac{k\theta_0^2}{2l}$

(3) $\frac{2k\theta_0^2}{l}$ (4) $\frac{k\theta_0^2}{l}$

22. A simple pendulum of length 1 m is oscillating with an angular frequency 10 rad/s. The support of the pendulum starts oscillating up and down with a small angular frequency of 1 rad/s and an amplitude of 10^{-2} m. The relative change in the angular frequency of the pendulum is best given by

(1) 10^{-3} rad/s (2) 10^{-1} rad/s

(3) 1 rad/s (4) 10^{-5} rad/s

23. A pendulum is executing simple harmonic motion and its maximum kinetic energy is K_1. If the length of the pendulum is doubled and it performs simple harmonic motion with the same amplitude as in the first case, its maximum kinetic energy is K_2. Then

(1) $K_2 = \frac{K_1}{4}$ (2) $K_2 = \frac{K_1}{2}$

(3) $K_2 = 2K_1$ (4) $K_2 = K_1$

24. A particle undergoing simple harmonic motion has time dependent displacement given by $x(t) = A\sin\frac{\pi t}{90}$. The ratio of kinetic to potential energy of this particle at $t = 210$ s will be

(1) 2 (2) $\frac{1}{9}$

(3) 3 (4) 1

25. A simple harmonic motion is represented by

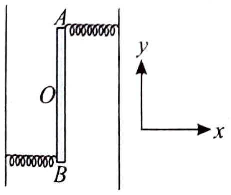
$$y = 5(\sin 3\pi t + \sqrt{3}\cos 3\pi t)\,\text{cm}$$

The amplitude and time period of the motion are

(1) $5\text{cm}, \frac{3}{2}\text{s}$ (2) $5\text{cm}, \frac{2}{3}\text{s}$

(3) $10\text{cm}, \frac{3}{2}\text{s}$ (4) $10\text{cm}, \frac{2}{3}\text{s}$

26. Two light identical springs of spring constant k are attached horizontally at the two ends of a uniform horizontal rod AB of length l and mass m. The rod is pivoted at its centre 'O' and can rotate freely in horizontal plane. The other ends of the two springs are fixed to rigid supports as shown in figure. The rod is gently pushed through a small angle and released. The frequency of resulting oscillation is



(1) $\frac{1}{2\pi}\sqrt{\frac{6k}{m}}$ (2) $\frac{1}{2\pi}\sqrt{\frac{2k}{m}}$

(3) $\frac{1}{2\pi}\sqrt{\frac{k}{m}}$ (4) $\frac{1}{2\pi}\sqrt{\frac{3k}{m}}$

27. A particle executes simple harmonic motion with an amplitude of 5 cm. When the particle is at 4 cm from the mean position, the magnitude of its velocity in SI units is equal to that of its acceleration. Then, its periodic time in seconds is

(1) $\frac{7}{3}\pi$ (2) $\frac{3}{8}\pi$

(3) $\frac{4\pi}{3}$ (4) $\frac{8\pi}{3}$

28. A cylindrical plastic bottle of negligible mass is filled with 310 ml of water and left floating in a pond with still water. If pressed downward slightly and released, it starts performing simple harmonic motion at angular frequency ω. If the radius of the bottle is 2.5 cm then ω close to: (density of water = 10^3 kg / m^3)

(1) 5.00 rad s^{-1} (2) 1.25 rad s^{-1}

(3) 3.75 rad s^{-1} (4) 2.50 rad s^{-1}

Chapter 6
Travelling Waves

29. A heavy ball of mass M is suspended from the ceiling of a car by a light string of mass m ($m << M$). When the car is at rest, the speed of transverse waves in the string is 60 ms^{-1}. When the car has acceleration 'a', the wave-speed increases to 60.5 ms^{-1}. The value of a, in terms of gravitational acceleration g, is closest to

(1) $\frac{g}{5}$ (2) $\frac{g}{20}$

(3) $\frac{g}{10}$ (4) $\frac{g}{30}$

30. An unknown metal of mass 192 g heated to a temperature of 100°C was immersed into a brass calorimeter of mass 128 g containing 240 g of water a temperature of 8.4°C. Calculate the specific heat of the unknown metal if water temperature stabilizes at 21.5°C (Specific heat of brass is 394 J kg^{-1} K^{-1})

(1) 1232 J kg^{-1} K^{-1} (2) 458 J kg^{-1} K^{-1}

(3) 654 J kg^{-1} K^{-1} (4) 916 J kg^{-1} K^{-1}

31. Half mole of an ideal monoatomic gas is heated at constant pressure of 1atm from 20°C to 90°C. Work done by gas i close to: (Gas constant R = 8.31 J /mol.K)

(1) 73 J (2) 291 J

(3) 581 J (4) 146 J

32. A closed organ pipe has a fundamental frequency c 1.5 kHz. The number of overtones that can be distinctl heard by a person with this organ pipe will be (Assume th the highest frequency a person can hear is 20,000 Hz)

(1) 7 (2) 5

(3) 6 (4) 4

33. Equation of travelling wave on a stretched string of linear density 5 g/m is $y = 0.03 \sin(450t - 9x)$ where distance and time are measured is SI units. The tension in the string is

(1) 10 N (2) 12.5 N

(3) 7.5 N (4) 5 N

34. A travelling harmonic wave is represented by the equation $y(x, t) = 10^{-3} \sin(50t + 2x)$, where x and y are in meter and t is in seconds. Which of the following is a correct statement about the wave? The wave is propagating along the

(1) negative x-axis with speed 25 ms^{-1}

(2) The wave is propagating along the positive x-axis with speed 25 ms^{-1}

(3) The wave is propagating along the positive x-axis with speed 100 ms^{-1}

(4) The wave is propagating along the negative x-axis with speed 100 ms^{-1}

Chapter 7
Superposition and Standing Waves

35. A musician using an open flute of length 50 cm produces second harmonic sound waves. A person runs towards the musician from another end of a hall at a speed of 10 km/h. If the wave speed is 330 m/s, the frequency heard by the running person shall be close

(1) 753 Hz (2) 500 Hz

(3) 333 Hz (4) 666 Hz

36. A string of length 1 m and mass 5 g is fixed at both ends. The tension in the string is 8.0 N. The siring is set into vibration using an external vibrator of frequency 100 Hz. The separation between successive nodes on the string is close to

(1) 16.6 cm (2) 20.0 cm

(3) 10.0 cm (4) 33.3 cm

37 A resonance tube is old and has jagged end. It is still used in the laboratory to determine velocity of sound in air. A tuning fork of frequency 512 Hz produces first resonance when the tube is filled with water to a mark 11 cm below a reference mark, near the open end of the tube. The experiment is repeated with another fork of frequency 256 Hz which produces first resonance when water reaches a mark 27 cm below the reference mark. The velocity of sound in air, obtained in the experiment, is close to

(1) 328 ms^{-1} (2) 322 ms^{-1}

(3) 341 ms^{-1} (4) 335 ms^{-1}

Chapter 8
Sound Waves and Doppler Effect

38. A train moves towards a stationary observer with speed 34 m/s. The train sounds a whistle and its frequency registered by the observer is f_1. If the speed of the train is reduced to 17 m/s, the frequency registered is f_2. If speed of sound is 340 m/s, then the ratio f_1/f_2 is

(1) 18/17 (2) 19/18

(3) 20/19 (4) 21/20

39. A person standing on an open ground hears the sound of a jet aeroplane, coming from north at an angle 60° with ground level. But he finds the aeroplane right vertically above his position. If v is the speed of sound, speed of the plane is

(1) $\frac{2v}{\sqrt{3}}$ (2) v

(3) $\frac{v}{2}$ 4) $\frac{\sqrt{3}}{2}v$

Answers Key

1. (2) **2.** (2) **3.** (2) **4.** (2) **5.** (4)
6. (1) **7.** (4) **8.** (1) **9.** (4) **10.** (3)
11. (3) **12.** (4) **13.** (1) **14.** (1) **15.** (2)
16. (2) **17.** (1) **18.** (4) **19.** (2) **20.** (3)
21. (4) **22.** (1) **23.** (3) **24.** (*) **25.** (4)
26. (1) **27.** (4) **28.** (*) **29.** (1) **30.** (4)
31. (2) **32.** (1) **33.** (2) **34.** (1) **35.** (4)
36. (2) **37.** (1) **38.** (2) **39.** (3)

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* No answer is matching.